

# Graph Coloring Based on Degree-Dominance Property

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## Abstract

Graphs are widely used to model complex systems in social, biological, and technological domains. Motivated by the need to incorporate hierarchical influence within such networks, this study introduces *domination coloring*, a new coloring framework based on degree–domination. In a graph  $G = (V, E)$ , each vertex  $v$  has degree  $d(v)$ , and a vertex  $x$  is said to dominate an adjacent vertex  $y$  if  $d(x) > d(y)$ . Under domination coloring, adjacent vertices must receive different colors whenever one dominates the other. This work establishes the theoretical foundations of domination coloring, proves essential structural properties, proposes efficient algorithms, and discusses potential applications in network analysis and decision-making environments.

**Keywords:** Graph, Domination, Coloring, Chromatic number, Degree

## 1. Introduction

Graph coloring is a classical technique in graph theory used to assign colors to vertices so that adjacent vertices receive different colors. Over the years, this method has become a fundamental tool for solving real-world problems such as scheduling, resource allocation, and frequency assignment. The basic idea is to use colors as labels that prevent conflicts between connected or interacting components of a system. As networks have grown more complex, coloring techniques have been extended and refined to capture additional structural features.

One important structural idea is *domination*, where the influence of a vertex is measured by its degree. A vertex with higher degree often plays a more significant role in the network. To address this, we introduce *domination coloring*, a new coloring approach that merges traditional graph coloring with degree-based domination. In a graph  $G = (V, E)$ , a vertex  $x$  dominates an adjacent vertex  $y$  if  $d(x) > d(y)$ . Under domination coloring, such vertex pairs must receive distinct colors. This technique brings an additional layer of structural awareness to the coloring process by incorporating local hierarchy within the network.

Existing literature does not examine how dominance relations influence coloring strategies or what advantages such combined rules may offer. Our work fills this gap by formally defining domination coloring, establishing its theoretical properties, and demonstrating its usefulness in several application areas. Applications include communication network design,

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social influence analysis, distributed resource allocation, and the study of dominance patterns in biological systems.

## 2. Related Work

Domination-based colorings and their variants have been extensively studied over the past two decades due to their usefulness in structural graph analysis, communication networks, and algorithmic applications. Early foundational work includes dominated colorings introduced by Merouane et al. [1], who formalized domination constraints and provided characterizations for several graph families. Henning [2] investigated total dominator colorings, establishing relations between total domination and coloring theory. Arumugam et al. [3] later expanded the theoretical basis for dominator colorings, offering new results on dominator chromatic numbers.

Several studies have explored domination coloring in specialized graph classes. Bagan et al. [4] examined domination colorings in general graphs, while Goddard and Henning [5] analyzed domination and dominator colorings specifically in planar graphs of small diameter. Zhou et al. [6] contributed further depth by investigating computational aspects and proving complexity bounds for domination coloring problems.

Applications of domination, coloring, and influence-based structures are well documented. Guellati and Kheddouci [7] surveyed distributed algorithms for domination and coloring in networks, a topic later extended by Parthasarathy and Gandhi [8] in the context of wireless ad hoc systems. Mishra and Senwar [9] proposed a dominance degree-based approach for efficient nondominated sorting, highlighting the relevance of hierarchical dominance in optimization. Recent work by Shi et al. [10] demonstrated the role of local dominance in uncovering clusters in complex networks.

Domination and coloring ideas have also been connected to probabilistic structures and decision sciences. Muller et al. [11] discussed stochastic dominance, while He and Lin [12] applied dominance metrics to monitor collective attention in social systems.

In parallel, fuzzy, neutrosophic, and competition-based graph models have strengthened applications of coloring and dominance rules. Samanta and Pal's contributions [13] include competition-based fuzzy models. Mahapatra and his colleagues have provided applications of coloring-based models in COVID-19 analysis [14], phishing detection [15], and fuzzy edge coloring [16]. These contributions demonstrate how structural color-based constraints can represent influence, uncertainty,

and interaction intensity in complex systems.

More recently, Maity et al. [17] studied dominating competition graphs in social networks, providing relevant insights into dominance-induced structural patterns. Yero et al. [18] explored domination and coloring in corona graphs, adding to the understanding of domination behavior in graph compositions.

Together, these studies highlight the increasing significance of domination-based coloring frameworks and provide the theoretical foundation that motivates the introduction of degree-dominance coloring as considered in the present work.

## 3. Domination Coloring in Graphs

Domination coloring introduces a hierarchy-aware refinement of classical vertex coloring by incorporating degree influence into adjacency constraints. Unlike standard graph coloring, where only adjacency determines color assignments, domination coloring adds a structural priority rule based on vertex degrees.

**Definition 1.** Let  $G = (V, E)$  be a simple, connected graph. For each vertex  $v \in V$ , let  $d(v)$  denote its degree. A vertex  $x$  is said to *dominate* an adjacent vertex  $y$  if

$$(x, y) \in E(G) \quad \text{and} \quad d(x) > d(y).$$

Let  $c : V \rightarrow \mathbb{N}$  be a vertex coloring function such that  $c(x)$  denotes the color assigned to vertex  $x$ .

A coloring  $c(x)$  is called a *domination coloring* of  $G$  if

$$(x, y) \in E(G), \quad d(x) > d(y) \Rightarrow c(x) \neq c(y).$$

Additionally,  $c(x)$  must satisfy the condition of a proper coloring, meaning

$$(x, y) \in E(G) \Rightarrow c(x) \neq c(y).$$

Thus, domination coloring strengthens the adjacency condition by enforcing degree-informed separation.

The minimum number of colors required to construct a domination coloring of  $G$  is called the *domination chromatic number* and is denoted by

$$\chi_{\text{dom}}(G).$$

### 3.1 Basic Bounds on $\chi_{\text{dom}}(G)$

We begin by establishing a fundamental bound on the domination chromatic number in terms of degree patterns.

**Theorem 1.** For any graph  $G$ , the domination chromatic number satisfies

$$1 \leq \chi_{\text{dom}}(G) \leq d_{\text{max}},$$

where  $d_{\text{max}}$  denotes the number of distinct vertex degrees in  $G$ .

*Proof.* If all vertices have equal degree, then  $d(x) = d(y)$  for all adjacent  $x, y$ ; hence no vertex dominates another and all vertices may receive the same color. Thus  $\chi_{\text{dom}}(G) = 1$ .

For the upper bound, suppose  $G$  contains  $d_{\text{max}}$  distinct degree values. Adjacent vertices with different degrees must receive different colors. Hence, vertices may be partitioned by degree class, and no two adjacent vertices from different classes can share a color. Therefore at most  $d_{\text{max}}$  colors are needed.  $\square$

### 3.2 Domination Coloring of Cycles

**Theorem 2.** Let  $C_n$  be a cycle of length  $n \geq 3$ . Then

$$\chi_{\text{dom}}(C_n) = 1.$$

*Proof.* Every vertex in a cycle has degree 2. Thus  $d(x) = d(y)$  for all adjacent pairs, and no vertex dominates another. Therefore only one color is required.  $\square$

### 3.3 Middle Graphs of Cycles

The middle graph  $M(G)$  introduces degree variation even when  $G$  is regular. For a cycle:

**Theorem 3.** For the middle graph of a cycle,

$$\chi_{\text{dom}}(M(C_n)) = 2.$$

*Proof.* The  $n$  new edge-vertices have degree 4 (each incident to two original vertices and two adjacent edge-vertices), while the original cycle vertices have degree 2. Every edge-vertex dominates its two adjacent original vertices. Thus at least two colors are required: one for the original vertices and another for all edge-vertices. No further distinctions are needed (see Figure 1).  $\square$

### 3.4 Degree-Dominance Partitioning

**Theorem 4.** Let  $G$  have  $d_{\text{max}}$  distinct degree values. Then every domination coloring induces a partition of  $V(G)$  into degree-monotone layers.

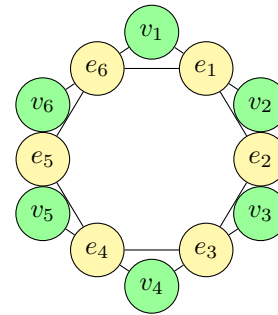


Figure 1. Domination coloring of  $M(C_6)$  with two colors. Edge vertices dominate original vertices.

*Proof.* If  $d(x) > d(y)$  and  $(x, y) \in E(G)$ , domination coloring requires  $c(x) \neq c(y)$ . Thus color classes cannot mix vertices that form dominating adjacency pairs. Vertices of identical degree may be colored together unless adjacency constraints force separation. Hence the layering corresponds exactly to the degree ordering.  $\square$

**Example 1.** Consider a graph with three degree categories: leaves (degree 1), intermediate vertices (degree 2–3), and hubs (degree  $\geq 4$ ). Domination coloring assigns:

- Color 1 for leaves,
- Color 2 for intermediate vertices,
- Color 3 for hub vertices.

This produces  $\chi_{\text{dom}}(G) = 3$ , illustrating the upper bound.

## 4. Algorithms and Computational Complexity

In this section, we analyse the computational aspects of domination coloring. Since domination coloring combines the constraints of both proper coloring and degree-based domination rules, determining  $\chi_{\text{dom}}(G)$  is generally more complex than traditional graph coloring. We present algorithmic strategies, complexity results, and a practical greedy algorithm suitable for real-world applications.

### 4.1 Greedy Domination-Coloring Algorithm

We propose a degree-aware greedy algorithm that assigns colors in descending order of influence (see Algorithm 1). This aligns with real-world scenarios such as social networks, where high-degree nodes represent influential individuals.

### Intuition

- Color vertices in order of decreasing degree.
- High-degree vertices dominate more neighbors, so they must be assigned unique or restrictive colors early.
- Lower-degree vertices can reuse colors more frequently.

**Algorithm 1.** Greedy domination coloring algorithm (degree-based).

**Require:** Simple connected graph  $G = (V, E)$

**Ensure:** A domination coloring  $c : V \rightarrow \mathbb{N}$  and an upper bound for  $\chi_{\text{dom}}(G)$

```

1: Choose an ordering of the vertices in non-increasing degree:
    $v_1, v_2, \dots, v_n$  such that  $d(v_1) \geq d(v_2) \geq \dots \geq d(v_n)$ 
2: Initialize color counter  $k \leftarrow 0$ 
3: for  $i \leftarrow 1$  to  $n$  do
4:   assigned  $\leftarrow$  false
5:   for each color  $c^* \in \{1, 2, \dots, k\}$  do
6:     valid  $\leftarrow$  true
7:     for each neighbor  $u$  of  $v_i$  do
8:       if  $d(u) \neq d(v_i)$  and  $c(u) = c^*$  then  $\triangleright$ 
         domination constraint: different degrees  $\Rightarrow$  different colors
9:         valid  $\leftarrow$  false
10:        break
11:      end if
12:    end for
13:    if valid then
14:      Assign  $c(v_i) \leftarrow c^*$ 
15:      assigned  $\leftarrow$  true
16:      break  $\triangleright$  go to next vertex
17:    end if
18:  end for
19:  if not assigned then
20:     $k \leftarrow k + 1$   $\triangleright$  introduce a new color
21:    Assign  $c(v_i) \leftarrow k$ 
22:  end if
23: end for
24: return Coloring  $c$  and  $k$  (an upper bound on  $\chi_{\text{dom}}(G)$ )
    
```

### 4.2 Correctness Discussion

The greedy algorithm is designed to enforce only the domination constraint.

Recall that a coloring  $c : V \rightarrow \mathbb{N}$  is a domination coloring if for every edge  $(x, y) \in E(G)$  with  $d(x) \neq d(y)$  we have  $c(x) \neq c(y)$ . Edges joining vertices of equal degree  $d(x) = d(y)$  impose no restriction on  $c(x)$  and  $c(y)$ .

Algorithm 1 processes the vertices in non-increasing order of degree. When a vertex  $v_i$  is considered, a candidate color  $c^*$  is accepted for  $v_i$  only if for every neighbor  $u$  of  $v_i$  with

$d(u) \neq d(v_i)$  we have  $c(u) \neq c^*$ . In other words, the algorithm explicitly forbids assigning the same color to adjacent vertices of different degrees.

Consequently, for every edge  $(u, v) \in E(G)$  with  $d(u) \neq d(v)$ , whichever endpoint is colored second will see the already colored neighbor in its neighborhood check and will be prevented from taking the same color. Thus, the final coloring  $c$  produced by the algorithm satisfies

$$d(u) \neq d(v) \Rightarrow c(u) \neq c(v),$$

for all edges  $(u, v) \in E(G)$ , and hence is a valid domination coloring.

The algorithm always terminates, since in the worst case a new color is introduced for the current vertex. Therefore, it produces a well-defined domination coloring and a (possibly non-optimal) upper bound on the domination chromatic number  $\chi_{\text{dom}}(G)$ .

### 4.3 Time Complexity

The greedy domination coloring algorithm consists of two main phases: vertex ordering and color assignment.

**Vertex ordering.** Sorting the vertices in non-increasing order of degree takes

$$O(|V| \log |V|).$$

**Color assignment.** For each vertex  $v_i$ , the algorithm may attempt up to  $k$  colors in the worst case, where  $k$  is the final number of colors used. For each trial color  $c^*$ , it scans the neighbors of  $v_i$  to ensure that no neighbor  $u$  with  $d(u) \neq d(v_i)$  already uses  $c^*$ . This costs  $O(k \cdot \deg(v_i))$  for vertex  $v_i$ .

Summed over all vertices, the total cost of the color assignment phase is

$$\begin{aligned} \sum_{i=1}^{|V|} O(k \cdot \deg(v_i)) &= O\left(k \sum_{i=1}^{|V|} \deg(v_i)\right) \\ &= O(k \cdot 2|E|) = O(k|E|). \end{aligned}$$

Thus, the overall time complexity is

$$O(|V| \log |V| + k|E|).$$

Using the bounds  $k \leq |V|$  and  $|E| \leq \frac{|V|\Delta}{2}$ , where  $\Delta$  is the maximum degree, we obtain the worst-case estimate

$$O(|V| \log |V| + |V|^2 \Delta).$$

In many sparse and moderately sized networks,  $|E|$  and  $k$  are much smaller than their worst-case bounds, so the algorithm remains practically efficient for social, communication, and infrastructure graphs.

#### 4.4 Example Execution on a Social Network Graph

We now illustrate the operation of the greedy domination coloring algorithm on the social network graph shown in Figure 2. Let  $G$  be the example graph with vertices  $A, B, C, D, E, F, G, H$ .

The degrees of the vertices are  $d(F) = d(C) = d(D) = 3$ ,  $d(A) = d(E) = d(G) = d(H) = 2$ ,  $d(B) = 1$ .

Sorting the vertices in non-increasing degree order gives

$$F, C, D, A, E, G, H, B.$$

The algorithm processes the vertices sequentially and assigns the smallest valid color while respecting the domination rule: whenever two adjacent vertices have different degrees, they must receive different colors; if their degrees are equal, no restriction is imposed.

- $F$  receives the first color:  $c(F) = 1$ .
- $C$  and  $D$ , having the same degree as  $F$ , may reuse the same color. Thus the algorithm assigns:  $c(C) = 1$ ,  $c(D) = 1$ .
- $A$  is adjacent to  $C$  and  $H$ , and since  $d(A) = 2 \neq d(C) = 3$ , it cannot use color 1. The algorithm assigns:  $c(A) = 2$ .
- $E$  is adjacent to  $C$  and  $D$ , both of degree 3, hence  $d(E) = 2 \neq 3$ . It must avoid color 1, so it receives:

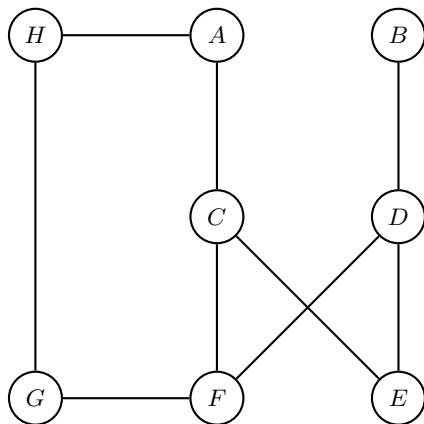


Figure 2. Example social network graph  $G$  illustrating domination coloring behavior under degree-based constraints.

$$c(E) = 2.$$

- $G$  is adjacent to  $F$  but has a different degree ( $2 \neq 3$ ); therefore it may not use color 1. It takes:  $c(G) = 2$ .
- $H$  is adjacent to  $A$  and  $G$ , but  $d(H) = 2$  equals their degree, so it may reuse their color. Thus:  $c(H) = 2$ .
- Finally,  $B$  is adjacent only to  $D$ , and since  $d(B) = 1 \neq 3$ , it cannot use color 1. It therefore takes:  $c(B) = 2$ .

The final coloring is

$$\text{Color 1: } \{F, C, D\}, \quad \text{Color 2: } \{A, E, G, H, B\}.$$

All domination constraints are satisfied, and no adjacent pair of unequal degree vertices shares the same color.

Therefore the algorithm produces a valid domination coloring using only two colors, and we obtain

$$\chi_{\text{dom}}(G) = 2.$$

**Theorem 5.** The greedy domination-coloring algorithm yields

$$\chi_{\text{dom}}(G) \leq |\{\text{distinct degree values in } G\}|$$

and achieves the best possible bound under degree-ordered strategies.

*Proof.* The algorithm assigns a color to a vertex only when a degree conflict occurs with one of its already colored neighbors. Since vertices with the same degree may reuse a color, only transitions between different degree classes force new colors.

In the worst case, each distinct degree class must receive a separate color. No greedy strategy respecting degree order can do better because adjacency between degree classes enforces separation. Thus the stated upper bound is tight for degree-ordered greedy strategies.  $\square$

**Theorem 6.** For any tree  $T$  with at least two vertices,

$$\chi_{\text{dom}}(T) = 2.$$

*Proof.* Trees have only two possible degree types relevant for domination: leaves ( $d = 1$ ) and non-leaf internal vertices ( $d \geq 2$ ). Any leaf is adjacent to a higher-degree vertex, so domination forces color separation between the two degree classes. Internal vertices, regardless of degree variation, may share color.

Thus two colors always suffice, and at least two are needed for any edge connecting a leaf to a non-leaf, proving equality.  $\square$

**Theorem 7.** Let  $G = (X, Y)$  be a bipartite graph. If all vertices in  $X$  share one degree value and all vertices in  $Y$  share another degree value with  $d(X) \neq d(Y)$ , then

$$\chi_{\text{dom}}(G) = 2.$$

*Proof.* All vertices within each part have equal degree and may share a color. Each edge connects vertices of different degree classes, forcing different colors across bipartition. Therefore only two colors are needed.  $\square$

**Corollary 1.** For the complete bipartite graph  $K_{m,n}$ ,

$$\chi_{\text{dom}}(K_{m,n}) = \begin{cases} 1, & m = n, \\ 2, & m \neq n. \end{cases}$$

**Theorem 8.** For a complete multipartite graph  $K_{n_1, n_2, \dots, n_k}$ ,

$$\begin{aligned} \chi_{\text{dom}}(K_{n_1, n_2, \dots, n_k}) \\ = |\{\text{distinct degree values among partitions}\}|. \end{aligned}$$

*Proof.* Vertices in the same part share the same degree and do not restrict color reuse. Vertices in distinct parts are adjacent and have different degrees unless their shares are equal. Only distinct degree groups require separate colors, proving the claim.  $\square$

**Theorem 9.** Every threshold graph  $G$  satisfies:

$$\chi_{\text{dom}}(G) = |\{\text{distinct degree values in } G\}|.$$

*Proof.* Threshold graphs are constructed by repetitive insertion of isolated and universal vertices, creating a strict hierarchy of degree values. Each new degree layer dominates (or is dominated by) all previously created layers, forcing a new color for each distinct degree value.  $\square$

**Theorem 10.** For the middle graph of a path  $P_n$  with  $n \geq 3$ :

$$\chi_{\text{dom}}(M(P_n)) = 3.$$

*Proof.* In  $M(P_n)$  three degree classes appear: original vertices (degree 2), interior edge-vertices (degree 4), and end edge-vertices (degree 3). Since all three types participate in adjacency relationships, domination constraints force three distinct colors.  $\square$

Table 1. Domination chromatic numbers for various graph classes

Graph class	$\chi_{\text{dom}}(G)$
Trees	2 (except $K_1$ which has 1)
Cycles $C_n$	1
Middle graph $M(C_n)$	2
Middle graph $M(P_n)$	3
Bipartite graph $K_{m,n}$	2 unless $m = n$ (then 1)
Regular graphs	1
Grid graphs $P_m \times P_n$	3
Threshold graphs	Distinct degree values
Split graphs	2 or 3
Complete multipartite graphs	Distinct degree values

**Theorem 11.** For the middle graph  $M(C_n)$  of a cycle  $C_n$ ,

$$\chi_{\text{dom}}(M(C_n)) = 2.$$

*Proof.* Vertices of the original cycle have degree 2, while edge-vertices have degree 4. Every adjacency connects a vertex of degree 2 with one of degree 4, forming a bipartition between two degree classes. Hence two colors are necessary and sufficient.  $\square$

**Theorem 12.** Let  $G = P_m \times P_n$  be an  $m \times n$  grid graph with  $m, n \geq 2$ . Then

$$\chi_{\text{dom}}(G) = 3.$$

*Proof.* Grid graphs contain three distinct degree classes: interior vertices (degree 4), boundary vertices excluding corners (degree 3), and corners (degree 2). Edges exist between degree classes in all required patterns, making three colors necessary.  $\square$

## 5. Applications and Conclusion

### 5.1 Social Network Influence Layers

Consider the social interaction network in Figure 3. Vertices represent individuals and edges represent frequent interactions.

From the structure in Figure 3 we can identify three practical types of individuals:

- **Influential core**

The vertex  $v_1$  is directly connected to four other individuals. It is a natural candidate for an influential or broadcast

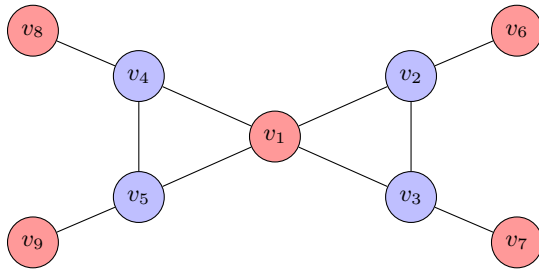


Figure 3. Domination coloring in a social network. Red and blue give a two color domination coloring.

node. We define the core set,

$$C_{\text{core}} = \{v_1\}.$$

• **Relay ring**

Vertices  $v_2, v_3, v_4, v_5$  sit between the core and the outer layer. Each relays information between  $v_1$  and one outer contact. These form the relay set,

$$R = \{v_2, v_3, v_4, v_5\}.$$

• **Peripheral layer**

Vertices  $v_6, v_7, v_8, v_9$  have only one connection each in Figure 3 and lie at the boundary of the network. They form the peripheral set,

$$P = \{v_6, v_7, v_8, v_9\}.$$

The domination coloring in Figure 3 uses two colors:

$$\text{Color 1 (red)} : \{v_1\} \cup P, \quad \text{Color 2 (blue)} : R.$$

This respects the domination rule, since every edge that connects different degree types joins a red and a blue vertex. Edges that join equal degree vertices (for example between  $v_2$  and  $v_3$ ) lie entirely inside the blue relay set and are allowed to share a color.

From a practical point of view:

- $C_{\text{core}}$  is the best set to start an information or awareness campaign.
- $R$  is the best set to monitor for early spread or to reinforce a message that originated at  $v_1$ .
- $P$  is the most vulnerable set to isolation if any relay fails.

The domination coloring therefore supports practical planning by grouping vertices into classes with clear social roles, while still satisfying a strict combinatorial constraint.

**5.2 Wireless and Communication Networks**

Now consider the wireless network shown in Figure 4. Vertices represent devices. Solid edges represent reliable direct communication. Dashed edges represent possible interference between client devices.

**5.2.1 Core devices and client layer**

From the structure of Figure 4 we can identify two practical types of devices.

• **Core communication devices**

Access points  $AP_1$  and  $AP_2$  each connect to many clients and also to each other. They coordinate and route traffic. We define the core set,

$$C_{\text{core}} = \{AP_1, AP_2\}.$$

• **Client devices**

Devices  $u_1, u_2, u_3, u_4, u_5, u_6$  attach to one access point each and may interfere with one neighbor at the client level. They form the client set,

$$U_{\text{client}} = \{u_1, u_2, u_3, u_4, u_5, u_6\}.$$

The domination coloring shown in Figure 4 uses

$$\text{Color 1 (red)} : C_{\text{core}}, \quad \text{Color 2 (blue)} : U_{\text{client}}.$$

Along every edge that joins a core device with a client device, the endpoints have different colors, which is required because core devices have strictly more connections than individual clients. Along dashed interference edges between clients (for

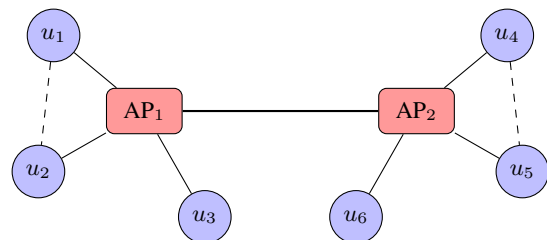


Figure 4. Domination coloring in a wireless network. Access points are red, clients are blue.

example  $u_1 u_2$ ) the devices have equal color and equal structural status, which is allowed by the domination rule.

### 5.2.2 Practical interpretation and simple performance counts

In this wireless system,

- Each access point in  $C_{\text{core}}$  directly serves three clients. Therefore two devices with Color 1 together cover six clients.
- Any failure in a core device would disconnect exactly three clients in this example. The set  $C_{\text{core}}$  therefore represents critical devices for fault tolerance planning.
- Interference relationships among clients do not force new colors, but they identify pairs of clients that require scheduling or channel separation when they are active at the same time.

For engineering practice, this allows the following:

- Assign higher priority in resource allocation and monitoring to  $C_{\text{core}}$ .
- Use the knowledge that  $U_{\text{client}}$  is a single color class to design simple rules for channel reuse among clients, with refinements only for the pairs that are connected by dashed edges.
- Plan redundancy by adding extra paths or backup devices inside  $C_{\text{core}}$  rather than among clients.

### 5.3 Concluding Remarks

In both application settings, the color classes obtained from domination coloring are not arbitrary labels. They correspond to interpretable groups:

- influential or central individuals versus relay and peripheral persons in a social network,
- core routing devices versus dependent clients in a wireless system.

### Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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