Ain Shams Engineering Journal 17 (2026) 103939

Contents lists available at ScienceDirect

Ain Shams Engineering Journal

journal homepage: www.sciencedirect.com

Full length article

Maximal product-based intuitionistic fuzzy line graphs for healthcare predictive analysis

A. Meenakshi [a,*], J. Shivangi Mishra [a,1], Leo Mršić [b,h,1], Antonios Kalampakas [c,1], Sovan Samanta [b,d,e,f,**], Tofigh Allahviranloo [d,g,1]

[a] Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R& D Institute of Science and Technology, Avadi, Chennai, 600062, Tamil Nadu, India
[b] Department of Technical Sciences, Algebra Bernays University, Gradišćanska 24, 10000, Zagreb, Croatia
[c] College of Engineering and Technology, American University of the Middle East, Egaila, 54200, Kuwait
[d] Research Center of Performance and Productivity Analysis, Istinye University, Istanbul, 34010, Turkey
[e] Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk, 721636, India
[f] Department of Technical Sciences, Western Caspian University, 1001, Baku, Azerbaijan
[g] Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
[h] Rudolfovo Science and Technology Centre, Podbreznik 15, Novo Mesto, 8000, Slovenia

ARTICLE INFO

MSC:
05C72
05C76

Keywords:
Intuitionistic fuzzy graphs
Intuitionistic fuzzy line graphs
Maximal product
Adjacency matrices
Correlation and regression coefficients

ABSTRACT

This paper explores the applications of Intuitionistic Fuzzy Graphs ($\mathscr{IFG}$) representing uncertainty and imprecision in complex systems through the analysis of correlation and regression coefficients ($\mathscr{CRC}s$) with focus on the maximal product. The study examines the relationships between the edges of the graph by analysing the line graph derived from $\mathscr{IFG}$, facilitating a deeper understanding of the network's dynamics. The construction of adjacency matrices that incorporate both membership and non-membership values enables the calculation of energy and weight scores, quantifying the strength and predictive correlations among variables. Furthermore, the study discusses the complement of Intuitionistic Fuzzy Line Graphs ($\mathscr{IFLG}$), using maximal product analysis to uncover concealed relationships within the network. MATLAB is used to generate heatmaps that visually represent the importance of correlation to critical network characteristics. The practical importance is demonstrated in a healthcare context, particularly in predicting diabetes risk by modelling factors of glucose levels, body mass index (BMI), and insulin. Heatmaps can be effectively visualized to show interrelationships between these features, aiding in the interpretation of network patterns.

1. Introduction

Fuzzy graph ($\mathcal{F}_{\mathscr{G}}$) theory is a generalisation of the classical graph theory, introduced by A. Rosenfeld [1] in 1975, which incorporates fuzziness to account for uncertainty and imprecise relationships among models in graph structures. The origin of Intuitionistic Fuzzy Graphs ($\mathscr{IFG}$) traces back to 1986, when Krassimir Atanassov [2] introduced intuitionistic fuzzy sets (IFS), which generalised Zadeh's [3] famous fuzzy sets. Its applications lie in addressing issues in social networks, decision-making, and optimisation problems. The key operations in graph theory, such as line graphs and complement graphs, are essential for understanding the relationships within networks. A line graph turns the edges into vertices of a graph, providing insights into edge adjacencies. Complement $\mathcal{F}_{\mathscr{G}}$, on the other hand, emphasises association by depicting edges that were not formed in a graph. This can only be realised while acquiring further significance along with $\mathcal{F}_{\mathscr{G}}$, as these reveal an uncertainty in some degree of ordinality among vertices. This paper will strive to discuss in detail the structure of Intuitionistic Fuzzy Line Graphs ($\mathscr{IFLG}$) and a complement of $\mathscr{IFG}$ in developing complex networks as representations under any uncertain scenario. Akula and Basha [4] have shown the regression coefficient measure of $\mathscr{IFG}$ and its applications in agricultural planning to determine soil choice for the superior paddy crop, with efficacy proven through practical, scenario-based applications. Akula and Shaik [5] investigated correlation coefficient measures of $\mathscr{IFG}$ and their applications to financial decision-making, such as money investment schemes, providing insights into applying $\mathcal{F}_{\mathscr{G}}$ in economics. Bajaj and Kumar [6] proposed a novel

* Corresponding author.
** Corresponding author at: Department of Technical Sciences, Algebra Bernays University, Gradišćanska 24, 10000, Zagreb, Croatia.
Email addresses: drameenakshi@veltech.edu.in (A. Meenakshi), shivangimishra1427@gmail.com (J.S. Mishra), leo.mrsic@algebra.hr (L. Mršić), Antonios.Kalampakas@aum.edu.kw (A. Kalampakas), ssamantavu@gmail.com (S. Samanta), tofigh.allahviranloo@istinye.edu.tr (T. Allahviranloo).

[1] These authors contributed equally to this work.

https://doi.org/10.1016/j.asej.2025.103939
Received 21 August 2025; Received in revised form 26 November 2025; Accepted 11 December 2025
Available online 7 January 2026

intuitionistic fuzzy correlation coefficient for multi-criteria decision-making, enhancing tools for analysing uncertainty in complex decision environments. Dey et al. [7] explored operations on the complement of $\mathcal{F}_{\mathscr{G}}$, contributing to the theoretical understanding of graph complements and advancing $\mathcal{F}_{\mathscr{G}}$ theory. Meenakshi and Mishra [8] studied the maximal product of cubic fuzzy graph structures, offering insights into the structural aspects of $\mathcal{F}_{\mathscr{G}}$ operations. Meenakshi et al. [9]-[10] studied optimal networks using neutrosophic graph products and their applications in disease prediction. Meenakshi and Shivangi [11] have studied the correlation and regression coefficients for SVNGS and their products with a focus on their MST. Mohamed and Ali [12] analysed the degree of a vertex in the complement of the maximal product of $\mathscr{IFG}$, providing a deeper understanding of vertex properties in $\mathcal{F}_{\mathscr{G}}$. Mohamed and Ali [13] also examined the complement of the max product of $\mathscr{IFG}$, contributing significantly to the theoretical understanding of $\mathcal{F}_{\mathscr{G}}$ complements. Mordeson and Peng [14] systematically studied the operations on $\mathcal{F}_{\mathscr{G}}$, laying the foundation for understanding the interplay of various graph operations. Nagoorgani et al. [15] researched double domination in $\mathscr{IFG}$, establishing a basis for optimisation within network structures. Reddy and Basha [16] proposed the concept of correlation coefficient of hesitancy $\mathcal{F}_{\mathscr{G}}$, which can play a significant role in decision-making situations and problem-solving in complications. Sahoo and Pal [17] have discussed in detail the various products for $\mathscr{IFG}$, which became a basis for further investigations on graph operations by Sandeep et al. [18]. It sheds light on the complement of $\mathcal{F}_{\mathscr{G}}$ and presents certain remarks that have been informative in subsequent research on $\mathcal{F}_{\mathscr{G}}$ properties. Talebi and Rashmanlou [19] illustrated complement, isomorphic, and similar bipolar $\mathcal{F}_{\mathscr{G}}$ by extending their applications in analysing and modelling complex systems. Ye [20] explored another correlation coefficient between single-valued neutrosophic sets. He proposed a method for multi-attribute decision-making and addressed uncertainty effectively by using the correlation coefficients between single-valued neutrosophic sets. Again, Ye [21] developed more effective correlation coefficients of IFS, which applied to real-life decision-making challenges, hence pushing the fuzzy set theory forward. Yahya Mohamed and Mohamed Ali [22] calculated the degree of a vertex in the complement of the maximal product of $\mathscr{IFG}$, providing crucial information about vertex properties in graph complements. Repalle et al. [23] studied interval-valued $\mathscr{IFLG}$ as an extension of fuzzy set theory and IFS for dealing with uncertainty in graph theory. It introduces new definitions, theorems, and assertions, making significant contributions to the theoretical foundation of $\mathcal{F}_{\mathscr{G}}$. The study contributes to a better understanding of isomorphic characteristics and homomorphisms in this setting, marking a new advance in the field. Repalle et al. [24] introduced interval-valued $\mathscr{IFLG}$ as a more generalised application of line graph theory to a fuzzy environment. It addresses structural features, linkages to existing $\mathcal{F}_{\mathscr{G}}$ models, and potential applications in uncertainty-based decision-making. Akram and Davvaz [25] presented strong $\mathscr{IFG}$, which are extensions of $\mathscr{IFG}$ that include strength metrics to improve decision-making. Akram [26] enhanced interval-valued fuzzy line graphs by developing a mathematical framework for investigating $\mathcal{F}_{\mathscr{G}}$ architectures in which edge uncertainty is expressed as interval values. Kosari et al. [27] studied the topological indices. Meenakshi and Babujee [28] studied equitable domination in graphs, which contributed to optimisation in graph-theoretic structures. Shi et al. [29] studied cubic fuzzy graph connectivity to determine zones of danger for tsunamis. Khan et al. [30] studied picture fuzzy hypergraphs in the context of decision-making. Tobaili et al. [31] introduced the edge hub number to characterise influential edges in fuzzy graphs under uncertain environments. Imran et al. [32] applied novel Sombor-based indices to $\mathscr{IFG}$ operations, showing their usefulness in modelling and optimising routing in uncertain networks. Talebi et al. [33] proposed the concept of interval-valued intuitionistic fuzzy soft graphs, thus providing a flexible framework that could represent uncertainty with higher accuracy. Similarly, Shi et al. [34] explored various structural properties of cubic fuzzy graphs and also proved the applicability of those in a case study. PK et al. [35] proposed a new concept of cubic graph and demonstrated its practical utility through applications. This paper extends the concepts in the graph theory and offers valuable insights in the methodological direction of our research. Furthermore, regular $\mathcal{F}_{\mathscr{G}}$ have been used to simulate psychology, and domination in $\mathcal{F}_{\mathscr{G}}$ has been applied in medical applications.

1.1. Motivation

The study examined the $\mathscr{CRC}s$ of the complement of the maximal product of an $\mathscr{IFLG}$ and found a positive correlation between the results. Thus, this approach has significant applications in social network analysis, where understanding the strength and dynamics of relationships is crucial. Representing relationships between individuals or groups using $\mathscr{IFG}$ is usually accomplished with a high degree of precision and involves both certainty and ambiguity. The complement of the maximal product of an $\mathscr{IFLG}$ allows one to see indirect or hidden interactions in the network that may not be obvious from the structure of the main graph. The $\mathscr{CRC}s$ under fuzzy set conditions have been analysed, and future research opportunities have been outlined. This work aims to contribute methods for network optimisation, predictive maintenance, resource allocation, and decision-making processes. The original contributions of this work are described below.

1.2. Novelty

This research adopts $\mathscr{CRC}s$ for application to $\mathscr{IFG}$ through an original approach to establish a new mathematical model that characterises dependencies in ambiguous and uncertain systems. The research applies the maximal product method to provide the network assessment capabilities and the prediction accuracy in complex systems. The present work introduces the line graph analysis of $\mathscr{IFG}$ together with its complementary content as an alternative method to explore edge interactions. This method reveals the sophisticated network behaviour that standard graph models often miss. Rephrasing this technique involves producing adjacency matrices that incorporate both membership and non-membership characteristics as a new approach to measuring network strength. The association and influence evaluation process becomes more reliable through weight and energy rating systems in network components. The maximum product framework extended to $\mathscr{IFLG}$ and its complements enables users to discover network patterns and secondary effects that remain invisible to direct graph analysis. The application proves most beneficial for analysing networks with high levels of indirect relationships. The importance of $\mathscr{IFG}$ and $\mathscr{CRC}s$ is highlighted by the application of this study to medicine, namely to diabetes risk prediction. A new computational approach for risk assessment and medical diagnosis modelling and analysis of these crucial components using $\mathcal{F}_{\mathscr{G}}$ theory. The proposed methodology is adaptable and can be applied to a variety of fields, including social network research, financial risk analysis, and cybersecurity. The use of maximal product analysis in $\mathscr{IFLG}$ opens up new possibilities for discovering relational patterns in various datasets. This paper advances the theoretical framework for $\mathscr{IFG}$ products and introduces a computationally feasible tool for exploring relational patterns in various datasets. Its methodology is versatile and can be applied to other areas, including social network analysis, financial risk analysis, and cybersecurity.

1.3. Structure of the paper

The paper aims to expand the theoretical framework and suggest potential solutions for network optimisation in uncertain environments. This manuscript is organised as follows: Section 2 presents the basic definitions relevant to our research. Section 3, examines the $\mathscr{CRC}s$ of $\mathscr{IFG}$ using maximal product. Section 4 focuses on a deeper analysis of $\mathscr{CRC}s$ regarding the maximal product of an $\mathscr{IFLG}$. Section 5 introduces the $\mathscr{CRC}s$ associated with the complement of an $\mathscr{IFLG}$ using the maximal product. Section 6 discusses the applications of social network analysis. Section 7 concludes the research and looks ahead to future efforts. Table 1 summarizes the key notations used in this work.

Table 1
List of Abbreviations.

Abbreviation	Expansion
$\mathcal{F}_{\mathscr{G}}$	Fuzzy Graph
$\mathscr{IFG}$	Intutionistic Fuzzy Graph
IFS	Intutionistic Fuzzy Set
$\mathscr{CRC}$	Correlation and Regression Coefficients
$\mathscr{IFLG}$	Intutionistic Fuzzy Line Graph

2. Preliminaries

This section presents definitions and some common working terminology through remarks and illustrations.

Definition 1. [3] A fuzzy set $\tilde{\mathcal{R}}$ in $\mathscr{X}$ is defined by a membership function $\mu_{\tilde{\mathcal{R}}} : w \to [0,1]$, which assigns to each element $w \in \mathscr{X}$ a real number $\mu_{\tilde{\mathcal{R}}}(w)$ representing the degree of membership of w in $\tilde{\mathcal{R}}$. The fuzzy set $\tilde{\mathcal{R}}$ is represented as

$$\tilde{\mathcal{R}} = \{(w, \mu_{\tilde{\mathcal{R}}}(w)) \mid w \in \mathscr{X}\},$$

where $\mu_{\tilde{\mathcal{R}}}(w)$ represents the value between 0 and 1.

Definition 2. [1] A Fuzzy graph ($\mathcal{F}_{\mathscr{G}}$), $\mathbb{G} = (\sigma, \hat{\mu})$ is a pair of functions, with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$. Let $\sigma : \mathscr{V} \to \{0,1\}$ and $\hat{\mu} : \mathcal{V} \times \mathcal{V} \to \{0,1\}$ such that $\hat{\mu}(u,v) \le \sigma(u) \wedge \sigma(v)$ for every $u, v \in \mathscr{V}$.

Definition 3. [2] An IFS $B = \langle x, \hat{\mu}_B(x), \hat{\nu}_B(x)\rangle_{x\in X}$ in a universe of discourse $\mathscr{X}$ is characterised by a membership function $\hat{\mu}_B$ and a non-membership function $\hat{\nu}_B$, as follows: $\hat{\mu}_B : \mathscr{X} \to [0,1], \quad \hat{\nu}_B : \mathscr{X} \to [0,1]$, and $\hat{\mu}_B(x) + \hat{\nu}_B(x) \le 1 \quad \text{for all } x \in \mathscr{X}$.

Definition 4. [7] The complement of a $\mathcal{F}_{\mathscr{G}}$ with $\mathbb{G} = (\sigma, \hat{\mu})$ is also a $\mathcal{F}_{\mathscr{G}}$ and it is denoted as $\overline{\mathbb{G}}$. The membership function for the complement $\overline{\mathbb{G}} = \overline{(\sigma, \hat{\mu})}$, where $\overline{\sigma(x_i)} = \sigma(x_i)$ and $\overline{\hat{\mu}(x_i, x_j)} = \sigma(x_i) \wedge \sigma(x_j) - \hat{\mu}(x_i, x_j) \quad \forall x_i, x_j \in \mathscr{E}$ where the membership values of the vertices x_i and x_j are represented by $\sigma(x_i)$ and $\sigma(x_j)$ respectively, and the membership values of the edge between vertices x_i and x_j are indicated by $\hat{\mu}(x_i, x_j)$.

Definition 5. [4] An Intuitionistic Fuzzy Graph ($\mathscr{IFG}$) is of the form $\mathbb{G} = (\mathscr{V}, \mathscr{E})$, where

1. $\mathscr{V} = \{x_1, x_2, \ldots, x_n\}$, such that $\sigma_1 : \mathscr{V} \to [0,1]$ and $\sigma_2 : \mathscr{V} \to [0,1]$ denote the degree of membership and non-membership of the element $x_i \in \mathscr{V}$, respectively. It holds that $0 \le \sigma_1(x_i) + \sigma_2(x_i) \le 1$ for all $x_i \in \mathscr{V}$ $(i = 1, 2, \ldots, n)$.
2. $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, such that $\hat{\mu}_1 : \mathscr{V} \times \mathscr{V} \to [0,1]$ and $\hat{\mu}_2 : \mathscr{V} \times \mathscr{V} \to [0,1]$ denote the degree of membership and degree of non-membership of the edge (x_i, x_j), respectively.

$$\hat{\mu}_1(x_i, x_j) \le \min(\hat{\mu}_1(x_i), \hat{\mu}_1(x_j)),$$

$$\hat{\mu}_2(x_i, x_j) \le \max(\hat{\nu}_1(x_i), \hat{\nu}_1(x_j)),$$

and $0 \le \hat{\mu}_1(x_i, x_j) + \hat{\mu}_2(x_i, x_j) \le 1$ for all $(x_i, x_j) \in \mathscr{E}$.

Definition 6. [11] The complement of an $\mathscr{IFG}$ $\mathbb{G} = (\mathscr{V}, \mathscr{E})$ is an $\mathscr{IFG}$ $\overline{\mathbb{G}} = (\overline{(\sigma_1, \sigma_2)}, \overline{(\hat{\mu}_1, \hat{\mu}_2)})$, where $\overline{(\sigma_1, \sigma_2)} = (\sigma_1, \sigma_2)$ and

$$\bar{\hat{\mu}}_1(xy) = \sigma_1(x) \wedge \sigma_1(y) - \hat{\mu}_1(xy),$$

$$\bar{\hat{\mu}}_2(xy) = \sigma_2(x) \vee \sigma_2(y) - \hat{\mu}_2(xy), \quad \forall xy \in \mathscr{E}.$$

Definition 7. [4] The intuitionistic energies of two $\mathscr{IFG}$ $\mathbb{G}_1$ and $\mathbb{G}_2$ are described as:

$$E_{\mathscr{IFG}}(\mathbb{G}_1) = \sum_{i=1}^{n} \left(\hat{\mu}^2_{\mathbb{G}_1}(x_i) + \hat{\nu}^2_{\mathbb{G}_1}(x_i)\right) = \sum_{j=1}^{n} \lambda_j^2(\mathbb{G}_1)$$

and

$$E_{\mathscr{IFG}}(\mathbb{G}_2) = \sum_{i=1}^{n} \left(\hat{\mu}^2_{\mathbb{G}_2}(x_i) + \hat{\nu}^2_{\mathbb{G}_2}(x_i)\right) = \sum_{j=1}^{n} \lambda_j^2(\mathbb{G}_2).$$

The covariance of the $\mathscr{IFG}$ $\mathbb{G}_1$ and $\mathbb{G}_2$ is defined as:

$$C_{\mathscr{IFG}}(\mathbb{G}_1, \mathbb{G}_2) = \sum_{i=1}^{n} \left[\hat{\mu}_{\mathbb{G}_1}(x_i)\hat{\mu}_{\mathbb{G}_2}(x_i) + \hat{\nu}_{\mathbb{G}_1}(x_i)\hat{\nu}_{\mathbb{G}_2}(x_i)\right].$$

Therefore, the correlation coefficient measure of $\mathscr{IFG}$ $\mathbb{G}_1$ and $\mathbb{G}_2$ is given by

$$K_{\mathscr{IFG}}(\mathbb{G}_1, \mathbb{G}_2) = \frac{C_{\mathscr{IFG}}(\mathbb{G}_1, \mathbb{G}_2)}{E_{\mathscr{IFG}}(\mathbb{G}_1)E_{\mathscr{IFG}}(\mathbb{G}_2)}$$
$$= \frac{\sum_{i=1}^{n}\left[\hat{\mu}_{\mathbb{G}_1}(x_i)\hat{\mu}_{\mathbb{G}_2}(x_i) + \hat{\nu}_{\mathbb{G}_1}(x_i)\hat{\nu}_{\mathbb{G}_2}(x_i)\right]}{\left(\sum_{i=1}^{n}\left(\hat{\mu}^2_{\mathbb{G}_1}(x_i) + \hat{\nu}^2_{\mathbb{G}_1}(x_i)\right)\sum_{i=1}^{n}\left(\hat{\mu}^2_{\mathbb{G}_2}(x_i) + \hat{\nu}^2_{\mathbb{G}_2}(x_i)\right)\right)}.$$

Definition 8. [12] Let $\mathbb{G}_1 = \left((\sigma_1^{\mathbb{G}_1}, \sigma_2^{\mathbb{G}_1}), (\hat{\mu}_1^{\mathbb{G}_1}, \hat{\mu}_2^{\mathbb{G}_1})\right)$ and $\mathbb{G}_2 = \left((\sigma_1^{\mathbb{G}_2}, \sigma_2^{\mathbb{G}_2}), (\hat{\mu}_1^{\mathbb{G}_2}, \hat{\mu}_2^{\mathbb{G}_2})\right)$ be two $\mathscr{IFG}$. The maximal product of $\mathbb{G}_1$ and $\mathbb{G}_2$, denoted by $\mathbb{G}_1 \times_m \mathbb{G}_2 = (\mathscr{V}_1 \times_m \mathscr{V}_2, \mathscr{E}_1 \times_m \mathscr{E}_2)$, is defined as follows:

$$\mathscr{V}_1 \times_m \mathscr{V}_2 = \{(x_1, y_1) \mid x_1 \in \mathscr{V}_1 \text{and } y_1 \in \mathscr{V}_2\}.$$

$$\mathscr{E}_1 \times_m \mathscr{E}_2 = \{((x_1, y_1)(x_2, y_2)) \mid x_1 = x_2, y_1y_2 \in \mathscr{E}_2 \text{or } y_1 = y_2, x_1x_2 \in \mathscr{E}_1\}.$$

$$\sigma_1^{\mathbb{G}_1\times_m\mathbb{G}_2}(x_1, y_1) = \sigma_1^{\mathbb{G}_1}(x_1) \vee \sigma_1^{\mathbb{G}_2}(y_1),$$

$$\sigma_2^{\mathbb{G}_1\times_m\mathbb{G}_2}(x_1, y_1) = \sigma_2^{\mathbb{G}_1}(x_1) \wedge \sigma_2^{\mathbb{G}_2}(y_1),$$

for all $(x_1, y_1) \in \mathscr{V}_1 \times \mathscr{V}_2$.

For any pair of edges $((x_1, y_1)(x_2, y_2)) \in \mathscr{E}_1 \times_m \mathscr{E}_2$:

$$\hat{\mu}_1^{\mathbb{G}_1\times_m\mathbb{G}_2}((x_1, y_1)(x_2, y_2)) = \begin{cases} \sigma_1^{\mathbb{G}_1}(x_1) \vee \hat{\mu}_1^{\mathbb{G}_2}(y_1y_2), & \text{if } x_1 = x_2,\ y_1y_2 \in \mathscr{E}_2, \\ \hat{\mu}_1^{\mathbb{G}_1}(x_1x_2) \vee \sigma_1^{\mathbb{G}_2}(y_1), & \text{if } y_1 = y_2,\ x_1x_2 \in \mathscr{E}_1. \end{cases}$$

$$\hat{\mu}_2^{\mathbb{G}_1\times_m\mathbb{G}_2}((x_1, y_1)(x_2, y_2)) = \begin{cases} \sigma_2^{\mathbb{G}_1}(x_1) \wedge \hat{\mu}_2^{\mathbb{G}_2}(y_1y_2), & \text{if } x_1 = x_2,\ y_1y_2 \in \mathscr{E}_2, \\ \hat{\mu}_2^{\mathbb{G}_1}(x_1x_2) \wedge \sigma_2^{\mathbb{G}_2}(y_1), & \text{if } y_1 = y_2,\ x_1x_2 \in \mathscr{E}_1. \end{cases}$$

Definition 9. [11] The complement of the max product of two $\mathscr{IFG}$ $\mathbb{G}_1 = ((\sigma_1^{\mathbb{G}_1}, \sigma_2^{\mathbb{G}_1}), (\hat{\mu}_1^{\mathbb{G}_1}, \hat{\mu}_2^{\mathbb{G}_1}))$ and $\mathbb{G}_2 = ((\sigma_1^{\mathbb{G}_2}, \sigma_2^{\mathbb{G}_2}), (\hat{\mu}_1^{\mathbb{G}_2}, \hat{\mu}_2^{\mathbb{G}_2}))$ is an $\mathscr{IFG}$ $\overline{\mathbb{G}_1 \times_m \mathbb{G}_2} = ((\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2}), (\hat{\mu}_1^{\mathbb{G}_1} \times_m \hat{\mu}_1^{\mathbb{G}_2})(\hat{\mu}_2^{\mathbb{G}_1} \times_m \hat{\mu}_2^{\mathbb{G}_2}))$ on $\mathbb{G}^* = (\mathscr{V}, \mathscr{E})$, where $\overline{\mathscr{V}_1 \times_m \mathscr{V}_2} = \mathscr{V}_1 \times_m \mathscr{V}_2$ and

$$\overline{\mathscr{E}_1 \times_m \mathscr{E}_2} = \begin{Bmatrix} x_1 = x_2, y_1y_2 \in \mathscr{E}_2 \text{or} \\ y_1 = y_2, x_1x_2 \in \mathscr{E}_1 \text{or} \\ x_1x_2 \in \mathscr{E}_1, y_1y_2 \notin \mathscr{E}_2 \text{or} \\ x_1x_2 \notin \mathscr{E}_1, y_1y_2 \in \mathscr{E}_2 \text{or} \\ x_1x_2 \in \mathscr{E}_1, y_1y_2 \in \mathscr{E}_2 \text{or} \\ x_1x_2 \notin \mathscr{E}_1, y_1y_2 \notin \mathscr{E}_2 \end{Bmatrix}$$

The membership functions are defined as:

$$(\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_1, y_1) = \sigma_1^{\mathbb{G}_1}(x_1) \vee \sigma_1^{\mathbb{G}_2}(y_1),$$

$(\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2})(x_1, y_1) = \sigma_2^{\mathbb{G}_1}(x_1) \wedge \sigma_2^{\mathbb{G}_2}(y_1)$,

where $x_1 \in \mathscr{V}_1$ and $y_1 \in \mathscr{V}_2$.

$$\overline{\hat{\mu}_1^{\mathbb{G}_1} \times_m \hat{\mu}_1^{\mathbb{G}_2}}((x_1, y_1), (x_2, y_2))$$

$$= \begin{cases} (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_1, y_1) \wedge (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_2, y_2) - \hat{\mu}_1^{\mathbb{G}_1} \\ \quad \times_m \hat{\mu}_1^{\mathbb{G}_2}((x_1, y_1), (x_2, y_2)), \qquad \text{if } x_1 = x_2, y_1 y_2 \in \mathscr{E}_2 \\ (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_1, y_1) \wedge (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_2, y_2) - \hat{\mu}_1^{\mathbb{G}_1} \\ \quad \times_m \hat{\mu}_1^{\mathbb{G}_2}((x_1, y_1), (x_2, y_2)), \qquad \text{if } y_1 = y_2, x_1 x_2 \in \mathscr{E}_1 \\ (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_1, y_1) \wedge (\sigma_1^{\mathbb{G}_1} \\ \quad \times_m \sigma_1^{\mathbb{G}_2})(x_2, y_2), \qquad \text{otherwise} \end{cases}$$

$$\overline{\hat{\mu}_2^{\mathbb{G}_1} \times_m \hat{\mu}_2^{\mathbb{G}_2}}((x_1, y_1), (x_2, y_2))$$

$$= \begin{cases} (\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2})(x_1, y_1) \vee (\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2})(x_2, y_2) - \hat{\mu}_2^{\mathbb{G}_1} \\ \quad \times_m \hat{\mu}_2^{\mathbb{G}_2}((x_1, y_1), (x_2, y_2)), \qquad \text{if } x_1 = x_2, y_1 y_2 \in \mathscr{E}_2 \\ (\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2})(x_1, y_1) \vee (\sigma_2^{\mathbb{G}_1} \times_m \sigma_2^{\mathbb{G}_2})(x_2, y_2) - \hat{\mu}_2^{\mathbb{G}_1} \\ \quad \times_m \hat{\mu}_2^{\mathbb{G}_2}((x_1, y_1), (x_2, y_2)), \qquad \text{if } y_1 = y_2, x_1 x_2 \in \mathscr{E}_1 \\ (\sigma_1^{\mathbb{G}_1} \times_m \sigma_1^{\mathbb{G}_2})(x_1, y_1) \vee (\sigma_1^{\mathbb{G}_1} \\ \quad \times_m \sigma_1^{\mathbb{G}_2})(x_2, y_2), \qquad \text{otherwise} \end{cases}$$

3. $\mathscr{CRCs}$ of $\mathscr{IFG}$ using maximal product

In this section, we conducted an in-depth analysis of correlation and regression coefficients ($\mathscr{CRCs}$) in the setting of $\mathscr{IFG}$, employing the maximal product in particular. This technique allowed us to simulate complicated interactions between nodes and edges while keeping the system's uncertainty intact. Using the maximal product, we investigated how the structural aspects of $\mathscr{IFG}$ affect network activity, providing a more nuanced means of assessing connectedness, strength, and influence in the graph. Our findings highlight the predictive potential of $\mathscr{IFG}$ and their applications in real-world complex systems, such as healthcare analytics and decision-making processes. Furthermore, we investigated the usage of adjacency matrices containing membership and non-membership values in the calculation of energy and weight scores, which improve network structure interpretability.

3.1. Working procedure

Below is a working procedure to find the $\mathscr{CRCs}$ of $\mathscr{IFG}$ using Maximal Product.

Step 1: Let $\mathbb{M}_1 = (\sigma_1, \hat{\mu}_1)$ and $\mathbb{M}_2 = (\sigma_2, \hat{\mu}_2)$ be two $\mathscr{IFG}$ of $\mathbb{G}_1 = (\mathscr{V}_1, \mathscr{E}_1)$ and $\mathbb{G}_2 = (\mathscr{V}_2, \mathscr{E}_2)$, respectively. Construct a maximal product of $\mathbb{M}_1$ and $\mathbb{M}_2$, denoted as $\mathbb{M}_1 \times_m \mathbb{M}_2$. Then, find the adjacency matrix of $\mathbb{M}_1 \times_m \mathbb{M}_2$.

Step 2: Compute the energy $\mathcal{E}(\mathbb{G}_i^{(r)})$ of an adjacency matrix $\mathbb{G}_i^{(r)}$ where $\mathcal{E}(\mathbb{G}_i^{(r)}) = \sum_{i=1}^{n} |\lambda_i|$

Step 3: Compute the weight scores $\mathcal{W}(\mathbb{S}_i^{(r)})$ determined by $\mathcal{E}(\mathbb{G}_i^{(r)})$

$$\mathcal{W}(\mathbb{S}_i^{(r)}) = \left(\frac{\mathcal{E}(\mathbb{G}_1^{(r)})}{\sum_{r=1}^{2} \mathcal{E}(\mathbb{G}_i^{(r)})}, \frac{\mathcal{E}(\mathbb{G}_2^{(r)})}{\sum_{r=1}^{2} \mathcal{E}(\mathbb{G}_i^{(r)})} \right)$$

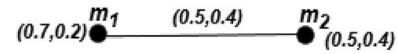


Fig. 1. Intuitionistic fuzzy graph $\mathbb{M}_1$.

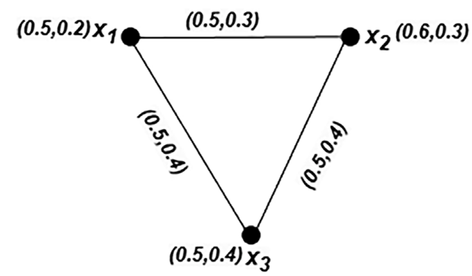


Fig. 2. Intuitionistic fuzzy graph $\mathbb{M}_2$.

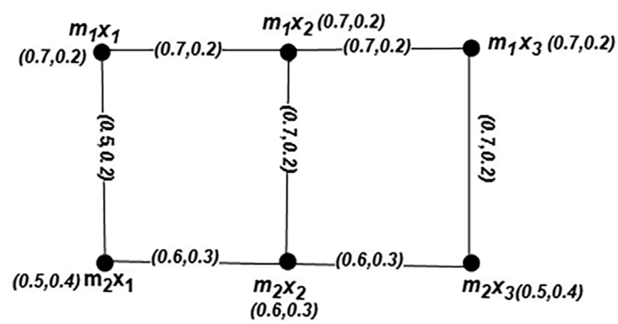


Fig. 3. Maximal product of Intuitionistic fuzzy graph $\mathbb{M}_1 \times_m \mathbb{M}_2$.

Step 4: Compute correlation coefficient $CC(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})$ between $\mathbb{G}_i^{(r)}$ and $\mathbb{S}_i^{(r)}$.

$$CC(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})) = \frac{\sum_{i=1}^{n} (\mathbb{T}_{\mathbb{G}_i^{(r)}}(t_i)\mathbb{T}_{\mathbb{S}_i^{(r)}}(t_i) + \mathbb{F}_{\mathbb{G}_i^{(r)}}(t_i)\mathbb{F}_{\mathbb{S}_i^{(r)}}(t_i))}{\sqrt{\sum_{i=1}^{n} (\mathbb{T}_{\mathbb{G}_i^{(r)}}(t_i)^2 + \mathbb{F}_{\mathbb{G}_i^{(r)}}(t_i)^2)} \sqrt{\sum_{i=1}^{n} (\mathbb{T}_{\mathbb{S}_i^{(r)}}(t_i)^2 + \mathbb{F}_{\mathbb{S}_i^{(r)}}(t_i)^2)}}$$

step 5: Find the regression coefficient $\mathbb{R}_{(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})}$ of $\mathbb{M}_1 \times_m \mathbb{M}_2$. The regression coefficient of $\mathbb{G}_i^{(r)}$ on $\mathbb{S}_i^{(r)}$ is defined as:

$$\mathbb{R}_{(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})} = \frac{\mathrm{Cov}(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})}{(\mathbb{G}_i^{(r)})^2}$$

The regression coefficient of $\mathbb{S}_i^{(r)}$ on $\mathbb{G}_i^{(r)}$ is defined as $\mathbb{R}_{(\mathbb{S}_i^{(r)}, \mathbb{G}_i^{(r)})} = \frac{\mathrm{Cov}(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})}{(\mathbb{S}_i^{(r)})^2}$

step 6: Calculate the relationship between the $\mathbb{CRC}s$

$$\mathcal{K}(\mathbb{S}_i^{(r)}, \mathbb{G}_i^{(r)}) = \sqrt{\mathbb{R}_{(\mathbb{G}_i^{(r)}, \mathbb{S}_i^{(r)})} \times \mathbb{R}_{(\mathbb{S}_i^{(r)}, \mathbb{G}_i^{(r)})}}$$

Example 1. Consider the $\mathscr{IFG}$ $\mathbb{M}_1$ and $\mathbb{M}_2$ as shown in Figs. 1 and 2 and their maximal product $\mathbb{M}_1 \times_m \mathbb{M}_2$ as shown in Fig. 3.

3.2. Illustration of $\mathbb{M}_1 \times_m \mathbb{M}_2$

Adjacency matrix of $\mathbb{M}_1 \times_m \mathbb{M}_2$

$$\mathbb{A}^{(r)}_{\hat{\mu}} = \begin{pmatrix} 0 & 0.7 & 0 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0.7 & 0 & 0.7 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0.7 \\ 0.7 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0.7 & 0 & 0.6 & 0 & 0.6 \\ 0 & 0 & 0.7 & 0 & 0.6 & 0 \end{pmatrix}$$

$$\mathbb{A}^{(r)}_{\hat{\nu}} = \begin{pmatrix} 0 & 0.2 & 0 & 0.2 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0.2 \\ 0.2 & 0 & 0 & 0 & 0.3 & 0 \\ 0 & 0.2 & 0 & 0.3 & 0 & 0.3 \\ 0 & 0 & 0.2 & 0 & 0.3 & 0 \end{pmatrix}$$

Energy of $\mathbb{M}_1 \times_m \mathbb{M}_2$

The energy $\mathcal{E}(\mathbb{G}_1^{(r)})$ of an adjacency matrix $\mathbb{G}_1^{(r)}$:

$$\mathcal{E}(\mathbb{G}_1^{(r)}) = \begin{pmatrix} 5.077 & 1.8142 \end{pmatrix}$$

Weight score of $\mathbb{M}_1 \times_m \mathbb{M}_2$

Score function of M1 star M2 The weight of the score function $\mathbb{S}_1^{(r)}$ is determined by $\mathcal{E}(\mathbb{G}_1^{(r)})$:

$$\mathcal{W}(\mathbb{S}_1^{(r)}) = \begin{pmatrix} 0.7367 & 0.263 \end{pmatrix}$$

Correlation coefficient of $\mathbb{M}_1 \times_m \mathbb{M}_2$

To compute correlation coefficient $CC(\mathbb{G}_1^{(r)}, \mathbb{S}_1^{(r)})$ between $\mathbb{G}_1^{(r)}$ and $\mathbb{S}_1^{(r)}$:

$$\mathrm{CC}(\mathbb{G}_1^{(r)}, \mathbb{S}_1^{(r)}) = \frac{4.2173}{4.2163} = 1.000$$

Regression coefficients of $\mathbb{M}_1 \times_m \mathbb{M}_2$

The regression coefficient measure of $\mathbb{G}_1^{(r)}$ in $\mathbb{S}_1^{(r)}$ is calculated as:

$$\mathbb{R}_{(\mathbb{G}_1^{(r)}, \mathbb{S}_1^{(r)})} = \frac{4.2173}{29.065} = 0.14509$$

The regression coefficient measure of $\mathbb{S}_1^{(r)})$ in $\mathbb{G}_1^{(r)}$ calculated as:

$$\mathbb{R}_{(\mathbb{S}_1^{(r)}), \mathbb{G}_1^{(r)})} = \frac{4.2173}{0.6118} = 6.8932$$

Relationship between $\mathscr{CRC}$*s of* $\mathbb{M}_1 \times_m \mathbb{M}_2$

The relationship between $\mathscr{CRC}s$ of $\mathbb{M}_1 \times_m \mathbb{M}_2$.

$$\mathcal{K}(\mathbb{G}_1^{(r)}, \mathbb{S}_1^{(r)}) = \sqrt{0.1450 \times 0.6118} = 1.000$$

4. $\mathscr{CRC}s$ of $\mathscr{IFLG}$ using maximal product

In this section, we present the maximal product of an $\mathscr{IFG}$, incorporating the concept of a line graph and its $\mathscr{CRC}s$, that characterise the relationship between the elements of the $\mathscr{IFG}$ and its corresponding line graph.

Definition Let $L(\mathbb{G}_1) = \left((\sigma^1_{L(\mathbb{G}_1)}, \sigma^2_{L(\mathbb{G}_1)}), (\hat{\mu}^1_{L(\mathbb{G}_1)}, \hat{\mu}^2_{L(\mathbb{G}_1)})\right)$ and $L(\mathbb{G}_2) = \left((\sigma^1_{L(\mathbb{G}_2)}, \sigma^2_{L(\mathbb{G}_2)}), (\hat{\mu}^1_{L(\mathbb{G}_2)}, \hat{\mu}^2_{L(\mathbb{G}_2)})\right)$ be two $\mathscr{IFLG}$, where $\mathbb{G}_1 = (\mathscr{V}_1, \mathscr{E}_1)$ and $\mathbb{G}_2 = (\mathscr{V}_2, \mathscr{E}_2)$ are the $\mathscr{IFG}$. The maximal product of the

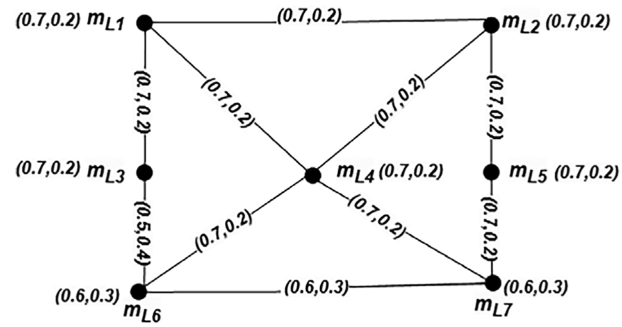


Fig. 4. Intuitionistic fuzzy Line graph $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$.

$\mathscr{IFLG}$ of $L(\mathbb{G}_1)$ and $L(\mathbb{G}_2)$, denoted by $L(\mathbb{G}_1) \times_m L(\mathbb{G}_2)$, is defined as follows:

$$\mathscr{V}_{L(\mathbb{G}_1)} \times_m \mathscr{V}_{L(\mathbb{G}_2)} = \{(\mathscr{s}_x, \mathscr{s}_y) \mid \mathscr{s}_x \in \mathscr{V}_{L(\mathbb{G}_1)}, \mathscr{s}_y \in \mathscr{V}_{L(\mathbb{G}_2)}\}.$$

$$\mathscr{E}_{L(\mathbb{G}_1)} \times_m \mathscr{E}_{L(\mathbb{G}_2)} = \{((\mathscr{s}_{x_1}, \mathscr{s}_{y_1})(\mathscr{s}_{x_2}, \mathscr{s}_{y_2})) \mid \mathscr{s}_{x_1} = \mathscr{s}_{x_2}, \mathscr{s}_{y_1}\mathscr{s}_{y_2} \in \mathscr{E}_{L(\mathbb{G}_2)} \text{or } \mathscr{s}_{y_1} = \mathscr{s}_{y_2}, \mathscr{s}_{x_1}\mathscr{s}_{x_2} \in \mathscr{E}_{L(\mathbb{G}_1)}\}.$$

$$\sigma^1_{L(\mathbb{G}_1)\times_m L(\mathbb{G}_2)}(\mathscr{s}_{x_1}, \mathscr{s}_{y_1}) = \sigma^1_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}) \vee \sigma^1_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}),$$

$$\sigma^2_{L(\mathbb{G}_1)\times_m L(\mathbb{G}_2)}(\mathscr{s}_{x_1}, \mathscr{s}_{y_1}) = \sigma^2_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}) \wedge \sigma^2_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}),$$

for all $(\mathscr{s}_{x_1}, \mathscr{s}_{y_1}) \in \mathscr{V}_{L(\mathbb{G}_1)} \times \mathscr{V}_{L(\mathbb{G}_2)}$.

For any pair of edges $((\mathscr{s}_{x_1}, \mathscr{s}_{y_1})(\mathscr{s}_{x_2}, \mathscr{s}_{y_2})) \in \mathscr{E}_{L(\mathbb{G}_1)} \times_m \mathscr{E}_{L(\mathbb{G}_2)}$:

$$\sigma^1_{L(\mathbb{G}_1)\times_m L(\mathbb{G}_2)}((\mathscr{s}_{x_1}, \mathscr{s}_{y_1})(\mathscr{s}_{x_2}, \mathscr{s}_{y_2})) = \begin{cases} \sigma^1_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}) \vee \hat{\mu}^1_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}\mathscr{s}_{y_2}), & \text{if } \mathscr{s}_{x_1} = \mathscr{s}_{x_2}, \mathscr{s}_{y_1}\mathscr{s}_{y_2} \in \mathscr{E}_{L(\mathbb{G}_2)}, \\ \hat{\mu}^1_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}\mathscr{s}_{x_2}) \vee \sigma^1_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}), & \text{if } \mathscr{s}_{y_1} = \mathscr{s}_{y_2}, \mathscr{s}_{x_1}\mathscr{s}_{x_2} \in \mathscr{E}_{L(\mathbb{G}_1)}. \end{cases}$$

$$\sigma^2_{L(\mathbb{G}_1)\times_m L(\mathbb{G}_2)}((\mathscr{s}_{x_1}, \mathscr{s}_{y_1})(\mathscr{s}_{x_2}, \mathscr{s}_{y_2})) = \begin{cases} \sigma^2_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}) \wedge \hat{\mu}^2_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}\mathscr{s}_{y_2}), & \text{if} \mathscr{s}_{x_1} = \mathscr{s}_{x_2}, \mathscr{s}_{y_1}\mathscr{s}_{y_2} \in \mathscr{E}_{L(\mathbb{G}_2)}, \\ \hat{\mu}^2_{L(\mathbb{G}_1)}(\mathscr{s}_{x_1}\mathscr{s}_{x_2}) \wedge \sigma^2_{L(\mathbb{G}_2)}(\mathscr{s}_{y_1}), & \text{if } \mathscr{s}_{y_1} = \mathscr{s}_{y_2}, \mathscr{s}_{x_1}\mathscr{s}_{x_2} \in \mathscr{E}_{L(\mathbb{G}_1)}. \end{cases}$$

Example 2. Consider the $\mathscr{IFG}$ of maximal product $\mathbb{M}_1 \times_m \mathbb{M}_2$ as shown in Fig. 3 and their $\mathscr{IFLG}$.

Adjacency matrix of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

$$\mathbb{A}^{L}_{\hat{\mu}} = \begin{pmatrix} 0 & 0.7 & 0.7 & 0.7 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0.7 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0.7 & 0 \\ 0.7 & 0.7 & 0 & 0 & 0 & 0.7 & 0.7 \\ 0 & 0.7 & 0 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0.7 & 0.7 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0.7 & 0.7 & 0.6 & 0 \end{pmatrix}$$

$$\mathbb{A}^{L}_{\hat{\nu}} = \begin{pmatrix} 0 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.2 & 0.2 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0.2 & 0 \\ 0.2 & 0.2 & 0 & 0 & 0 & 0.2 & 0.2 \\ 0 & 0.2 & 0 & 0 & 0 & 0 & 0.2 \\ 0 & 0 & 0.2 & 0.2 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0.2 & 0.2 & 0.3 & 0 \end{pmatrix}$$

Energy of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

The energy $\mathcal{E}(\mathbb{G}_{\mathbb{L}_1}^{(r)})$ of an adjacency matrix $\mathbb{G}_{\mathbb{L}_1}^{(r)}$:

$$\mathcal{E}(\mathbb{G}_{\mathbb{L}_1}^{(r)}) = \begin{pmatrix} 6.8764 & 2.1362 \end{pmatrix}$$

Weight score of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

Score function of M1 star M2 The weight of the score function $\mathbb{S}_{\mathbb{L}_1}^{(r)}$ is determined by $\mathcal{E}(\mathbb{G}_{\mathbb{L}_1}^{(r)})$:

$$\mathcal{W}_{\mathcal{L}}(\mathbb{S}_{\mathbb{L}_1}^{(r)}) = \begin{pmatrix} 0.7629 & 0.2370 \end{pmatrix}$$

Correlation coefficient of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

To compute correlation coefficient $CC(\mathbb{G}_{\mathbb{L}_1}^{(r)}, \mathbb{S}_{\mathbb{L}_1}^{(r)})$ between $\mathbb{G}_{\mathbb{L}_1}^{(r)}$ and $\mathbb{S}_{\mathbb{L}_1}^{(r)}$:

$$\mathrm{CC}(\mathbb{G}_{\mathbb{L}_1}^{(r)}, \mathbb{S}_{\mathbb{L}_1}^{(r)}) = \frac{2.655}{2.652} = 1.000$$

Regression coefficients of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

The regression coefficient measure of $\mathbb{G}_{\mathbb{L}_1}^{(r)}$ on $\mathbb{S}_{\mathbb{L}_1}^{(r)}$ is calculated as:

$$\mathbb{R}_{(\mathbb{G}_{\mathbb{L}_1}^{(r)}, \mathbb{S}_{\mathbb{L}_1}^{(r)})} = \frac{2.655}{215.748} = 0.01230$$

The regression coefficient measure of $\mathbb{S}_{\mathbb{L}_1}^{(r)})$ on $\mathbb{G}_{\mathbb{L}_1}^{(r)}$ is computed as:

$$\mathbb{R}_{(\mathbb{S}_{\mathbb{L}_1}^{(r)}), \mathbb{G}_{\mathbb{L}_1}^{(r)})} = \frac{2.655}{0.03265} = 81.316$$

Relationship between $\mathscr{CRC}$*s of* $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$

We defined the relationship between $\mathscr{CRCs}$ of $\mathbb{M}_{\mathbb{L}1} \times_m \mathbb{M}_{\mathbb{L}2}$.

$$\mathcal{K}(\mathbb{G}_{\mathbb{L}_1}^{(r)}, \mathbb{S}_{\mathbb{L}_1}^{(r)}) = \sqrt{0.0123 \times 81.316} = 1.000$$

5. $\mathscr{CRCs}$ of complement of $\mathscr{IFLG}$ using the maximal product

In this section, we present the concept of the complement of the maximal product of an $\mathscr{IFLG}$ and provide calculations for the $\mathscr{CRCs}$. We define the complement of the maximal product of an $\mathscr{IFLG}$. The complement of a maximal product graph is useful for studying structural and connectivity changes when edge interactions are reversed. The complement allows us to discuss how dominance, connectedness, and general graph attributes are influenced in a fuzzy framework.

Furthermore, we present step-by-step calculations of the ($\mathscr{CRCs}$) from the complement of the maximal product of $\mathscr{IFLG}$. This includes the computation of adjacency matrices, energy values, weight scores, and other relevant statistical measurements such as correlation and regression coefficients. These factors help quantify the impact of complement structures on graph-based prediction models. The results of these computations provide insights into the relative performance of the initial maximal product and its complement under various decision dilemmas.

Example 3. Consider a complement of the maximal product of an $\mathscr{IFLG}$ Fig. 4 using definition as shown in Fig. 5.

Adjacency matrix of $\mathbb{M}_{\mathbb{L}_1}^{\ c} \times_m \mathbb{M}_{\mathbb{L}_2}^{\ c}$

$$\mathbb{A}_{\hat{\mu}}^{c(L)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.7 & 0.6 & 0.6 \\ 0 & 0 & 0.7 & 0 & 0 & 0.6 & 0.6 \\ 0 & 0.7 & 0 & 0.7 & 0.7 & 0 & 0.6 \\ 0 & 0 & 0.7 & 0 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0.7 & 0.7 & 0 & 0.7 & 0 \\ 0.6 & 0.6 & 0 & 0 & 0.7 & 0 & 0 \\ 0.6 & 0.6 & 0.6 & 0 & 0 & 0 & 0 \end{pmatrix}$$

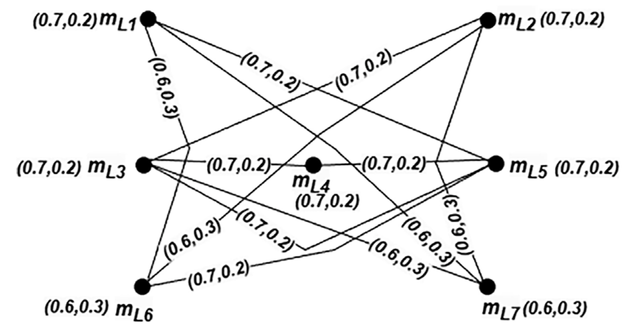


Fig. 5. Complement Intuitionistic fuzzy Line graph $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$.

$$\mathbb{A}_{\hat{\nu}}^{c(L)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.2 & 0.3 & 0.3 \\ 0 & 0 & 0.2 & 0 & 0 & 0.3 & 0.3 \\ 0 & 0.2 & 0 & 0.2 & 0.2 & 0 & 0.3 \\ 0 & 0 & 0.2 & 0 & 0.2 & 0 & 0 \\ 0.2 & 0 & 0.2 & 0.2 & 0 & 0.2 & 0 \\ 0.3 & 0.3 & 0 & 0 & 0.2 & 0 & 0 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Energy of $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$

The energy $\mathcal{E}(\mathbb{G}_{\mathbb{L}1}^{c\,(r)})$ of an adjacency matrix $\mathbb{G}_{\mathbb{L}1}^{c\,(r)}$:

$$\mathcal{E}(\mathbb{G}_{\mathbb{L}1}^{c\,(r)}) = \begin{pmatrix} 6.8752 & 2.5709 \end{pmatrix}$$

Weight score of $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$

The weight of the score function $\mathbb{S}_{\mathbb{L}1}^{c\,(r)}$ is determined by $\mathcal{E}(\mathbb{G}_{\mathbb{L}1}^{c\,(r)})$:

$$\mathcal{W}_{\mathcal{L}}^{c}(\mathbb{S}_{\mathbb{L}1}^{c\,(r)}) = \begin{pmatrix} 0.7278 & 0.2721 \end{pmatrix}$$

Correlation coefficient of $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$

To compute correlation coefficient $CC(\mathbb{G}_{\mathbb{L}1}^{c\,(r)}, \mathbb{S}_{\mathbb{L}1}^{c\,(r)})$ between $\mathbb{G}_{\mathbb{L}1}^{c\,(r)}$ and $\mathbb{S}_{\mathbb{L}1}^{c\,(r)}$:

$$\mathrm{CC}(\mathbb{G}_{\mathbb{L}1}^{c\,(r)}, \mathbb{S}_{\mathbb{L}1}^{c\,(r)}) = \frac{3.5000}{5.7025} = 0.61$$

Regression coefficients of $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$

The regression coefficient measure of $\mathbb{G}_{\mathbb{L}1}^{c\,(r)}$ on $\mathbb{S}_{\mathbb{L}1}^{c\,(r)}$ is calculated as:

$$\mathbb{R}^{c}_{(\mathbb{G}_{\mathbb{L}1}^{c\,(r)}, \mathbb{S}_{\mathbb{L}1}^{c\,(r)})} = \frac{3.5000}{53.877} = 0.0649$$

The regression coefficient measure of $\mathbb{S}_{\mathbb{L}1}^{c\,(r)})$ on $\mathbb{G}_{\mathbb{L}1}^{c\,(r)}$ computed as:

$$\mathbb{R}^{c}_{(\mathbb{S}_{\mathbb{L}1}^{c\,(r)}), \mathbb{G}_{\mathbb{L}1}^{c\,(r)})} = \frac{3.5000}{0.7769} = 4.5050$$

Relationship between $\mathscr{CRC}$*s of* $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$

We defined the relationship between $\mathscr{CRCs}$ of $\mathbb{M}_{\mathbb{L}1}^{c} \times_m \mathbb{M}_{\mathbb{L}2}^{c}$.

$$\mathcal{K}(\mathbb{G}_{\mathbb{L}_1}^{c(r)}, \mathbb{S}_{\mathbb{L}_1}^{c(r)}) = \sqrt{0.0649 \times 4.5050} = 0.5406$$

Theorem 1. *The maximal product of* $\mathscr{IFG}$ *and its complement* ($\mathscr{IFG}^{c}$) *satisfies:*

$$\mathbb{E}(\mathscr{IFG} \times_{\max} \mathscr{IFG}^{c}) = \emptyset$$

if and only if the maximal product of $\mathscr{IFG}$ *is a complete* $\mathscr{IFG}$.

Proof. Let $\mathcal{G} = (\mathbb{V}, \mathbb{E}, \hat{\mu}, \hat{\nu})$ be an $\mathcal{IFG}$ where $\hat{\mu} : \mathbb{E} \to [0,1]$ represents the membership function and $\hat{\nu} : \mathbb{E} \to [0,1]$ represents the non-membership function. The complement of maximal product of $\mathcal{IFG}^c$ is defined such that:

$$\hat{\mu}^c(e) = 1 - \hat{\mu}(e), \quad \hat{\nu}^c(e) = 1 - \hat{\nu}(e), \quad \forall e \in \mathbb{E}.$$

The maximal product of $\mathcal{IFG}$ and $\mathcal{IFG}^c$ is defined with the vertex set: $\mathbb{V}(\mathcal{IFG} \times_{\max} \mathcal{IFG}^c) = \mathbb{V} \times_{\max} \mathbb{V}$, there exists an edge between the two vertices (u_1, v_1) and (u_2, v_2) in $\mathcal{IFG} \times_{\max} \mathcal{IFG}^c$ if and only if:

$$(u_1, u_2) \in \mathbb{E}(\mathcal{IFG}) \quad \text{or} \quad (v_1, v_2) \in \mathbb{E}(\mathcal{IFG}^c).$$

Assume $\mathcal{IFG}$ is a complete $\mathcal{IFG}$. Then, every pair of distinct vertices in $\mathcal{IFG}$ is connected by an edge: $\mathbb{E}(\mathcal{IFG}) = \{(x, y) \mid x, y \in \mathbb{V}, x \neq y\}$. Since $\mathcal{IFG}^c$ is its complement, it contains no edges: $\mathcal{E}(\mathcal{IFG}^c) = \emptyset$. Now, there is an edge between the maximal product (u_1, v_1) and (u_2, v_2) if $(u_1, u_2) \in \mathbb{E}(\mathcal{IFG})$, or $(v_1, v_2) \in \mathbb{E}(\mathcal{IFG}^c)$. Since $\mathbb{E}(\mathcal{IFG}^c) = \emptyset$, the converse is true. However, since $\mathcal{IFG}$ is complete, for any pair (u_1, u_2), an edge always exists in $\mathcal{IFG}$. Hence $\mathbb{E}(\mathcal{IFG} \times_{\max} \mathcal{IFG}^c) = \emptyset$.

Conversely, let $\mathbb{E}(\mathcal{IFG} \times_{\max} \mathcal{IFG}^c) = \emptyset$. This means no edges exist in the maximal product of $\mathcal{IFG}$ so for every pair (u_1, v_1) and (u_2, v_2):

$$(u_1, u_2) \notin \mathbb{E}(\mathcal{IFG}) \quad \text{and} \quad (v_1, v_2) \notin \mathbb{E}(\mathcal{IFG}^c).$$

This condition holds for all vertex pairs if $\mathcal{IFG}$ is complete, $\mathbb{E}(\mathcal{IFG}^c) = \emptyset$.

Thus, $\mathbb{E}(\mathcal{IFG} \times_{\max} \mathcal{IFG}^c) = \emptyset$ if and only if $\mathcal{IFG}$ is a complete $\mathcal{IFG}$, proving the theorem.

Theorem 2. *The maximal product of two connected intuitionistic fuzzy graphs is always a connected intuitionistic fuzzy graph.*

Proof. Let $\mathcal{G}_1 = (\mathbb{V}_1, \mathbb{E}_1, \hat{\mu}_1, \hat{\nu}_1)$ and $\mathcal{G}_2 = (\mathbb{V}_2, \mathbb{E}_2, \hat{\mu}_2, \hat{\nu}_2)$ be two connected $\mathcal{IFG}$, where: $\hat{\mu}_1 : \mathbb{E}_1 \to [0,1]$ and $\hat{\nu}_1 : \mathbb{E}_1 \to [0,1]$ and $\hat{\mu}_2 : \mathbb{E}_2 \to [0,1]$ and $\hat{\nu}_2 : \mathbb{E}_2 \to [0,1]$ represent the membership and non-membership functions in $\mathcal{G}_1$and and $\mathcal{G}_2$ respectively. The underlying crisp graphs of $\mathcal{G}_1$ and $\mathcal{G}_2$ are denoted by $\mathcal{G}_1^* = (\mathbb{V}_1, \mathbb{E}_1)$ and $\mathcal{G}_2^* = (\mathbb{V}_2, \mathbb{E}_2)$, respectively.

Let $\mathbb{V}_1 = \{u_1, u_2, \ldots, u_m\}$ and $\mathbb{V}_2 = \{v_1, v_2, \ldots, v_n\}$. Since $\mathcal{G}_1$ and $\mathcal{G}_2$ are connected, we have:

$$\hat{\mu}_1^{\max}(u_i, u_j) > 0, \quad \forall u_i, u_j \in \mathbb{V}_1$$

$$\hat{\mu}_2^{\max}(v_i, v_j) > 0, \quad \forall v_i, v_j \in \mathbb{V}_2$$

Consider the maximal product of $\mathcal{G}_1$ and $\mathcal{G}_2$, denoted as $\mathcal{G} = (\mathbb{V}, \mathbb{E}, \hat{\mu}, \hat{\nu})$. The vertex set of $\mathcal{G}$ is given by:

$$\mathbb{V} = \mathbb{V}_1 \times \mathbb{V}_2 = \{(u_i, v_j) \mid u_i \in \mathbb{V}_1, v_j \in \mathbb{V}_2\}$$

$$\hat{\mu}((u_i, v_j), (u_k, v_l)) = \max\{\min(\hat{\mu}_1(u_i, u_k), \hat{\mu}_2(v_j, v_l))\}$$

$$\hat{\nu}((u_i, v_j), (u_k, v_l)) = \min\{\max(\hat{\nu}_1(u_i, u_k), \hat{\nu}_2(v_j, v_l))\}$$

Now, consider the m subgraphs of $\mathcal{G}$, where each subgraph has vertex sets:

$$\mathbb{V}_i = \{(u_i, v_1), (u_i, v_2), \ldots, (u_i, v_n)\}, \quad i = 1, 2, \ldots, m.$$

Every subgraph is connected by the same first component, u_i. Since $\mathcal{G}_2$ is connected, every v_j is adjacent to at least one other vertex in $\mathbb{V}_2$.

Similarly, since $\mathcal{G}_1$ is connected, each u_i is adjacent to at least one vertex in $\mathbb{V}_1$. Thus, there exists at least one edge between any two subgraphs, ensuring that:

$$\hat{\mu}((u_i, v_j), (u_k, v_l)) > 0, \quad \forall (u_i, v_j), (u_k, v_l) \in \mathbb{E}.$$

Therefore, $\mathcal{G}$ is a connected $\mathcal{IFG}$.

Theorem 3. *Let $\mathcal{G}_1 = (\mathbb{V}_1, \mathbb{E}_1, \hat{\mu}_1, \hat{\nu}_1)$ be a partially regular intuitionistic fuzzy graph and $\mathcal{G}_2 = (\mathbb{V}_2, \mathbb{E}_2, \hat{\mu}_2, \hat{\nu}_2)$ be an intuitionistic fuzzy graph such that*

$$\sigma_1 \leq \hat{\mu}_2, \quad \textit{and} \quad \sigma_2 \textit{is a constant function with value } c.$$

Then, the maximal product $\mathcal{G}_1 \times_M \mathcal{G}_2$ is regular if and only if $\mathcal{G}_2$ is regular.

Proof. Let $\mathcal{G}_1 = (\mathbb{V}_1, \mathbb{E}_1, \hat{\mu}_1, \hat{\nu}_1)$ and $\mathcal{G}_2 = (\mathbb{V}_2, \mathbb{E}_2, \hat{\mu}_2, \hat{\nu}_2)$ be two $\mathcal{IFG}$. The maximal product of $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as:

$$\mathbb{V}(\mathcal{G}_1 \times_M \mathcal{G}_2) = \mathbb{V}_1 \times \mathbb{V}_2.$$

The edge set is determined by the condition that there is an edge between (u_1, v_1) and (u_2, v_2) if and only if:

$$(u_1 = u_2) \text{and } (v_1, v_2) \in \mathbb{E}_2, \quad \text{or} \quad (v_1 = v_2) \text{and } (u_1, u_2) \in \mathbb{E}_1.$$

Since $\mathcal{G}_1$ is partially regular, every vertex $u \in \mathbb{V}_1$ has a constant degree in its regular subgraph. Given that $\sigma_1 \leq \hat{\mu}_2$, the degree of any vertex (u, v) in $\mathcal{G}_1 \times_M \mathcal{G}_2$ depends on the degrees of u in $\mathcal{G}_1$ and v in $\mathcal{G}_2$.

Now, assume that $\mathcal{G}_1 \times_M \mathcal{G}_2$ is regular. This means that for any two vertices (u_1, v_1) and (u_2, v_2), their degrees must be equal:

$$d_{\mathcal{G}_1 \times_M \mathcal{G}_2}(u_1, v_1) = d_{\mathcal{G}_1 \times_M \mathcal{G}_2}(u_2, v_2).$$

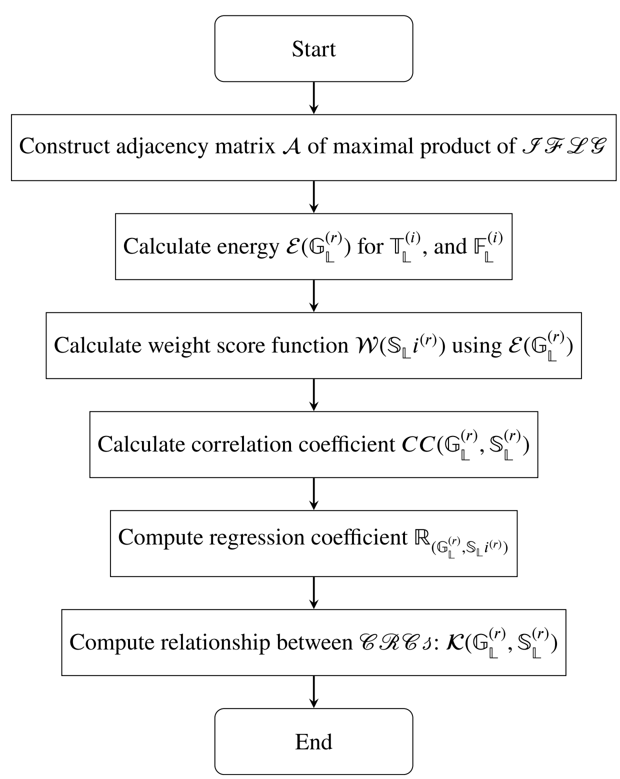
All vertices in $\mathbb{V}_1$ have the same degree contribution since σ_2 is a constant function with value of c.

Thus, for $\mathcal{G}_1 \times_M \mathcal{G}_2$ to be regular, the degree of each vertex $v \in \mathbb{V}_2$ in $\mathcal{G}_2$ must likewise be constant, i.e. $\mathcal{G}_2$ must be regular.

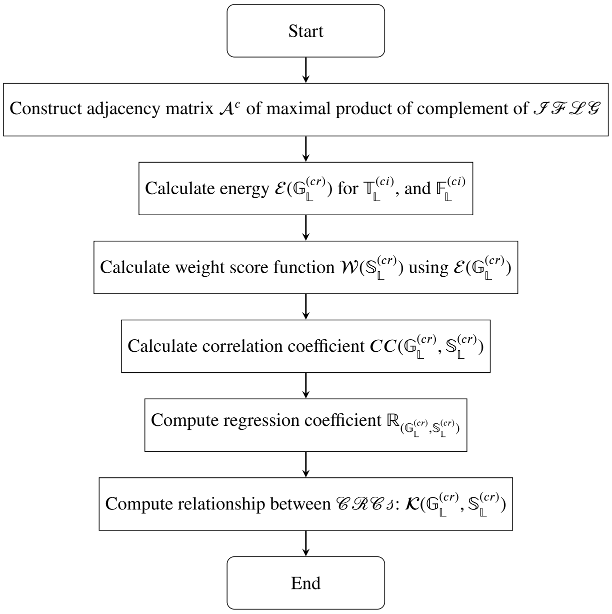
Conversely, if $\mathcal{G}_2$ is regular, then the degree $d_{\mathcal{G}_2}(v)$ is constant for every $v \in \mathbb{V}_2$. The degree of each $u \in \mathbb{V}_1$ in its regular subgraph is constant. Since $\mathcal{G}_1$ is partially regular. As a result, $\mathcal{G}_1 \times_M \mathcal{G}_2$ is a regular $\mathcal{IFG}$ with a constant degree for each vertex.

Hence, the maximal product of $\mathcal{G}_1 \times_M \mathcal{G}_2$ is regular if and only if $\mathcal{G}_2$ is regular.

5.1. Flowchart for $\mathcal{CRCS}$ of $\mathcal{IFLG}$



5.2. Flowchart for $\mathscr{CRC}s$ of complement of $\mathscr{IFLG}$



6. Applications

The applications of $\mathscr{CRC}s$ in the analysis of line $\mathscr{IFG}$ and their complements of line $\mathscr{IFG}$ offer researchers a powerful tool for exploring and modelling complex relationships between variables. This methodology is particularly valuable in healthcare research, especially in predicting diabetes risk, as it provides a deeper understanding of the inter-dependencies between factors such as glucose levels, blood pressure, BMI, and other key health indicators. Researchers can use $\mathscr{IFLG}$ to visually and mathematically depict positive correlations between attributes. For instance, a high correlation between glucose and BMI helps confirm that patients with elevated glucose levels are likelier to have a higher BMI, which is crucial for identifying primary risk factors. Regression coefficients on the $\mathscr{IFLG}$ also aid in building predictive models, allowing researchers to predict outcomes like diabetes risk based on key attributes. When the regression coefficient between glucose and the outcome is high, glucose is a strong predictor, helping researchers design algorithms for early diagnosis and intervention. Furthermore, analysing direct relationships through the line $\mathscr{IFG}$ allows researchers to study causality, such as how higher glucose levels lead to a higher likelihood of developing diabetes, and therefore focus on managing glucose levels in at-risk populations. The complement of $\mathscr{IFLG}$ enables researchers to identify indirect relationships, which can be equally important. Sometimes, the connection between attributes like BMI and blood pressure may not be straightforward. The complements of $\mathscr{IFLG}$ highlight instances where expected relationships break down, such as when a person with a high BMI does not show elevated blood pressure, helping researchers understand the anomalies and exceptions. Moreover, the complement $\mathscr{IFLG}$ uncovers complex interactions that might not be immediately visible in the original graph. For example, attributes like skin thickness, which may show weak relationships in $\mathscr{IFLG}$, can reveal hidden risk factors when analysed in the complement of $\mathscr{IFLG}$. This deeper exploration of indirect relationships can uncover new or overlooked risk factors for conditions like diabetes, improving understanding of the disease's complex aetiology. Anomaly detection is another significant application of the complement of line $\mathscr{IFG}$. In healthcare datasets, cases often deviate from the expected patterns, such as patients with high glucose but normal BMI. Using non-membership values in the complement of $\mathscr{IFLG}$, researchers can identify these anomalies, which may point to previously unrecognised risk factors or conditions. This can lead to the early detection of new patterns or conditions, such as individuals with low insulin levels and high BMI exhibiting higher diabetes risk, which might not be immediately apparent through standard correlation analysis. Table 2 provides the dataset of the five diabetes patients.

Table 2
Intuitionistic fuzzy table for the given dataset.

Row	BMI $(\hat{\mu}, \hat{\nu})$	Glucose $(\hat{\mu}, \hat{\nu})$	Blood Pressure $(\hat{\mu}, \hat{\nu})$	Skin Thickness $(\hat{\mu}, \hat{\nu})$	Insulin $(\hat{\mu}, \hat{\nu})$
1	(1.0, 0.0)	(0.975, 0.025)	(0.8, 0.2)	(0.4, 0.6)	(0.7, 0.3)
2	(0.5, 0.5)	(0.8, 0.2)	(0.6, 0.4)	(0.3, 0.7)	(0.0, 1.0)
3	(0.7, 0.3)	(0.9, 0.1)	(0.7, 0.3)	(0.2, 0.8)	(0.6, 0.4)
4	(0.2, 0.8)	(0.85, 0.15)	(0.4, 0.6)	(0.0, 1.0)	(0.0, 1.0)
5	(0.2, 0.8)	(0.95, 0.05)	(0.6, 0.4)	(0.5, 0.5)	(0.0, 1.0)

$$\text{membership values} = \begin{pmatrix} 1.0 & 0.975 & 0.8 & 0.4 & 0.7 \\ 0.5 & 0.8 & 0.6 & 0.3 & 0.0 \\ 0.7 & 0.9 & 0.7 & 0.2 & 0.6 \\ 0.2 & 0.85 & 0.4 & 0.0 & 0.0 \\ 0.2 & 0.95 & 0.6 & 0.5 & 0.0 \end{pmatrix}$$

$$\text{non- membership values} = \begin{pmatrix} 0.0 & 0.025 & 0.2 & 0.6 & 0.3 \\ 0.5 & 0.2 & 0.4 & 0.7 & 1.0 \\ 0.3 & 0.1 & 0.3 & 0.8 & 0.4 \\ 0.8 & 0.15 & 0.6 & 1.0 & 1.0 \\ 0.8 & 0.05 & 0.4 & 0.5 & 1.0 \end{pmatrix}$$

To compute the correlation coefficient between membership and non-membership functions using energy and weight score:

$$= \frac{3.191}{3.191} = 1.00$$

Relationship between $\mathscr{CRC}$s of membership and non-membership functions:

$$\sqrt{0.156 \times 6.383} = 0.99$$

The $\mathscr{IFLG}$ allows researchers to identify key predictors of health outcomes, such as BMI, glucose, blood pressure, skin thickness, and insulin. This has been efficiently represented using heatmaps that show the significance of features and their relationships as shown in Figs. 6 and 7. Analysing the $\mathscr{CRC}s$ of $\mathscr{IFLG}$ and their complement, is highly beneficial for optimising predictive models. Researchers can prioritise the most influential features, leading to more accurate predictive models. The complement of $\mathscr{IFLG}$ is valuable in assessing secondary predictors that may not be immediately apparent. For example, suppose that skin thickness and insulin function appear weakly related to the outcome in $\mathscr{IFLG}$ but show a stronger relationship in the complement. Researchers can refine their predictive models to incorporate these secondary influences. This iterative model evaluation and adjustment process ensures that predictive models are robust and can adapt to new insights, improving prediction accuracy in real-world scenarios. The visual representation provided by $\mathscr{IFG}$ helps researchers better understand complex data relationships. These graphs show both the strengths (membership) and weaknesses (nonmembership) of connections between attributes, providing an intuitive and comprehensive framework for analysis. The combination of $\mathscr{IFLG}$ and its complement provides researchers with a complete understanding of the data, showcasing both significant direct relationships and subtle indirect interactions.

Finally, the insights gained from the correlation and regression analysis of $\mathscr{IFLG}$ have significant implications for healthcare interventions. By understanding both strong and weak relationships between attributes, researchers can design more targeted and effective healthcare

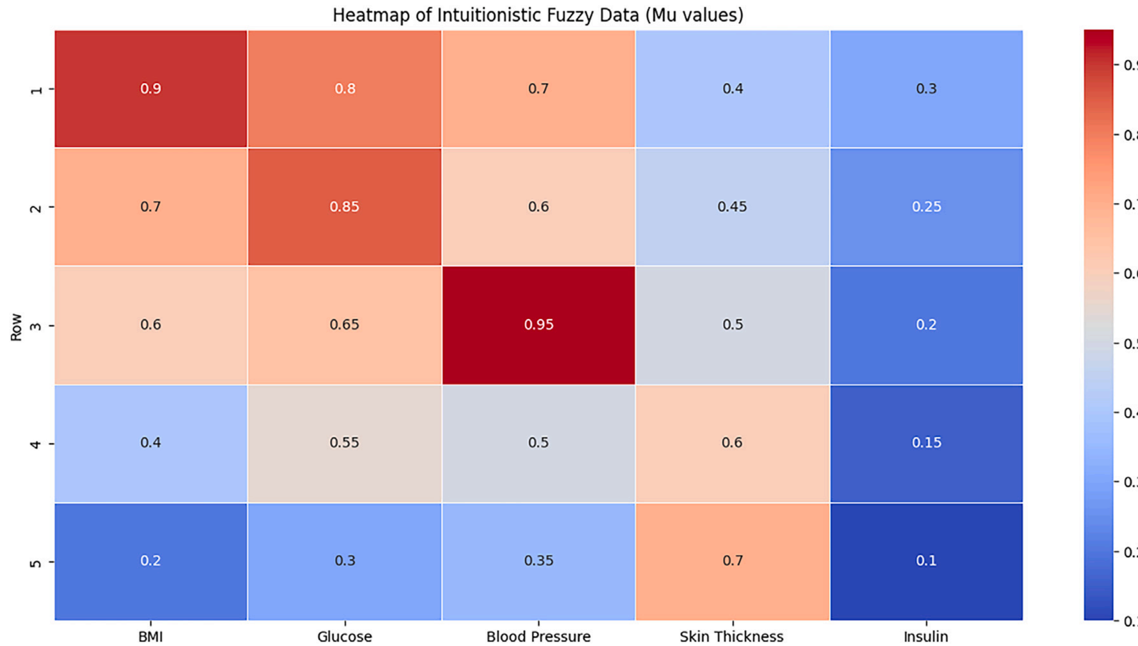


Fig. 6. Heatmap of $\mathcal{IFLG}$ using membership values.

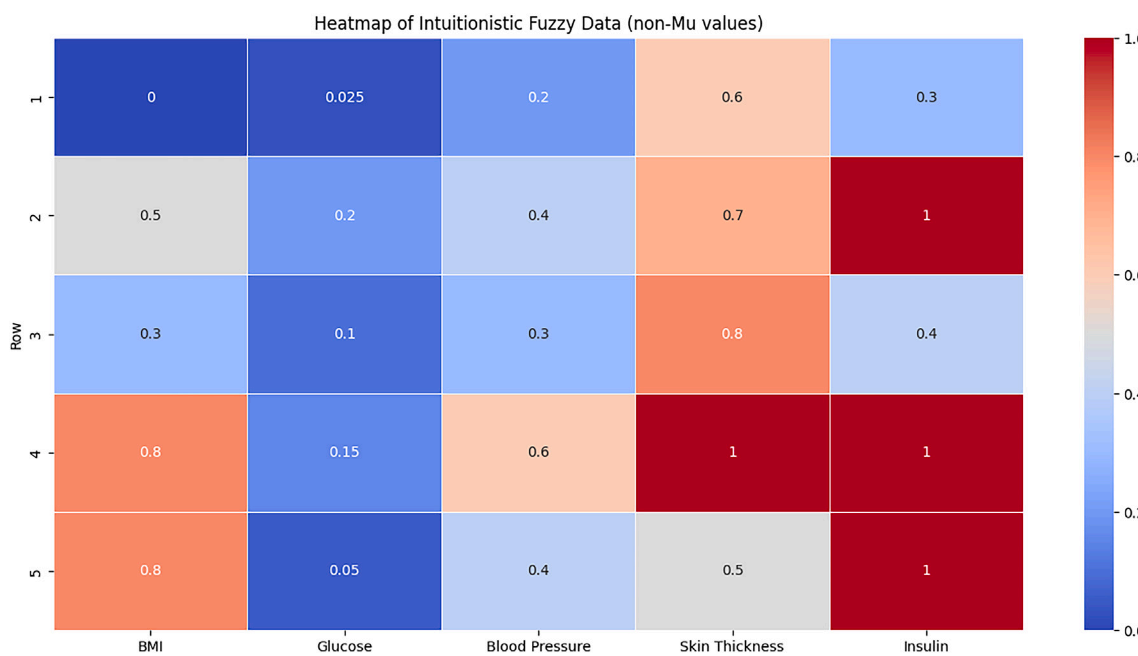


Fig. 7. Heatmap of $\mathcal{IFLG}$ using non-membership values.

interventions. For example, if glucose and skin thickness are strongly correlated, interventions can focus on managing both glucose levels and weight to prevent diabetes. Insights from the complement graph might suggest that factors like age or diabetes pedigree function are more critical in certain subgroups, allowing for personalised healthcare strategies.

This heatmap can be used to visualize the correlation matrix of membership $\mathcal{IFG}$, between a selection of the medical features Insulin, BMI, Skin Thickness, Blood Pressure and others, and the Outcome (presence of diabetes) as shown in Fig. 8. The strength of correlations is reflected according to the colour gradient, with lighter tones showing stronger positive correlations. As an example, the BMI is strongly correlated with Outcome, but Insulin and Skin Thickness demonstrate a low correlation. Though in this case some of the features were selected, other features (including Glucose, Age, and Pregnancies) can be utilized to conduct a broader analysis. Furthermore, the insights provided by these graphs can inform resource allocation decisions. Researchers can prioritise resources for patients at higher risk based on the most influential attributes identified from the analysis, ensuring that interventions are both effective and efficiently targeted. Overall, the applications of $\mathcal{CRCs}$ in $\mathcal{IFLG}$ and their complements offer a comprehensive, insightful, and actionable approach to healthcare research, particularly in the area of diabetes risk prediction and management.

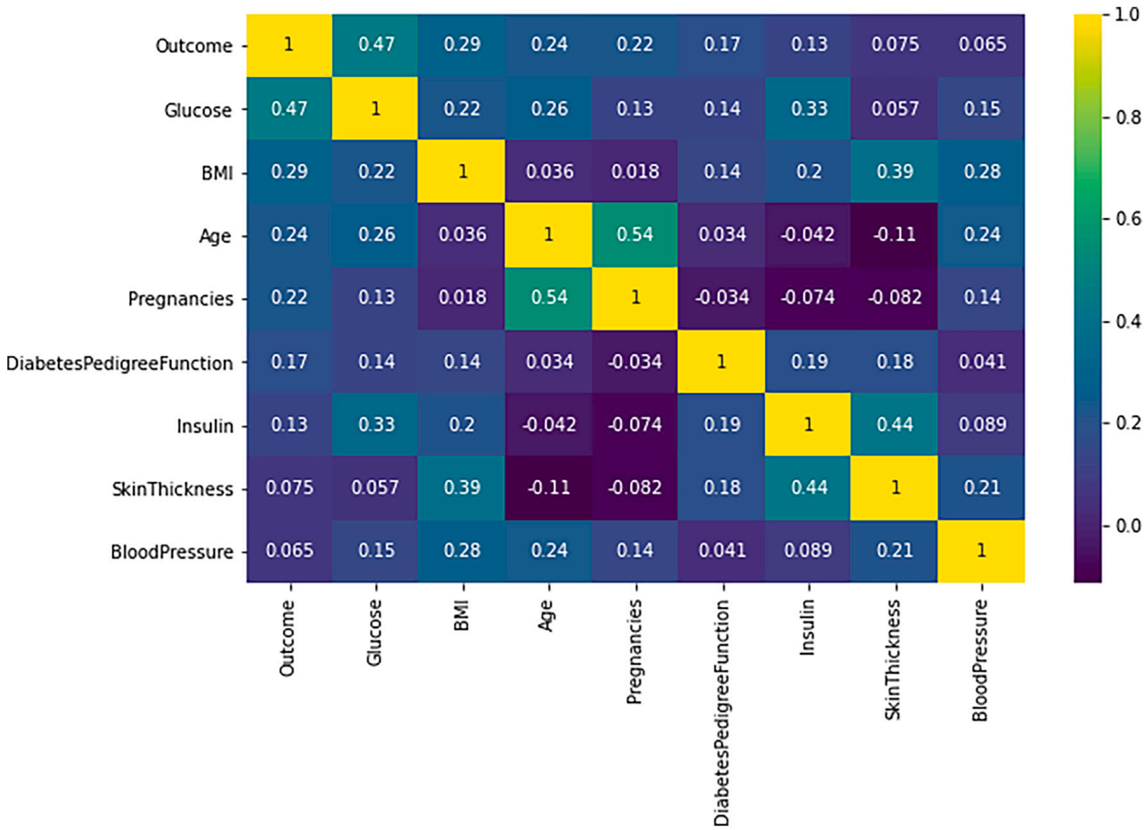


Fig. 8. Correlation heatmap showing relationships among selected medical parameters in the $\mathscr{IFG}$.

7. Conclusion

The study of $\mathscr{CRCs}$ of $\mathscr{IFLG}$ and their complement can successfully evaluate complex networks utilizing the maximal product. Such models help identify weak connections, leading to targeted network improvement efforts. This model can be applied to specialised marketing tasks such as identifying influential users, improving campaign techniques, and analysing public health and group behaviour. Using $\mathscr{IFLG}$, firms can derive insights into consumer behaviour, sentiment analysis, and disease dissemination patterns, thereby optimising decision-making. The framework $\mathscr{IFLG}$ identifies the most important predictors of every health outcome, including glucose levels, blood pressure, skin thickness, and insulin levels. The complement of $\mathscr{IFLG}$ is useful in evaluating secondary predictors that may not be easily identified. The complement of $\mathscr{IFLG}$ allows one to analyse secondary predictors, which may not be visible at first glance but have a major effect on improving prediction models. The effectiveness of this strategy has been demonstrated successfully using heatmaps, which display the relevance of features and their relationship to one another, offering a visual representation of complex data. Future research opportunities include developing complex algorithms to govern large-scale social networks while enhancing computational efficiency and scalability. Exploring dynamic $\mathscr{IFLG}$ may enable real-time network interactions, leading to more flexible and responsive decision-making paradigms. Additionally, incorporating machine learning approaches into $\mathscr{IFLG}$ can improve forecast accuracy and bring new insights into dynamic network architecture. Although the proposed approach effectively analyses the correlation and regression coefficients of $\mathscr{IFG}$, it has several drawbacks. The model has been evaluated using a small-scale healthcare dataset, and its accuracy depends on the correctness of the membership and non-membership characteristics. Furthermore, the framework currently addresses static systems, and computational complexity increases with vast networks. This study may be expanded upon in future research to include large-scale application areas and dynamic networks.

CRediT authorship contribution statement

A. Meenakshi: Writing – review & editing, Supervision, Methodology, Formal analysis. **J. Shivangi Mishra:** Writing – review & editing, Writing – original draft, Validation, Methodology, Formal analysis, Conceptualization. **Leo Mršić:** Validation, Methodology, Investigation, Formal analysis. **Antonios Kalampakas:** Visualization, Methodology, Formal analysis, Conceptualization. **Sovan Samanta:** Validation, Investigation, Conceptualization. **Tofigh Allahviranloo:** Visualization, Validation, Resources, Investigation, Formal analysis, Conceptualization.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data used in this study is available from the corresponding author upon reasonable request.

References

[1] Rosenfeld A. Fuzzy graphs. In *fuzzy sets and their applications to cognitive and decision processes*. Academic Press; 1975. pp. 77–95.
[2] Atanassov KT. Intuitionistic fuzzy sets. Intuitionistic fuzzy sets: theory and applications, *Heidelberg: Physica-Verlag HD* 1999:1–137.
[3] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338–53.
[4] Akula NK, Basha SS. Regression coefficient measure of intuitionistic fuzzy graphs with application to soil selection for the best paddy crop. AIMS Math 2023;8(8):17631–50.
[5] Akula NK, Shaik SB. Correlation coefficient measure of intuitionistic fuzzy graphs with application in money investing schemes. Computing Inform 2023;42(2):436–56.
[6] Bajaj J, Kumar S. A new intuitionistic fuzzy correlation coefficient approach with applications in multi-criteria decision-making. Decis Anal J 2023;9: 100340.
[7] Dey A, Ghosh D, Pal A. Operations on the complement of a fuzzy graph. In: Proceedings of the National Seminar on recent advances in Mathematics and its applications in engineering sciences; 2012. p. 43–50.
[8] Meenakshi A, Mishra SJ. The modular product of two cubic fuzzy graph structures. In: First international conference on Advances in electrical, electronics and computational intelligence (ICAEECI); 2023. https://doi.org/10.1109/ICAEECI58247.2023.10370796
[9] Meenakshi A, Mythreyi O, Bramila M, Kannan A, Senbagamalar J. Application of neutrosophic optimal network using operations. J Intell Fuzzy Syst 2023;45(1):421–33. https://doi.org/10.3233/JIFS-223718
[10] Meenakshi A, Mishra JS, Lee JG, Kalampakas A, Samanta S. Advanced risk prediction in healthcare: neutrosophic graph neural networks for disease transmission. Complex Intell Syst 2025;11(9):413.
[11] Meenakshi A, Shivangi Mishra J. Correlation and regression coefficients of neutrosophic graph products using minimum spanning tree for enhancing healthcare strategies. Neutrosophic Sets and Systems 2025;78:105–26.
[12] Mohamed SY, Ali AM. Degree of a vertex in the complement of maximal product of intuitionistic fuzzy graph. J Phys Sci 2019;24:115–24.
[13] Mohamed SY, Ali AM. Complement of max product of intuitionistic fuzzy graphs. Complex Intell Syst 2021;7(6):2895–905.
[14] Mordeson JN, Peng CS. Operations on fuzzy graphs. Inf Sci 1994;79:159–70.
[15] Nagoorgani A, Akram M, Anupriya S. Double domination on intuitionistic fuzzy graphs. J Appl Math Comput 2016;52:515–28.
[16] Reddy NR, Basha SS. The correlation coefficient of hesitancy fuzzy graphs in decision making. Comput Syst Sci Eng 2023;46:579–96.
[17] Sahoo S, Pal M. Different types of products on intuitionistic fuzzy graphs. Pac Sci Rev A Nat Sci Eng 2015;17(3):87–96.
[18] Sandeep KR, Narayan MS, Sunitha MS. Some remarks on the complement of fuzzy graph. IOSR J Math 2013, No.2. https://doi.org/10.9790/5728-0747577
[19] Talebi AA, Rashmanlou H. Complement and isomorphism on bipolar fuzzy graph. Fuzzy Inf Eng 2014;6(4):505–22.
[20] Ye J. Another form of correlation coefficient between single-valued neutrosophic sets and its multiple attribute decision-making method. Neutrosophic Sets and Systems 2013;1(1):8–12.
[21] Ye J. An improved correlation coefficient between intuitionistic fuzzy sets and its applications to real-life decision-making problems. Notes on Intuitionistic Fuzzy Sets 2020;26(2):1–14. https://doi.org/10.7546/nifs2020.26.2.1-14
[22] Yahya Mohamed S, Mohamed Ali A. Degree of a vertex in complement of maximal product of intuitionistic fuzzy graph. J Phys Sci 2019;24:115–24.
[23] Repalle VSR, Tola KA, Ashebo MA. Interval valued intuitionistic fuzzy line graphs. BMC Res Notes 2022;15(1):250.
[24] Repalle VNSR, Tola KA, Ashebo MA. Extended intuitionistic fuzzy line graphs: theory and properties.*in coding theory essentials*. IntechOpen 2023.
[25] Akram M, Davvaz B. Strong intuitionistic fuzzy graphs. Filomat 2012;26(1):177–96.
[26] Akram M. Interval-valued fuzzy line graphs. Neural Comput Appl 2012;21: 145–50.
[27] Kosari S, Qiang X, Kacprzyk J, Ain QT, Rashmanlou H. A study on topological indices in fuzzy graphs with application in decision making problems. J Mult Valued Logic Soft Comput 2024:42.
[28] Meenakshi A, Babujee JB. Paired equitable domination in graphs. Int J Pure Appl Math 2016;109(7):75–81.
[29] Shi X, Kosari S, Hameed S, Shah AG, Ullah S. Application of connectivity index of cubic fuzzy graphs for identification of danger zones of tsunami threat. PLoS One 2024;19(1):e0297197.
[30] Khan WA, Arif W, Rashmanlou H, Kosari S. Interval-valued picture fuzzy hypergraphs with application towards decision making. J Appl Math Comput 2024;70(2):1103–25.
[31] Tobaili S, Ahmed H, Alsharafi M. Edge HUB number of fuzzy graphs.*open*. J Discrete Appl Math 2024;7(1):11–20.
[32] Imran M, Azeem M, Jamil MK, Deveci M. Some operations on intuitionistic fuzzy graphs via novel versions of the Sombor index for internet routing. Granular Comput 2024;9(2):53.
[33] Talebi AA, Rashmanlou H, Sadati SH. Interval-valued intuitionistic fuzzy soft graph. TWMS J Appl Eng Math 2023.
[34] Shi X, Akhoundi M, Talebi AA, Sadati SH. Some properties of cubic fuzzy graphs with an application. Symmetry 2022;14(12):2623.
[35] PK, K.K, Talebi Y, Rashmanlou H, Talebi A, Mofidnakhaei F. New concept of cubic graph with application. J Mult Val Log Soft Comput 2019;33.

Author biography

Dr. A. Meenakshi is working as a professor in the Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R& D Institute of Science and Technology. She received her Ph.D. degree in 2018 from Anna University in Chennai. Her areas of research interest include graph theory, domination, labeling, chemical graph theory, and fuzzy domination. She has published 40 research articles in SCI-indexed and Scopus-indexed journals and authored three research books.

J. Shivangi Mishra is a Research Scholar in the Department of Mathematics at Vel Tech Rangarajan Dr. Sagunthala R& D Institute of Science and Technology, in Chennai, India. Her research interests include domination in fuzzy digraphs, fuzzy graph theory, and the novel applications based on these mathematical frameworks.

Prof. Leo Mršić is a full university professor, active in the scientific fields of computing, information sciences and economics. He is the author and co-author of over 100 scientific papers and 4 international patents, IPMA A certified project director, IEEE Senior Member, and a holder of the ISO 42001:2024 Foundation certificate. He is also a Tableau Academic Ambassador, Vice Rector for Science and Research at Algebra Bernays University, Head of BDV DARIO (Data, AI and Robotics) i-Gold Data Center, researcher at Rudolfovo Science and Technology Center Novo Mesto, Slovenia and Vice President for Technological Development and Innovation at the National Council for Higher Education, Science and Technological Development of the Republic of Croatia.

Antonios Kalampakas received his Bachelor's degree in Mathematics in 1999 and his Ph.D. in Mathematics in 2004 from Aristotle University of Thessaloniki, Greece. In 2005, he received the Algebra Award from the Academy of Athens, Greece. He served as an Assistant Professor of Mathematics at Democritus University of Thrace, Greece, from 2007 to 2013 and as an Assistant Professor of Mathematics at the American University of the Middle East, Kuwait, from 2013 to 2019. Since 2019 he has been an Associate Professor of Mathematics at the American University of the Middle East, Kuwait. His research interests include Fuzzy Graph Theory, Graph Automata, Discrete Mathematics, Algebraic Hyperstructures and Network Optimization.

Dr. Sovan Samanta is an Assistant Professor in the Department of Mathematics at Tamralipta Mahavidyalaya, West Bengal, India. He also holds the roles of Adjunct Professor in the Department of Technical Sciences, Western Caspian University, Azerbaijan, and Visiting Professor at the Research Center of Performance and Productivity Analysis, Istinye University, Istanbul, Türkiye. Additionally, he serves as a Scientific Advisor in the Department of Technical Sciences at Algebra University, Zagreb, Croatia. Previously, he served as an Assistant Professor in the Department of Basic Sciences at the Indian Institute of Information Technology, Nagpur. Dr. Samanta earned his Ph.D. from Vidyasagar University, India, and completed a Postdoctoral Fellowship at Hanyang University, South Korea. With over 100 publications in international journals, including 85 in SCIE/SCI journals, he has also authored and edited several books and book chapters with Springer and IGI Global. He is an Associate Editor for the Journal of Applied Mathematics and Computing (SCIE, I.F-2.4, Scopus), Springer; Fuzzy Information and Engineering (ESCI, I.F.-1.3, Scopus); Neutrosophic Sets and Systems (SCOPUS, ESCI); Transactions on Fuzzy Sets and Systems (SCOPUS); Journal of Intelligent & Fuzzy Systems (SCIE, I.F-1.72), IOS Press (2020-2024); and Mathematical Problems in Engineering (SCIE, I.F-1.305), Hindawi (2020-2024). Dr. Samanta is recognized among the top 2% of scientists globally according to the Stanford University ranking based on the Scopus citation database for 2021-2025 (current).

Tofigh Allahviranloo is a Professor of Applied Mathematics at Istinye University in Istanbul, Türkiye. An accomplished mathematician and computer scientist, Prof. Allahviranloo is sincerely dedicated to multi- and interdisciplinary research efforts. His expertise lies primarily in fundamental research in applied fuzzy mathematics, with a special focus on dynamical systems and pioneering applications in applied biological sciences. He is the main editor of the book series 'Uncertainty, Computational Techniques and Decision Intelligence' published by ELSEVIER. In addition to his extensive writing activities, Prof. Allahviranloo plays an important role in the academic community as Associate Editor and Editorial Board Member of several prestigious journals. These include 'Information Sciences (ELSEVIER)', 'Fuzzy Sets and Systems (ELSEVIER)', 'Journal of Intelligent and Fuzzy Systems (IOS Press)', 'Iranian Journal of Fuzzy Systems', 'Mathematical Sciences (Springer)', 'Granular Computing (Springer)', 'Journal of Mathematics and Computer Science (ISRP)' and 'Journal of Computational Methods for Differential Equations (University of Tabriz)'.

He is currently Executive Editor-in-Chief of 'Information Sciences (ELSEVIER)', Editor-in-Chief of the 'Transactions on Fuzzy Sets and Systems', Editor-in-Chief of the 'International Journal of Industrial Mathematics', Chairman of the 'International Conference on Decision Sciences (IDS)' and Managing Editor of 'The Journal of Mathematics and Computer Science' published by International Scientific Research Publications.