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# The Möbius–Kantor graph is a faithful unit-distance graph

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***To Tom Tucker on the happy occasion of his 80th birthday.***

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**Abstract**

In this paper, it has been shown that the generalized Petersen graph $\mathrm{GP}(8,3)$, also known as the Möbius–Kantor graph, admits a faithful unit-distance representation in the plane.

*Keywords: Polycirculant, faithful unit-distance graph, Möbius–Kantor graph, generalized Petersen graph.*

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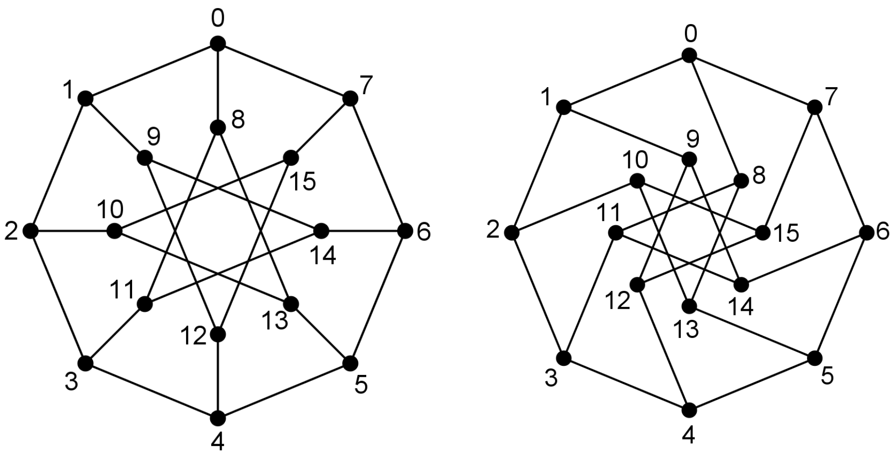
The generalized Petersen graph $\mathrm{GP}(8, 3)$, also known as the Möbius–Kantor graph, appears in many mathematical themes [5]. Its name comes from the fact that it is the Levi graph of the Möbius–Kantor $(8_3)$ configuration [6]. Its automorphism group is of order 96 and is the only group of genus two, as shown by Thomas W. Tucker [8].

It is well known that the Möbius–Kantor graph, as any other generalized Petersen graph, is a unit-distance graph [9]. The drawing depicted in Figure 1(b) is usually taken as a prime example of a unit-distance representation that is not *faithful* or *strong*. In practice, this means that all edges have length 1, however, there exist pairs of non-adjacent vertices that are placed at a distance 1.




(a) (b)

Figure 1: The Möbius–Kantor graph. (a) Standard drawing with vertex labels as used in Table 1. (b) Unit-distance representation that is not faithful. For each vertex there is a non-adjacent vertex at a distance 1 from it; for example, the distance between vertices 0 and 10 is 1.

A formal definition, taken from [1], is as follows. A *faithful unit-distance graph* in $\mathbb{R}^d$ is a graph whose set of vertices is a finite subset of the $d$-dimensional Euclidean space, where two vertices are adjacent if and only if the Euclidean distance between them is exactly 1. A *unit-distance graph* in $\mathbb{R}^d$ is any subgraph of such a graph.

Note that one has to be precise when addressing possible degeneracies [4, 7]. That is, even in the case of faithful unit-distance graphs, a vertex may be mapped to the interior of an interval representing an edge to which it does not belong. In general, there exist unit-distance graphs with a partial overlap of certain represented edges.

Recently, we discovered the representation of $\mathrm{GP}(8, 3)$ that is shown in Figure 2. It can be proven that this representation is a unit-distance representation with a rhombus-shaped outermost 8-cycle; moreover, it can easily be checked that the distances between its non-adjacent vertices are not close to 1. Hence, the representation depicted in Figure 2 is in fact a faithful unit-distance representation. It turns out that there exist infinitely many such representations. The context of this finding will be published elsewhere.

Note that the novel representation has a dihedral symmetry of order 4 and is polycirculant with respect to the group $\mathbb{Z}_2$; compare [2].

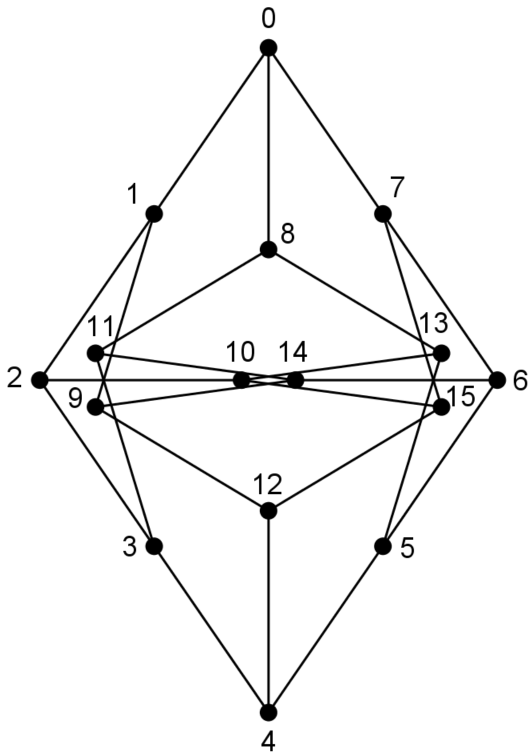



Figure 2: Faithful unit-distance representation of the Möbius–Kantor graph. The vertex labels refer to our Table 1; also cf. Figure 1(a).

If we insist on that the symmetry is dihedral $\mathbb{D}_2$ and require that the outer shape be a rhombus, choose the following coordinates for three vertex representations:

$$\begin{aligned} 0&\colon (0,k), \\ 6&\colon (h,0), \\ 13&\colon (p,q), \end{aligned}$$

which then define all other vertex coordinates. The values $h, k, p$ and $q$ are determined by the following system of quadratic equations, each of them obtained by using Pythagoras' theorem:

$$\begin{aligned} h^2 + k^2 &= 2^2, \\ p^2 + (q-k+1)^2 &= 1, \\ q^2 + (p+h-1)^2 &= 1, \\ (p-h/2)^2 + (q+k/2)^2 &= 1. \end{aligned} \tag{1}$$

The full set of coordinates is given in Table 1.

| Vertex | Coordinates |
|---|---|
| 0 | $(0,k)$ |
| 1 | $(-h/2,k/2)$ |
| 2 | $(-h,0)$ |
| 3 | $(-h/2,-k/2)$ |
| 4 | $(0,-k)$ |
| 5 | $(h/2,-k/2)$ |
| 6 | $(h,0)$ |
| 7 | $(h/2,k/2)$ |
| 8 | $(0,k-1)$ |
| 9 | $(-p,-q)$ |
| 10 | $(1-h,0)$ |
| 11 | $(-p,q)$ |
| 12 | $(0,1-k)$ |
| 13 | $(p,q)$ |
| 14 | $(h-1,0)$ |
| 15 | $(p,-q)$ |

Table 1: The coordinates of the 16 vertices of the graph $\mathrm{GP}(8,3)$, where $h, k, p$ and $q$ satisfy the system of equations (1).

The system has two non-degenerate real solutions. One solution is:

$$\begin{aligned} h &\approx 1.133693, \\ k &\approx 1.647647, \\ p &\approx 0.857420, \\ q &\approx 0.133029, \end{aligned}$$

with the corresponding drawing presented in Figure 2. The other is just a reflection in the line $y = x$.

As we mentioned above, it is easy to verify that all edges of $\mathrm{GP}(8, 3)$ are indeed of length $1$ and that no non-adjacent vertices are at a distance $1$ from each other.

We conclude our paper by briefly mentioning a novel consequence of our result in the context of configurations, which follows from the construction invented in [3].

It is well known that the Möbius–Kantor configuration cannot be geometrically realized with points and (straight) lines in the real Euclidean plane [6]. However, a faithful unit-distance representation of the Möbius–Kantor graph provides the possibility to realize it with points and circles, in the following way. Since the graph is bipartite and unit-distance, any of the bipartition classes can be taken as the set of centres of unit circles, while the other class serves as the set of configuration points. Furthermore, since the graph is faithful, this guarantees that no false incidences occur in the configuration. By exchanging the role of the two bipartition classes one obtains a second configuration such that the two configurations are dual to each other (see Figure 3). As the radius for all the circles is of unit length, we call such configurations *isometric* point-circle configurations [3].

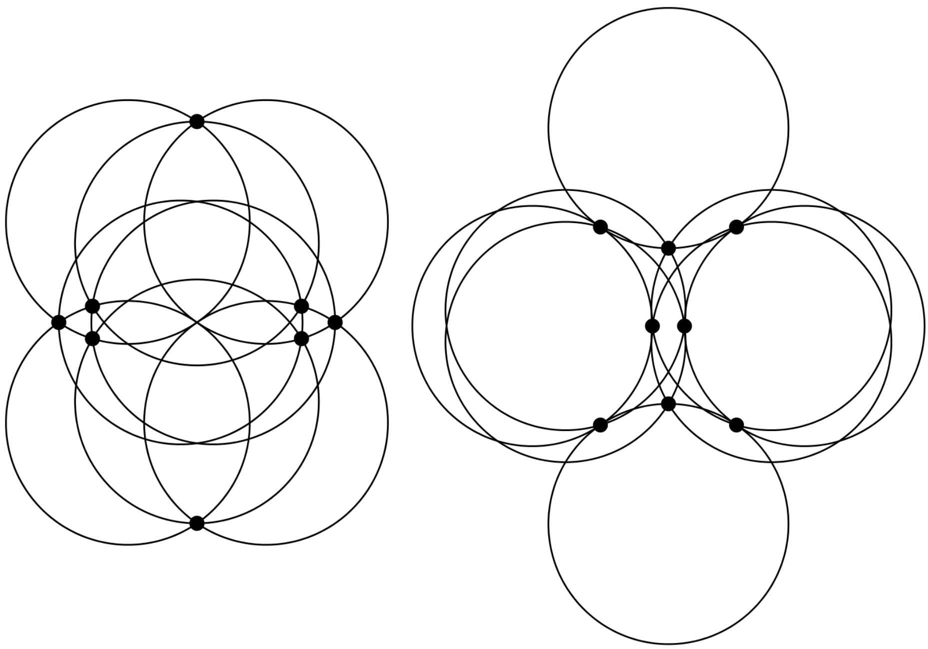


Figure 3: Isometric point-circle realizations of the Möbius–Kantor configuration derived from the faithful unit-distance representation of the Möbius–Kantor graph in Figure 2. They are dual to each other.

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