

Search for a Dark Higgs Boson Produced in Association with Inelastic Dark Matter at the Belle II Experiment

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Inelastic dark matter models that have two dark matter particles and a massive dark photon can reproduce the observed relic dark matter density without violating cosmological limits. The mass splitting between the two dark matter particles χ_1 and χ_2 , with $m(\chi_2) > m(\chi_1)$, is induced by a dark Higgs field and a corresponding dark Higgs boson h' . We present a search for dark matter in events with two vertices, at least one of which must be displaced from the interaction region, and missing energy. Using a 365 fb^{-1} data sample collected at Belle II, which operates at the SuperKEKB e^+e^- collider, we observe no evidence for a signal. We set upper limits on the product of the production cross section $\sigma(e^+e^- \rightarrow h'\chi_1\chi_2)$, and the product of branching fractions $\mathcal{B}(\chi_2 \rightarrow \chi_1 e^+e^-) \times \mathcal{B}(h' \rightarrow x^+x^-)$, where x^+x^- indicates $\mu^+\mu^-$, $\pi^+\pi^-$, or K^+K^- , as functions of h' mass and lifetime at the level of 10^{-1} fb . We set model-dependent upper limits on the dark Higgs mixing angle at the level of 10^{-5} and on the dark photon kinetic mixing parameter at the level of 10^{-3} . This is the first search for dark Higgs bosons in association with inelastic dark matter.

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Despite clear observations of the gravitational effects of dark matter (DM) [1,2], the mass of dark matter particles is unknown, and they have not yet been shown to interact with standard model (SM) particles. Direct detection experiments are sensitive to elastically scattering DM particles with masses typically in the GeV to TeV range [3–6]. At colliders, searches for particles mediating interactions with dark sector particles are sensitive to mediator masses in the MeV to TeV range [7,8]. These searches cover both promptly decaying and long-lived mediators decaying after a macroscopically large distance. A nonminimal class of models introduces inelastic DM (iDM), where DM couples inelastically to SM states, depending on the mass difference $\Delta m = m(\chi_2) - m(\chi_1)$ between the two DM mass eigenstates χ_1 and χ_2 [9]. The simplest iDM models introduce the inelastic coupling via a massive dark photon A' that couples off diagonally to the two DM states. The A' kinetically mixes with SM photons via a mixing parameter ϵ [10–12] and decays predominantly via $A' \rightarrow \chi_1\chi_2$. A small Δm or small coupling to the A' makes the heavier state χ_2 long lived before it decays into a pair of SM particles and the lighter state χ_1 . The relic DM candidate χ_1 , which

contributes to the DM abundance observed today, is stable and escapes detection [10]. These models can be extended to explain the mass splitting Δm and the A' mass by introducing an additional dark Higgs boson h' [13]. The h' would mix with the SM Higgs boson through a mixing angle θ [14,15]. In total, the model has seven free parameters: the masses $m(h')$, $m(A')$, $m(\chi_1)$, and the mass splitting Δm ; the mixing angle θ ; the kinetic mixing parameter ϵ ; and the coupling $g_D = \sqrt{4\pi\alpha_D}$ between DM and the A' . The coupling $k \approx g_D\Delta m/m(A')$ between DM and the h' is fixed by the other parameters [13]. We restrict the search to parameter combinations that correspond to the perturbative regime and that evade existing constraints from observations of the cosmic microwave background by Planck [1] by requiring $m(h') < m(\chi_1) < m(A')$. In this scenario, the DM relic density would be predominantly determined by the process $\chi_1\chi_1 \rightarrow h'h'$. The h' lifetime increases for decreasing values of θ , making the h' long-lived at small θ .

This model is already constrained by searches for h' or A' mediators without specific assumptions about an iDM model. Searches for scalars exclude $\sin\theta$ larger than 10^{-3} to 10^{-4} for h' masses up to about $5 \text{ GeV}/c^2$ (see Ref. [16] for a review) for $m(h') < 2m(\chi_1)$, while for higher h' masses the limits are considerably weaker. For the direct production of an A' through kinetic mixing with a photon and subsequent decay into iDM, the CMS experiment excludes $y = \epsilon^2\alpha_D(m(\chi_1)/m(A'))^4$ larger than 10^{-7} to 10^{-8} for $m(\chi_1) \gtrsim 3 \text{ GeV}/c^2$ [17]. Reinterpretations

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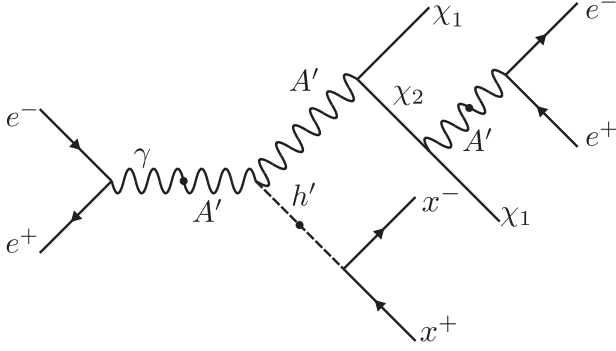


FIG. 1. Feynman diagram depicting the search channel for A' production in association with a h' with subsequent decays into both visible and dark sector states. Here x^+x^- indicates $\mu^+\mu^-$, $\pi^+\pi^-$, or K^+K^- . Mixing between dark sector and visible states is indicated by black dots.

[12,18,19] of searches for invisible decays of A' at BABAR [20], and of searches for long-lived A' decays at NuCal [21,22], CHARM [23], and NA64 [24] exclude y larger than about 10^{-9} for $m(\chi_1) \lesssim 3 \text{ GeV}/c^2$ and y larger than 10^{-12} below $1 \text{ GeV}/c^2$.

We present the first search for a dark Higgs boson in association with iDM. We use events with up to two displaced vertices and missing energy, produced in e^+e^- collisions via $e^+e^- \rightarrow h'(\rightarrow x^+x^-)A'[\rightarrow \chi_1\chi_2(\rightarrow \chi_1e^+e^-)]$, where x^+x^- indicates $\mu^+\mu^-$, $\pi^+\pi^-$, or K^+K^- . The corresponding Feynman diagram is shown in Fig. 1. We search for the signal as a narrow enhancement in the $m(x^+x^-)$ distribution. We present our results as model-independent limits on the product of the production cross section $\sigma_{\text{prod}} = \sigma(e^+e^- \rightarrow h'\chi_1\chi_2)$ and the branching fractions $\mathcal{B}(h' \rightarrow x^+x^-) \times \mathcal{B}(\chi_2 \rightarrow \chi_1e^+e^-)$. In addition to the model-independent search, we interpret our results as a model-dependent limit on the mixing angle θ as a function of the h' mass, and as a limits on y as a function of the χ_1 mass.

We use a 365 fb^{-1} data sample [25] collected at a center-of-mass (c.m.) energy of $\sqrt{s} = 10.58 \text{ GeV}$ by the Belle II experiment [26] at the SuperKEKB e^+e^- collider [27]. The beam energies are 7 GeV for e^- and 4 GeV for e^+ , resulting in a boost $\beta\gamma = 0.28$ of the c.m. frame relative to the laboratory frame.

The Belle II detector consists of a variety of subdetectors surrounding the interaction point (IP) in a cylindrical manner. The trajectories of charged particles (tracks) are reconstructed by a combination of a two-layer silicon-pixel detector, a four-layer silicon-strip detector, and a central drift chamber (CDC). The tracking detectors are surrounded by time-of-propagation and aerogel ring-imaging Cherenkov detectors used for particle identification (PID). The PID detectors cover an angular region of $14^\circ < \theta_{\text{polar}} < 124^\circ$. Photons are reconstructed by an electromagnetic calorimeter (ECL) that also serves in the identification of electrons covering $12^\circ < \theta_{\text{polar}} < 155^\circ$. The ECL is surrounded by a 1.5 T superconducting solenoid. The

outermost subdetector is a K_L^0 and muon detector (KLM) which is installed in the iron flux return of the solenoid. The longitudinal direction, the transverse plane, and the polar angle θ_{polar} are defined with respect to the detector's solenoidal axis in the direction of the electron beam. In the following, quantities are defined in the laboratory frame unless specified otherwise.

We use simulated events to optimize the event selection, and to determine efficiencies and signal resolutions. Signal events are generated using a combination of MadGraph5@NLO [28] and EvtGen [29] taking into account effects of initial state radiation (ISR) [30]. Furthermore, we consider effects from electromagnetic final state radiation in the decay of the h' using PHOTOS [31,32]. To correct for efficiency differences caused by different beam background conditions, we generate signal simulations for a variety of different data-taking conditions using beam-induced backgrounds sampled from data overlaid with simulated signal events and find an approximately linear correlation between background level and signal efficiency. We use the efficiency obtained from a linear fit at the luminosity-weighted average beam background level of our dataset. Motivated by Ref. [13] and a previous search for a similar model [17], we consider values of $m(A') = 3m(\chi_1)$ and $4m(\chi_1)$, $\Delta m = 0.2m(\chi_1), 0.4m(\chi_1)$ and $1.0m(\chi_1)$, and $\alpha_D = 0.1$ and 0.5 . For all possible combinations of these values, we generate events for h' masses between $0.2 \text{ GeV}/c^2$ and $3.0 \text{ GeV}/c^2$ in about 45 steps of varying size and various lifetimes $0.1 < c\tau(h') < 10000 \text{ cm}$ in steps that are approximately equidistant on a logarithmic scale; we generate events for $m(\chi_1)$ between $0.2 \text{ GeV}/c^2$ and $3.0 \text{ GeV}/c^2$ in 30 steps of $0.1 \text{ GeV}/c^2$, and various lifetimes $0.01 < c\tau(\chi_2) < 1000 \text{ cm}$ in variable steps. Since $m(A') > m(\chi_1) + m(\chi_2)$, the A' in the χ_2 decay is always off shell, while the A' produced in association with the h' can be either on shell or off shell with A' masses up to $12 \text{ GeV}/c^2$. The lifetime of the A' is negligible for all values of ϵ probed in this analysis.

We simulate the following background processes: $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ with EvtGen; $e^+e^- \rightarrow q\bar{q}(\gamma)$, where $q = u, d, s, c$ with KKMC [33] interfaced with Pythia8 [34] and EvtGen; $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ and $e^+e^- \rightarrow \tau^+\tau^-(\gamma)$ with KKMC; $e^+e^- \rightarrow e^+e^-e^+e^-$, $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$, $e^+e^- \rightarrow \mu^+\mu^-\mu^+\mu^-$, $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$, and $e^+e^- \rightarrow \mu^+\mu^-\tau^+\tau^-$ with AAFH [35]; $e^+e^- \rightarrow \tau^+\tau^-\tau^+\tau^-$ with KoralW [36]; $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$, $e^+e^- \rightarrow e^+e^-K^+K^-$, and $e^+e^- \rightarrow e^+e^-p\bar{p}$ with TREPS [37]; $e^+e^- \rightarrow e^+e^-(\gamma)$ and $e^+e^- \rightarrow \gamma\gamma(\gamma)$ with Babayaga.NLO [38]; $e^+e^- \rightarrow K_S^0 K_L^0 \gamma$, $e^+e^- \rightarrow \pi^+\pi^-\gamma$, $e^+e^- \rightarrow K^+K^-\gamma$, and $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma$ with PHOKHARA [39]. Decays of τ leptons are simulated with TAUOLA [40] for KKMC, and using Pythia8 for all other event generators. The detector geometry and interactions of final-state particles with detector material are simulated using Geant4 [41]. Both experimental and simulated events are reconstructed and analyzed using the Belle II software

[42,43]. To avoid experimenter's bias, we examine the experimental data only after finalizing the analysis selection. All selection criteria are chosen by iteratively optimizing the figure of merit for a discovery with a significance of five standard deviations [44]. To avoid additional complexity of the analysis we chose a single set of selections for all model parameter combinations.

We use events selected by a calorimeter-only trigger, which requires the sum of energy depositions in the polar angle region $22^\circ < \theta_{\text{polar}} < 128^\circ$ of the ECL to be above 1 GeV. We require that the total deposited energy in this angular region exceeds 1.5 GeV, which ensures nearly 100% trigger efficiency for events that pass this selection, to avoid systematic uncertainties introduced by the trigger requirement.

We reconstruct h' and χ_2 candidates by combining pairs of oppositely charged particles reconstructed from tracks. We require each track to have at least 20 tracking detector hits. The track pairs are separately constrained by fits to originate from common vertices. The opening angle between the two tracks must be greater than 0.1 rad to suppress background from photon conversions, and the h' pointing angle $\Delta\alpha_{\vec{x},\vec{p}}$ between the h' momentum $\vec{p}$ and vertex position vectors $\vec{x}$ must have $\Delta\alpha_{\vec{x},\vec{p}} < 3.1^\circ$, so the h' vertex points back to the IP. To reduce promptly decaying SM backgrounds, at least one of the two vertices must have a transverse displacement with respect to the IP d_v of at least 0.2 cm. To reject contributions from track-pairs produced in hadronic interactions in the ECL, both vertices must have $d_v < 110$ cm.

PID information from all relevant subdetectors is combined to classify final states [45]. At least one track from the h' decay must have an extrapolated polar angle $37^\circ < \theta_{\text{polar}}^{\text{ext}} < 122^\circ$, calculated by extrapolating the track helix to the KLM inner surface to ensure high muon identification efficiency. To further reduce the backgrounds in the final state with $h' \rightarrow \pi^+\pi^-$ decays, all four tracks must be in the range $18^\circ < \theta_{\text{polar}}^{\text{ext}} < 155^\circ$. To ensure high electron PID purity, we require the ratio between the energy deposition in the calorimeter and the momentum of the corresponding track to be larger than 0.6.

We reject events with h' candidates with $0.467 < M(\pi^+\pi^-) < 0.529$ GeV/ c^2 to reduce background from K_S^0 decays; we remove events in the $h' \rightarrow \pi^+\pi^-$ final state with $1.06 < M(p\pi^-) < 1.15$ GeV/ c^2 to reduce background from Λ -baryon decays; we also remove events in the $h' \rightarrow K^+K^-$ final state with $0.977 < M(K^+K^-) < 1.061$ GeV/ c^2 to reduce background from ϕ decays produced in $e^+e^- \rightarrow \phi(\rightarrow K^+K^-)\gamma(\rightarrow e^+e^-)$. The total missing energy in the c.m. frame, calculated from the momenta of the four charged particles and the known initial e^+e^- kinematics, must be greater than 0.4 GeV which is twice the minimal χ_1 mass we consider. The missing momentum direction must be separated from any energy deposition in the KLM detector by at least 0.5 rad to

reject neutral hadron backgrounds. To reduce backgrounds from nonreconstructed particles, the missing momentum direction must point into a more restrictive tracking region ($23^\circ < \theta_{\text{polar}}^{\text{miss}} < 149^\circ$) to avoid the CDC edges where data-simulation agreement is less reliable. The reconstructed electron pair mass from the $\chi_2 \rightarrow \chi_1 e^+ e^-$ decay must be less than 2.5 GeV/ c^2 , which corresponds to the maximal Δm for which we provide model-dependent interpretations. We require that no other tracks are reconstructed, and that the total deposited energy in the calorimeter not matched to tracks satisfies $E_{\text{extra}} < 1.0$ GeV. We require that the opening angle of the tracks from the h' vertex be less than 3.0 rad to suppress background from cosmic muons crossing the detector that are incorrectly reconstructed as two back-to-back tracks.

If multiple signal candidates in the same event pass the selections, which occurs in less than 3% of events, we choose the candidate with the smallest h' pointing angle.

The overall signal selection efficiency is typically a few percent up to 20%. It is generally higher for large Δm and small displacements of the h' .

We determine the signal mass resolution by fitting a double-sided Crystal Ball (DSCB) function [46,47] to simulated $m(h')$ distributions. The resolution $\sigma_{\text{sig}}^{\text{DSCB}}$ increases smoothly from about 1 MeV/ c^2 for a light h' to about 7 MeV/ c^2 for a heavy h' and depends only slightly on the h' lifetime or final state. Mass hypotheses that lack a simulation sample are interpolated from adjacent simulated samples.

We extract the signal yield by counting events in narrow windows of $M(x^+x^-)$ with a width of $\pm 2\sigma_{\text{sig}}^{\text{DSCB}}$ in steps of $\sigma_{\text{sig}}^{\text{DSCB}}/2$. Based on studies with simulated samples, we assume a uniform background as a reasonable approximation given the size of our sample. We determine the background level from data by counting all events in $M(x^+x^-)$ sidebands (SBs). In the K^+K^- and $\mu^+\mu^-$ final state the SB is the full mass range excluding the respective signal window; for the $\pi^+\pi^-$ final state we split the mass region at 1 GeV/ c^2 and determine different uniform background levels below and above this value.

We evaluate systematic uncertainties affecting selection efficiency, integrated luminosity, the limited number of simulated events, and the background model. The dominant systematic uncertainties are associated with the signal efficiency, and depend on combinations of the h' and χ_2 masses and lifetimes. Relative uncertainties are typically around 4% for most parameter configurations, but can reach 10% for the lightest h' masses and large displacements. For large displacements, the dominant systematic uncertainty on the signal efficiency arises from data-simulation differences in track finding for displaced tracks. We correct for this with an auxiliary measurement using K_S^0 decays from the process $D^{*+} \rightarrow D^0(\rightarrow K_S^0\pi^+\pi^-)\pi^+$, and assign an uncertainty obtained by varying the nominal correction within the total uncertainty on the correction. For

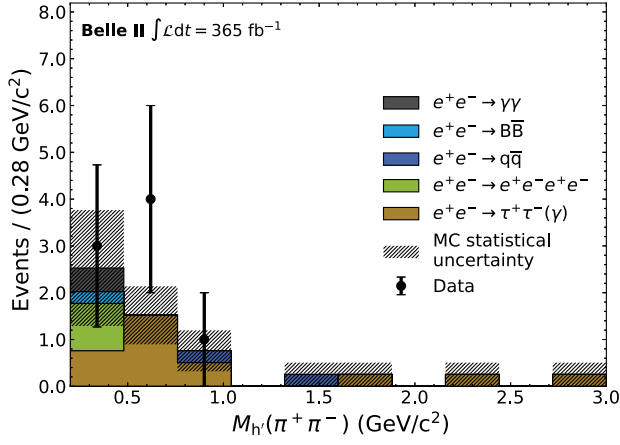


FIG. 2. Distribution of $M(\pi^+\pi^-)$ together with the stacked contributions from the various simulated SM background samples for $h' \rightarrow \pi^+\pi^-$ candidates. Simulation is normalized to a luminosity of 365 fb^{-1} .

tracks close to the IP we instead correct for momentum-dependent efficiency differences between data and simulation, resulting in uncertainties at the level of 0.5% [48]. The uncertainties arising from PID are evaluated using the processes $e^+e^- \rightarrow e^+e^-e^+e^-$, $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$, and $e^+e^- \rightarrow \mu^+\mu^-\gamma$, as well as decays of K_S^0 or J/Ψ . For electrons these uncertainties are typically at the level of 3%, while uncertainties for μ , π , and K are below 1%. We account for a lifetime-dependent effect on PID by introducing an additional systematic uncertainty, evaluated using K_S^0 and Λ decays. For very displaced vertices, these uncertainties can reach up to 10%. The uncertainty on the luminosity is 0.47% [25]. The limited number of simulated events for each signal configuration introduces systematic uncertainties at the level of 1%–2% for most parameter configurations but can reach up to 10% for very long lifetimes. We verify that our interpolation procedure between simulated mass points does not introduce a significant additional uncertainty. We estimate the uncertainty introduced by splitting the mass region in the $\pi^+\pi^-$ final state by varying the split point to $0.9 \text{ GeV}/c^2$ and $1.2 \text{ GeV}/c^2$, respectively, and take the maximum deviation from the nominal background level as the uncertainty δ .

We find no events in the $\mu^+\mu^-$ final state, 8 events in the $\pi^+\pi^-$ final state, and one event in the K^+K^- final state. The $M(\pi^+\pi^-)$ distribution in the $h' \rightarrow \pi^+\pi^-$ final state is shown in Fig. 2, while the distributions for $h' \rightarrow \mu^+\mu^-$ and $h' \rightarrow K^+K^-$ are shown in the Supplemental Material [49]. The statistical model used to compute the signal significances and p values is discussed in Appendix A. The largest local significance for the model-independent search is 2.9σ , including systematic uncertainties, found near $m(h') = 0.531 \text{ GeV}/c^2$ for the $\pi^+\pi^-$ final state for a lifetime of $c\tau = 1.0 \text{ cm}$. Taking into account the look-elsewhere effect [50], this excess has a global significance of 1.1σ .

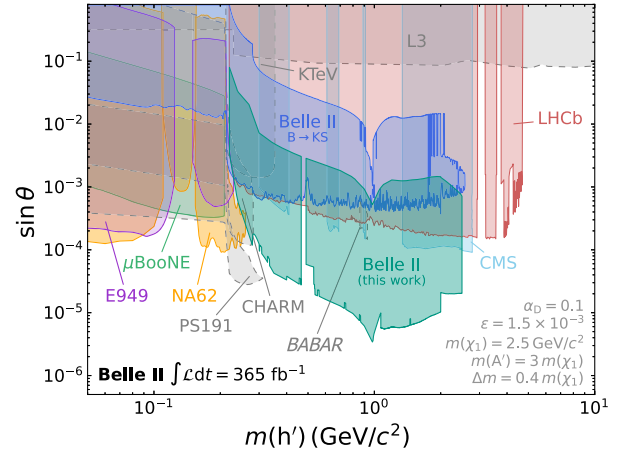


FIG. 3. Exclusion regions at 95% credibility level in the plane of the sine of the mixing angle θ and dark Higgs mass $m(h')$ from this work (teal) together with existing constraints at 90% confidence level from PS191 [55], E949 [56], NA62 [57,58], KOTO [16,59], KTeV [60], and BABAR [54,61], and at 95% confidence level from MicroBooNE [16,62,63], L3 [16,64], CHARM [54,65], LHCb [54,66,67], Belle II [68], and CMS [69] for $\alpha_D = 0.1$, $m(A') = 3m(\chi_1)$, $\Delta m = 0.4m(\chi_1)$, $\epsilon = 1.5 \times 10^{-3}$, and $m(\chi_1) = 2.5 \text{ GeV}/c^2$. Constraints colored in gray with dashed outline are reinterpretations not performed by the experimental collaborations. All constraints except for the one from this work do not require the presence of a dark photon or iDM.

With the method described in Appendix B, we compute 95% Bayesian credibility level upper limits on $\sigma_{\text{sig}} = \sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-) \times \mathcal{B}(h' \rightarrow x^+ x^-)$ using the Bayesian Analysis Toolkit software package [51,52]. The observed upper limits, including systematic uncertainties, are shown in Supplemental Material [49]. Using a Jeffreys prior [53] would decrease the upper limits on σ_{sig} by up to 30% with respect to the uniform prior. The systematic uncertainties weaken the limits, with the largest increase of 2.5% occurring for heavy h' with small lifetimes.

For the model-dependent interpretations, we multiply the p values in all relevant and kinematically accessible analysis channels, again separately for various lifetimes.

For the calculation of the model-dependent upper limits on $\sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-)$ we multiply the individual likelihoods weighted by the theoretical h' branching fractions from Ref. [54]. For each h' mass value, we determine the value of $\sin \theta$ such that the resulting predicted value of $\sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-)$ equals the 95% excluded $\sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-)$. To calculate the prediction, we fix σ_{prod} and the χ_2 branching fractions to the theoretical values from Ref. [13] taking into account ISR. Figure 3 shows the observed upper limit on $\sin \theta$ for one specific choice of model parameters. Similarly, for each $m(\chi_1)$, we determine the value of y such that the resulting predicted value of $\sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-)$ equals the 95% excluded $\sigma_{\text{prod}} \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+ e^-)$. Figure 4 shows the observed upper limit on y for a specific choice of model parameters.

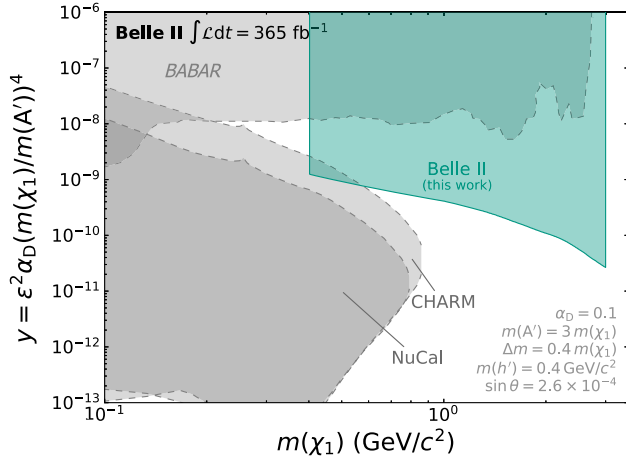


FIG. 4. Exclusion regions at 95% credibility level in the plane of the dimensionless variable $y = \epsilon^2 \alpha_D (m(\chi_1)/m(A'))^4$ and DM mass $m(\chi_1)$ from this work (teal) together with existing constraints at 90% confidence level from CHARM [18,23], NuCal [18,21,22], and BABAR [12,20] for $\alpha_D = 0.1$, $m(A') = 3m(\chi_1)$, $\Delta m = 0.4m(\chi_1)$, $\sin \theta = 2.6 \times 10^{-4}$, and $m(h') = 0.4 \text{ GeV}/c^2$. Constraints colored in gray with dashed outline are reinterpretations not performed by the experimental collaborations. All constraints except for the one from this work do not require the presence of a dark Higgs boson or iDM.

In general, σ_{prod} increases with ϵ^2 , the lifetime of the h' increases with $1/(\sin \theta)^2$, and the lifetime of the χ_2 increases with $1/\epsilon^2$. Additional plots and detailed numerical results for many more parameter combinations can be found in Supplemental Material [49].

In conclusion, we report the first search for a dark Higgs in association with inelastic DM, using 365 fb^{-1} of Belle II e^+e^- data. We do not observe a significant excess above the background. We set 95% credibility level upper limits on $\sigma(e^+e^- \rightarrow h'\chi_1\chi_2) \times \mathcal{B}(\chi_2 \rightarrow \chi_1 e^+e^-) \times \mathcal{B}(h' \rightarrow x^+x^-)$. Depending on the combination of model parameters, the limits improve over existing searches by up to 2 orders of magnitude.

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Data availability—The data that support the findings of this Letter are not publicly available upon publication

because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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- [1] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
 - [2] A. Arbey and F. Mahmoudi, Dark matter and the early Universe: A review, *Prog. Part. Nucl. Phys.* **119**, 103865 (2021).
 - [3] J. Aalbers *et al.* (LZ Collaboration), Dark matter search results from 4.2 tonne-years of exposure of the LUX-ZEPLIN (LZ) experiment, *Phys. Rev. Lett.* **135**, 011802 (2025).
 - [4] E. Aprile *et al.* (XENON Collaboration), First dark matter search with nuclear recoils from the XENONnT experiment, *Phys. Rev. Lett.* **131**, 041003 (2023).
 - [5] Z. Bo *et al.* (PandaX Collaboration), Dark matter search results from 1.54 tonne-year exposure of PandaX-4T, *Phys. Rev. Lett.* **134**, 011805 (2025).
 - [6] P. Agnes *et al.* (DarkSide-50 Collaboration), Search for low-mass dark matter WIMPs with 12 ton-day exposure of DarkSide-50, *Phys. Rev. D* **107**, 063001 (2023).
 - [7] A. Hayrapetyan *et al.* (CMS Collaboration), Dark sector searches with the CMS experiment, *Phys. Rep.* **1115**, 448 (2025).
 - [8] G. Aad *et al.* (ATLAS Collaboration), Exploration at the high-energy frontier: ATLAS Run 2 searches investigating the exotic jungle beyond the Standard Model, *Phys. Rep.* **1116**, 301 (2025).
 - [9] D. Smith and N. Weiner, Inelastic dark matter, *Phys. Rev. D* **64**, 043502 (2001).
 - [10] E. Izaguirre, G. Krnjaic, and B. Shuve, Discovering inelastic thermal-relic dark matter at colliders, *Phys. Rev. D* **93**, 063523 (2016).
 - [11] E. Izaguirre, Y. Kahn, G. Krnjaic, and M. Moschella, Testing light dark matter coannihilation with fixed-target experiments, *Phys. Rev. D* **96**, 055007 (2017).
 - [12] M. Duerr, T. Ferber, C. Hearty, F. Kahlhoefer, K. Schmidt-Hoberg, and P. Tunney, Invisible and displaced dark matter signatures at Belle II, *J. High Energy Phys.* **02** (2020) 039.
 - [13] M. Duerr, T. Ferber, C. Garcia-Cely, C. Hearty, and K. Schmidt-Hoberg, Long-lived dark Higgs and inelastic dark matter at Belle II, *J. High Energy Phys.* **04** (2021) 146.
 - [14] D. O'Connell, M. J. Ramsey-Musolf, and M. B. Wise, Minimal extension of the standard model scalar sector, *Phys. Rev. D* **75**, 037701 (2007).
 - [15] J. Beacham *et al.*, Physics beyond colliders at CERN: Beyond the standard model working group report, *J. Phys. G* **47**, 010501 (2020).
 - [16] T. Ferber, A. Grohsjean, and F. Kahlhoefer, Dark Higgs bosons at colliders, *Prog. Part. Nucl. Phys.* **136**, 104105 (2024).
 - [17] A. Hayrapetyan *et al.* (CMS Collaboration), Search for inelastic dark matter in events with two displaced muons and missing transverse momentum in proton-proton collisions at $\sqrt{s} = 13$ TeV, *Phys. Rev. Lett.* **132**, 041802 (2024).

- [18] Y.-D. Tsai, P. deNiverville, and M. X. Liu, Dark photon and muon $g - 2$ inspired inelastic dark matter models at the high-energy intensity frontier, *Phys. Rev. Lett.* **126**, 181801 (2021).
- [19] M. Mongillo, A. Abdullahi, B. B. Oberhauser, P. Crivelli, M. Hostert, D. Massaro, L. M. Bueno, and S. Pascoli, Constraining light thermal inelastic dark matter with NA64, *Eur. Phys. J. C* **83**, 391 (2023).
- [20] J. P. Lees *et al.* (BABAR Collaboration), Search for invisible decays of a dark photon produced in e^+e^- collisions at BABAR, *Phys. Rev. Lett.* **119**, 131804 (2017).
- [21] J. Blümlein and J. Brunner, New exclusion limits for dark gauge forces from beam-dump data, *Phys. Lett. B* **701**, 155 (2011).
- [22] J. Blümlein and J. Brunner, New exclusion limits on dark gauge forces from proton Bremsstrahlung in beam-dump data, *Phys. Lett. B* **731**, 320 (2014).
- [23] S. N. Gninenko, Constraints on sub-GeV hidden sector gauge bosons from a search for heavy neutrino decays, *Phys. Lett. B* **713**, 244 (2012).
- [24] D. Banerjee *et al.* (NA64 Collaboration), Improved limits on a hypothetical $X(16.7)$ boson and a dark photon decaying into e^+e^- pairs, *Phys. Rev. D* **101**, 071101 (2020).
- [25] I. Adachi *et al.* (Belle II Collaboration), Measurement of the integrated luminosity of data samples collected during 2019-2022 by the Belle II experiment, *Chin. Phys. C* **49**, 013001 (2025).
- [26] T. Abe *et al.* (Belle II Collaboration), Belle II technical design report, [arXiv:1011.0352](https://arxiv.org/abs/1011.0352).
- [27] K. Akai, K. Furukawa, and H. Koiso (SuperKEKB Accelerator Team), SuperKEKB Collider, *Nucl. Instrum. Methods Phys. Res., Sect. A* **907**, 188 (2018).
- [28] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H.-S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, *J. High Energy Phys.* **07** (2014) 079.
- [29] D. J. Lange, The EvtGen particle decay simulation package, *Nucl. Instrum. Methods Phys. Res., Sect. A* **462**, 152 (2001).
- [30] S. Frixione *et al.*, Lepton collisions in MadGraph5_aMC@NLO, [arXiv:2108.10261](https://arxiv.org/abs/2108.10261).
- [31] E. Barberio, B. van Eijk, and Z. Waś, PHOTOS: A universal Monte Carlo for QED radiative corrections in decays, *Comput. Phys. Commun.* **66**, 115 (1991).
- [32] E. Barberio and Z. Waś, PHOTOS: A Universal Monte Carlo for QED radiative corrections. Version 2.0, *Comput. Phys. Commun.* **79**, 291 (1994).
- [33] S. Jadach, B. F. L. Ward, and Z. Waś, The precision Monte Carlo event generator KK for two-fermion final states in e^+e^- collisions, *Comput. Phys. Commun.* **130**, 260 (2000).
- [34] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, An Introduction to Pythia 8.2, *Comput. Phys. Commun.* **191**, 159 (2015).
- [35] F. A. Berends, P. H. Daverveldt, and R. Kleiss, Complete lowest-order calculations for four-lepton final states in electron-positron collisions, *Nucl. Phys.* **B253**, 441 (1985).
- [36] S. Jadach, W. Placzek, M. Skrzypek, B. F. L. Ward, and Z. Waś, Monte Carlo program KoralW 1.42 for all four-fermion final states in e^+e^- collisions, *Comput. Phys. Commun.* **119**, 272 (1999).
- [37] S. Uehara, TREPS: A Monte-Carlo event generator for two-photon processes at e^+e^- colliders using an equivalent photon approximation, [arXiv:1310.0157](https://arxiv.org/abs/1310.0157).
- [38] G. Balossini, C. Bignamini, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, Photon pair production at flavour factories with per mille accuracy, *Phys. Lett. B* **663**, 209 (2008).
- [39] F. Campanario, H. Czyz, J. Gluza, T. Jelinski, G. Rodrigo, S. Tracz, and D. Zhuridov, Standard model radiative corrections in the pion form factor measurements do not explain the a_μ anomaly, *Phys. Rev. D* **100**, 076004 (2019).
- [40] S. Jadach, J. H. Kühn, and Z. Waś, TAUOLA—a library of Monte Carlo programs to simulate decays of polarized τ leptons, *Comput. Phys. Commun.* **64**, 275 (1990).
- [41] S. Agostinelli *et al.* (Geant4 Collaboration), Geant4—a simulation toolkit, *Nucl. Instrum. Methods Phys. Res., Sect. A* **506**, 250 (2003).
- [42] T. Kuhr *et al.* (Belle II Framework Software Group), The Belle II Core Software, *Comput. Software Big Sci.* **3**, 1 (2019).
- [43] Belle II Analysis Software Framework (basf2) (release-06-00-08), [10.5281/zenodo.16267708](https://zenodo.org/record/16267708).
- [44] G. Punzi, Sensitivity of searches for new signals and its optimization, eConf C **030908**, MODT002 (2003), <https://inspirehep.net/literature/634798>.
- [45] E. Kou *et al.*, The Belle II physics book, *Prog. Theor. Exp. Phys.* **2019**, 123C01 (2019); **2020**, 029201(E) (2020).
- [46] J. Gaiser, Charmonium spectroscopy from radiative decays of the J/ψ and ψ' , Ph.D. thesis, Stanford University, 1982.
- [47] T. Skwarnicki, A study of the radiative CASCADE transitions between the Upsilon-Prime and Upsilon resonances, Ph.D. thesis, Cracow, INP, 1986.
- [48] I. Adachi *et al.* (Belle II Collaboration), Observation of the decay $B^0 \rightarrow J/\psi\omega$ at Belle II, *Phys. Rev. D* **111**, 032012 (2025).
- [49] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/37w5-glpp> for additional plots and numerical results.
- [50] E. Gross and O. Vitells, Trial factors for the look elsewhere effect in high energy physics, *Eur. Phys. J. C* **70**, 525 (2010).
- [51] A. Caldwell, D. Kollar, and K. Kroninger, BAT: The Bayesian Analysis Toolkit, *Comput. Phys. Commun.* **180**, 2197 (2009).
- [52] F. Beaujean *et al.*, BAT release, version 1.0.0, [10.5281/zenodo.1322675](https://zenodo.org/record/1322675).
- [53] H. Jeffreys, An invariant form for the prior probability in estimation problems, *Proc. R. Soc. A* **186**, 453 (1946).
- [54] M. W. Winkler, Decay and detection of a light scalar boson mixing with the Higgs boson, *Phys. Rev. D* **99**, 015018 (2019).
- [55] D. Gorbunov, I. Krasnov, and S. Suvorov, Constraints on light scalars from PS191 results, *Phys. Lett. B* **820**, 136524 (2021).

- [56] A. V. Artamonov *et al.* (BNL-E949 Collaboration), Study of the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the momentum region $140 < P_\pi < 199$ MeV/c, *Phys. Rev. D* **79**, 092004 (2009).
- [57] E. Cortina Gil *et al.* (NA62 Collaboration), Search for π^0 decays to invisible particles, *J. High Energy Phys.* **02** (2021) 201.
- [58] E. Cortina Gil *et al.* (NA62 Collaboration), Measurement of the very rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, *J. High Energy Phys.* **06** (2021) 093.
- [59] J. K. Ahn *et al.* (KOTO Collaboration), Study of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay at the J-PARC KOTO experiment, *Phys. Rev. Lett.* **126**, 121801 (2021).
- [60] A. Alavi-Harati *et al.*, Search for the decay $K_L \rightarrow \pi^0 \mu^+ \mu^-$, *Phys. Rev. Lett.* **84**, 5279 (2000).
- [61] J. P. Lees *et al.* (BABAR Collaboration), Search for long-lived particles in e^+e^- collisions, *Phys. Rev. Lett.* **114**, 171801 (2015).
- [62] P. Abratenko *et al.* (MicroBooNE Collaboration), Search for a Higgs portal scalar decaying to electron-positron pairs in the MicroBooNE detector, *Phys. Rev. Lett.* **127**, 151803 (2021).
- [63] P. Abratenko *et al.* (MicroBooNE Collaboration), Search for long-lived heavy neutral leptons and Higgs portal scalars decaying in the MicroBooNE detector, *Phys. Rev. D* **106**, 092006 (2022).
- [64] M. Acciarri *et al.* (L3 Collaboration), Search for neutral Higgs boson production through the process $e^+e^- \rightarrow Z^* H^0$, *Phys. Lett. B* **385**, 454 (1996).
- [65] F. Bergsma *et al.* (CHARM Collaboration), Search for axion-like particle production in 400 GeV proton-copper interactions, *Phys. Lett. B* **157**, 458 (1985).
- [66] R. Aaij *et al.* (LHCb Collaboration), Search for hidden-sector bosons in $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decays, *Phys. Rev. Lett.* **115**, 161802 (2015).
- [67] R. Aaij *et al.* (LHCb Collaboration), Search for long-lived scalar particles in $B^+ \rightarrow K^+ \chi(\mu^+ \mu^-)$ decays, *Phys. Rev. D* **95**, 071101 (2017).
- [68] I. Adachi *et al.* (Belle II Collaboration), Search for a long-lived spin-0 mediator in $b \rightarrow s$ transitions at the Belle II experiment, *Phys. Rev. D* **108**, L111104 (2023).
- [69] A. Tumasyan *et al.* (CMS Collaboration), Search for long-lived particles decaying into muon pairs in proton-proton collisions at $\sqrt{s} = 13$ TeV collected with a dedicated high-rate data stream, *J. High Energy Phys.* **04** (2022) 062.

End Matter

Appendix A: Signal extraction—In the hypothesis of uniform background, the expected number of background events in the signal window is μ/f , where the nuisance parameter μ is the expected number of background events in the SB and f the ratio between the widths of the SB and the signal window. The likelihood for observing n events in the signal window with the background-only hypothesis is

$$\mathcal{L}(n, \mu) = \frac{(\mu/f)^n}{n!} e^{-(\mu/f)} \times \frac{\mu^{N_{\text{obs}}^{\text{SB}}}}{N_{\text{obs}}^{\text{SB}}!} e^{-\mu}, \quad (\text{A1})$$

with $N_{\text{obs}}^{\text{SB}}$ being the number of observed events in the sideband. Equation (A1) incorporates all the relevant statistical fluctuations through Poissonian priors. For the $\pi^+ \pi^-$ final state, we include the additional systematic uncertainty of the background model by substituting $\mu \rightarrow \mu(1 + x\delta)$, with the nuisance parameter x , that is constrained by an additional Gaussian prior. The dependency on the nuisance parameters μ and x is removed by marginalization of $\mathcal{L}$

$$\mathcal{L}(n) = \int_0^\infty d\mu \int_{-\infty}^\infty dx \mathcal{L}(n, \mu, x). \quad (\text{A2})$$

We calculate the p value, which describes the probability of observing at least N_{obs} events in the signal window given the background expectation, as

$$p_0 = \sum_{n=N_{\text{obs}}}^\infty \mathcal{L}(n). \quad (\text{A3})$$

In the case of multiple final states, as in the background-only hypothesis the final states are independent of each other, for each scan point the combined p value is given by the product of the individual p values. From the p value the significance Z is calculated via

$$Z = \Phi^{-1}(1 - p_0), \quad (\text{A4})$$

where Φ^{-1} is the quantile of the standard Gaussian.

Appendix B: Upper limit calculation—In the presence of signal, the likelihood of observing N_{obs} events in the signal window with a background expectation is given by

$$\mathcal{L}(n_{\text{sig}}, \mu) = \frac{(n_{\text{sig}} + \mu/f)^{N_{\text{obs}}}}{N_{\text{obs}}!} e^{-(n_{\text{sig}} + \mu/f)} \times \frac{\mu^{N_{\text{obs}}^{\text{SB}}}}{N_{\text{obs}}^{\text{SB}}!} e^{-\mu}. \quad (\text{B1})$$

The number of signal events n_{sig} can be expressed in terms of the signal cross section σ_{sig} via

$$n_{\text{sig}} = \sigma_{\text{sig}} \epsilon_{\text{sig}} L \quad (\text{B2})$$

with the signal efficiency ϵ_{sig} and the integrated luminosity L . Systematic uncertainties on ϵ_{sig} and L are incorporated into the likelihood by adding additional nuisance parameters y and z , respectively. These nuisance parameters are again constrained by Gaussian priors. The dependence on all nuisance parameters μ and $\vec{\theta} = (x, y, z)$ is removed by marginalization

$$\mathcal{L}(\sigma_{\text{sig}}) = \int_0^\infty d\mu \int_{-\infty}^\infty d\vec{\theta} \mathcal{L}(\sigma_{\text{sig}}, \mu, \vec{\theta}). \quad (\text{B3})$$

Using this marginalized likelihood we compute $\alpha = 95\%$ CL upper limits on σ_{sig} via

$$\alpha = \int_0^{\sigma_{\text{sig}}^{\text{up}}} d\sigma_{\text{sig}} \mathcal{L}(\sigma_{\text{sig}}) \Theta(\sigma_{\text{sig}}) \quad (\text{B4})$$

with a uniform prior $\Theta(\sigma_{\text{sig}})$ on the cross section.