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An analytical framework for sustainability assessment under stochastic conditions

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ABSTRACT

Measuring sustainability as an efficient tool to achieve sustainable development and improve economic, social, and environmental aspects is always fraught with complications. In this sense, developing a suitable approach for evaluating and recognizing the strengths and weaknesses across these dimensions is paramount. Given the inherent uncertainty in data for many real-world applications, the primary aim of this paper is to present a data envelopment analysis (DEA) model for evaluating sustainability within a stochastic environment. The proposed model is non-radial and incorporates undesirable outputs, enabling the assessment of overall sustainability as well as each of the economic, social, and environmental dimensions simultaneously. This multi-dimensional evaluation capability is a key advantage of the proposed model. Additionally, the proposed model is based on input excesses and output shortfalls. Another notable advantage is the incorporation of the assumption of managerial disposability when dealing with undesirable outputs. To demonstrate the applicability of the proposed model, data from 59 diverse countries across Africa, Europe, North America, and Asia were analyzed over a 12-year period (2010–2022). The country selection was designed to capture global heterogeneity in development levels, policies, and environmental conditions, allowing for robust cross-continental comparisons. Key findings reveal that: (1) Europe achieves the highest stochastic sustainability scores, while North America performs poorest; (2) Environmental sustainability shows the most success cases globally, whereas social sustainability lags; (3) Significant trade-offs exist between economic growth and environmental protection.

1. Introduction

Sustainability has emerged as a central concept in addressing global challenges related to balancing economic growth, social equity, and environmental preservation. It emphasizes the need to ensure that systems and processes can persist over time without depleting natural resources or compromising the well-being of future generations. The concept gained widespread recognition with the publication of the Brundtland Commission's report in 1987, which defined sustainable development as "meeting the needs of the present without compromising the ability of future generations to meet their own needs". This definition remains a cornerstone in sustainability discourse and has guided numerous studies and policies aimed at achieving sustainable development.

In recent decades, sustainability assessment has become a critical tool for evaluating the performance of systems and processes across economic, environmental, and social dimensions [1]. One of the most widely used methods for such assessments is Data Envelopment Analysis (DEA), a non-parametric technique that evaluates the relative efficiency of decision-making units (DMUs) without requiring a predefined production function. DEA's flexibility in handling multiple inputs and outputs has made it a popular choice for sustainability research, particularly in contexts where traditional parametric methods may not be applicable.

However, sustainability assessment at the country level is inherently affected by temporal variability and uncertainty. Economic indicators fluctuate due to business cycles and policy changes, environmental indicators are influenced by climatic and ecological shocks, and

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social indicators evolve gradually but unevenly over time. Relying solely on deterministic average values may therefore obscure important information regarding the stability, resilience, and vulnerability of sustainability performance.

Incorporating stochasticity into sustainability assessment allows for distinguishing between countries that exhibit consistently stable performance and those whose apparent efficiency is driven primarily by favorable average outcomes but remains vulnerable to fluctuations. This distinction is particularly relevant for policy makers; as long-term sustainability depends not only on performance levels but also on their robustness over time.

Despite the growing body of literature, significant gaps remain in the comprehensive evaluation of sustainability, particularly in stochastic environments where uncertainty and variability are inherent. Ignoring uncertainty may lead to efficiency rankings that are insensitive to volatility, thereby limiting their relevance for policy design and long-term sustainability planning. As will be discussed in detail in Section 2, many studies have focused on individual dimensions of sustainability, such as environmental efficiency [2], economic performance [3], or social aspects [4], while neglecting the interconnectedness of these dimensions. Furthermore, the handling of undesirable outputs — often byproducts of industrial activities — has received limited attention, despite their critical role in achieving sustainable development.

The primary objective of this study is to address these gaps by proposing a novel model for assessing sustainability in stochastic environments, incorporating undesirable outputs across economic, environmental, and social dimensions. Using the Slacks-Based Measure (SBM) model and the managerial disposability assumption, this research provides a unified approach that captures both efficiency levels and their sensitivity to uncertainty. By explicitly accounting for stochastic variation, the proposed framework enhances the interpretability of sustainability scores and offers a more realistic representation of country-level performance.

To operationalize this objective, the study addresses the following research questions:

– How can an SBM-DEA-based framework capture sustainability performance under uncertainty while explicitly accounting for undesirable outputs?
– How do economic, environmental, and social sustainability interact under stochastic conditions, and how can they be evaluated simultaneously?
– What role does the managerial disposability assumption play in improving the accuracy of sustainability assessments when undesirable outputs are present?
– What regional disparities in sustainability performance emerge when applying this model to 59 countries across diverse continents?

The rest of the paper is structured as follows: Section 2 provides a comprehensive literature review, highlighting the evolution of sustainability assessment and the role of DEA in this domain. Section 3 offers a concise review of the managerial disposability assumption, the SBM model, and the stochastic BCC model. In Section 4, we introduced the proposed model for assessing sustainability in a stochastic environment, incorporating undesirable outputs. Section 5 delves into an analysis of the stochastic sustainability of 59 countries across four continents, demonstrating the proposed approach. Finally, Section 6 concludes the paper.

2. Literature review

Surveys on the applications of DEA in sustainability indicate that many studies in the field of sustainability assessment have focused solely on one of the environmental, economic, or social dimensions. Among these dimensions, the environmental dimension has received the most attention. For example, Sueyoshi and Goto [2] introduced models to assess environmental sustainability using the DEA approach. Their methodology incorporated radial and non-radial models under managerial and natural disposability assumptions. Similarly, Sueyoshi and Goto [5] employed managerial and natural disposability assumptions to develop a radial model for assessing the environmental efficiency of Japanese industries. Building on these works, Sueyoshi and Yuan [4] combined radial and non-radial models to create an intermediate model, which they applied to measure the social sustainability of 30 Chinese provinces. Additionally, Sueyoshi et al. [6] conducted a comprehensive review of DEA-based studies on environmental performance over four decades.

In the context of social sustainability, Zhang et al. [7] employed both radial and non-radial DEA approaches to assess the social sustainability of Chinese provinces from 2005 to 2014. Their methodology incorporated managerial and natural disposability concepts. On the other hand, Mahdiloo et al. [8] formulated a multi-objective programming model to evaluate environmental and eco-efficiency, applying their method to suppliers of Hyundai steel companies. Omrani et al. [9] later modified this model, presenting a version with fewer calculations. Their common weight DEA-based model not only assesses environmental efficiency but also evaluates social efficiency, as demonstrated in their application to the Iranian railway. More recently, Shu et al. [3] evaluated energy efficiency across 168 economies using a super-efficiency SBM-DEA model, highlighting the importance of energy performance in achieving global sustainability goals. Similarly, Yang et al. [10] reassessed industrial eco-efficiency in China using a meta two-stage parallel entropy dynamic DDF-DEA model, aligning their analysis with the Sustainable Development Goals (SDGs) and providing insights into improving industrial sustainability. D'Adamo et al. [11] applied a multiple criteria analysis approach for assessing regional and territorial progress toward achieving the Sustainable Development Goals in Italy.

While the above studies have primarily focused on the environmental dimension, some research has examined sustainability from all three perspectives—economic, environmental, and social. For instance, Chang et al. [12] evaluated the sustainability of 16 industrial sectors in North America over a three-year period from 2003 to 2005. Galán-Martín et al. [13] introduced a model for evaluating the sustainability of units and making comparisons, which simultaneously investigates sustainability from economic, environmental, and social perspectives. They applied their model to assess the sustainability of electricity generation in the United Kingdom. Li et al. [14] categorized social, economic, and environmental indicators to assess the sustainability of oil refining enterprises. Amirteimoori et al. [15] introduced a multi-period sustainability assessment model and applied it to gas companies, employing the weak disposability assumption to account for undesirable outputs. Jahani Sayyad Noveiri and Kordrostami [16] developed a model for evaluating multi-period sustainability, considering discrete and bounded data, and applied it to Iranian gas companies from 2013 to 2015. Tajbakhsh and Shamsi [17] examined the sustainability of 133 countries, considering social, economic, and environmental aspects. Moghaddas et al. [18] introduced a model for evaluating urban transportation systems, incorporating non-optional, negative, non-negative, and undesirable data.

Given the significant role of network structures in real-world applications, many studies have focused on sustainability assessment in such contexts. Tajbakhsh and Hassini [19] introduced a model to assess the sustainability of supply chains, considering social, economic, and environmental aspects, and applied it to a case study in the beverage industry. Badiezadeh et al. [20] defined supply chain sustainability based on optimistic and pessimistic efficiency scores. Izadikhah and Saen [21] introduced cooperative and non-cooperative models for a two-stage structure that includes undesirable outputs. They later reformulated these models in a stochastic environment and applied them to evaluate the sustainability of the pasta supply chain, incorporating economic, social, and environmental indicators. Zhao et al. [22] investigated sustainability using a two-stage structure, considering series and parallel forms, with the first stage encompassing economic and environmental dimensions and the second stage focusing on the social dimension. They applied their model to calculate the sustainability of 30 Chinese cities. Fallah and Hosseinzadeh Lotfi [23] assessed the financial efficiency and sustainability of Iranian petrochemical companies with network structures during 2015–2016. Nematizadeh and Nematizadeh [24] employed the directional distance function to evaluate the multi-period sustainability of a two-stage network structure in the presence of undesirable outputs. Izadikhah and Farzipoor Saen [25] introduced a centralized approach to assess the sustainability of a two-stage structure, applying it to the welding industry. Fathi and Saen [26] proposed a novel approach combining network DEA with a common set of weights and Shannon's entropy to evaluate sustainability in transportation supply chains, addressing challenges such as zero inputs in the data.

The above-mentioned studies have primarily been conducted in deterministic environments, ignoring the stochasticity and uncertainty inherent in real-world data. However, real-world applications often involve uncertain or stochastic data, necessitating the use of chance-constrained programming. Sengupta [27] pioneered the development of the stochastic edition of the DEA model in this field. For further insight into stochastic DEA, readers are referred to Banker [28], Cooper et al. [29,30], Land et al. [31], Olesen and Petersen [32], Simar and Wilson [33], Azadi and Saen [34], Heesche and Asmild [35], Costa et al. [36], and Amirteimoori et al. [37].

In the last decade, researchers have explored sustainability assessment in stochastic environments. For example, Tavassoli et al. [38] introduced four models for evaluating the efficiency of units using deterministic, stochastic, and fuzzy data, encompassing economic, environmental, and social factors. Jahani Sayyad Noveiri et al. [39] introduced a stochastic model from both optimistic and pessimistic perspectives to assess the sustainability of gas companies. Sarkar et al. [40] developed a Z-Number SBM-DEA model for sustainable supplier selection, offering a robust framework for decision-making under uncertainty. Wang [41] focused on long-term sustainability by measuring power consumption efficiency in electromechanical systems using a combination of fuzzy DEA and TOPSIS, demonstrating the applicability of DEA in dynamic and uncertain environments.

In the realm of economics, enhancing efficiency is often associated with increasing production. However, certain industrial activities may generate undesirable outputs at higher production levels, leading to economic losses. The identification and mitigation of these undesirable factors within the production process have become critical concerns and research priorities. Various approaches have been proposed to address these undesirable factors. For example, Hailu and Veeman [42] introduced the concept of treating undesirable outputs as inputs. Seiford and Zhu [43] proposed a mathematical programming model for performance evaluation that considered the inverse of undesirable outputs. Färe and Grosskopf [44] contested the approach of Hailu and Veeman [42], citing its contradiction with physical laws, and proposed an alternative approach. Kuosmanen [45] introduced a modified model to address the limitations of previous methods when dealing with undesirable outputs. Korhonen and Luptacik [46] defined efficiency as the ratio of the weighted sum of desirable and undesirable outputs to the weighted sum of inputs, assigning negative weights to undesirable outputs. Sueyoshi and Goto [47] introduced the concept of managerial disposability, which allows for increasing inputs to enhance desirable outputs while simultaneously reducing undesirable outputs. This study leverages the assumption of managerial disposability to address undesirable outputs in the proposed model. For further references on efficiency analysis in the presence of undesirable outputs, readers are referred to Zoroufchi et al. [48], Azadi et al. [49], and Amirteimoori et al. [50].

Despite these advances, three important limitations remain in the existing literature. First, most stochastic DEA-based sustainability studies are conducted at the firm, industry, or supply-chain level, while country-level sustainability assessment under stochastic conditions remains relatively underexplored. Second, many studies focus on a single sustainability dimension or provide only an aggregate sustainability score, thereby limiting insights into the heterogeneous behavior of economic, environmental, and social dimensions under uncertainty. Third, undesirable outputs are typically incorporated only within the environmental dimension, whereas their presence in economic and social dimensions is rarely modeled explicitly.

These gaps suggest the need for a unified stochastic framework that simultaneously evaluates sustainability across all three dimensions at the country level, while allowing both an overall assessment and a dimension-specific analysis that accounts for undesirable outputs and performance volatility over time.

3. Theoretical and methodological foundations

In this section, we will briefly review the managerial disposability assumption, the SBM model and stochastic BCC model.

3.1. Managerial disposability

In economic strategies, the primary objective is often to boost income, which requires increasing production. The general expectation is that higher input consumption leads to greater output and improved efficiency. However, certain industrial activities present scenarios where an increase in output results in economic losses, referred to as undesirable outputs. In 2012, Sueyoshi and Goto introduced the concept of managerial disposability to address these outputs. According to this assumption, it becomes feasible to augment inputs to enhance desirable outputs while simultaneously diminishing undesirable outputs.

Suppose there are J decision-making units (DMUs), and each DMU_j employs I inputs to yield R desirable outputs and N undesirable outputs. The production possibility set under the managerial disposability assumption is defined as follows:

$$T = \Big\{ (x, w, v) | \sum_{j=1}^{J} \lambda_j x_{ij} \geq x_{io}, \quad i = 1, \dots, I, \\ \sum_{j=1}^{J} \lambda_j v_{rj} \geq v_{ro}, \quad r = 1, \dots, R, \\ \sum_{j=1}^{J} \lambda_j w_{nj} \leq w_{to}, \quad n = 1, \dots, N, \\ \sum_{j=1}^{J} \lambda_j = 1, \\ \lambda_j \geq 0, \qquad j = 1, \dots, J \Big\}. \tag{1}$$

The first I constraints are related to the inputs and indicate that inputs must be increased to reduce undesirable outputs. The second R constraints stand for desirable outputs and indicate that the desirable outputs must be increased. The third N constraints are related to undesirable outputs. Finally, the last constraint is convexity constraint. The decision to use greater and smaller symbols for inputs and undesirable outputs is influenced by the assumption of managerial disposability.

3.2. Slack-based measure model

The slack-based measure (SBM) model of Tone [51] is introduced as an alternative efficiency analysis model to evaluate the technical efficiency of the DMUs. The mathematical formulation of SBM model of Tone [51] is:

$$\rho_o^* = Min \frac{1 - \frac{1}{I} \sum_{i=1}^{I} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{R} \sum_{r=1}^{R} \frac{s_r^+}{v_{ro}}}$$

$$
\begin{aligned}
&s.t.\\
&\sum_{j=1}^{J}\lambda_j x_{ij}+s_i^-=x_{io},\ i=1,\dots,I,\\
&\sum_{j=1}^{J}\lambda_j \upsilon_{rj}-s_r^+=\upsilon_{ro},\ r=1,\dots,R,\\
&\lambda_j,s_i^-,s_r^+\geq 0\ \forall j,i,r.
\end{aligned}
\tag{2}
$$

In the above model, s_i^-, s_r^+, and λ_j for each i,r,j are unknown variables. Additionally, inputs $(x_{ij}: i=1,\dots,I, j=1,\dots,J)$ and outputs $(\upsilon_{rj}: r=1,\dots,R, j=1,\dots,J)$ are non-negative parameters of the model. Model (2) is given in constant returns to scale environment and by imposing the convexity constraint $\sum_{j=1}^{J}\lambda_j=1$, it can be transformed into a model that operates under variable returns to scale conditions. The objective function of the model remains unchanged with respect to a unit change in inputs and outputs. Furthermore, it is monotonically increasing for inputs or outputs. In simpler terms, any increase in output variables enhances efficiency, while any decrease in input variables diminishes efficiency.

Definition 1. DMU_o is SBM-efficient if and only if $\rho_o^*=1$ which is equivalent to $s_i^-=s_r^+=0$.

3.3. Stochastic BCC model

Suppose there are J DMUs, and each $DMU_j: j=1,\dots,J$ uses the random inputs $\tilde{x}_{ij}: i=1,\dots,I$ to produce random outputs $\tilde{y}_{rj}: r=1,\dots,R$. All random input/output variables are assumed to be distributed normally with known mean and variance. The following envelopment DEA model, introduced by Land et al. [31], estimates the relative efficiency of a specific DMU_o:

$$
\begin{aligned}
&Min\,\theta\\
&s.t.\\
&P\left\{\sum_{j=1}^{J}\lambda_j\tilde{x}_{ij}\leq\theta\tilde{x}_{io}\right\}\geq 1-\alpha,\ i=1,\dots,I,\\
&P\left\{\sum_{j=1}^{J}\lambda_j\tilde{y}_{rj}\geq\tilde{y}_{ro}\right\}\geq 1-\alpha, r=1,\dots,R,\\
&\sum_{j=1}^{J}\lambda_j=1,\\
&\lambda_j\geq 0,\quad\forall j.
\end{aligned}
\tag{3}
$$

where $\alpha\in[0,1]$ is a user-defined parameter that indicates the confidence level. Assume $x_{ij}: i=1,\dots,I$ and $y_{rj}: r=1,\dots,R$ are the mean values of inputs and outputs for the jth DMU. Additionally, ϕ represents the cumulative standard normal distribution function, and ϕ^{-1} is its inverse. Hence, by applying the central limit theorem, the deterministic form of model (3) can be expressed as follows:

$$
\begin{aligned}
&Min\,\theta\\
&s.t.\\
&\sum_{j=1}^{J}\lambda_j x_{ij}-\phi^{-1}(\alpha)\sigma_i(\lambda,\theta)\leq\theta x_{io}, i=1,\dots,I,\\
&\sum_{j=1}^{J}\lambda_j y_{rj}+\phi^{-1}(\alpha)\sigma_r(\lambda)\geq y_{ro}, r=1,\dots,R,\\
&\sum_{j=1}^{J}\lambda_j=1,\\
&\lambda_j\geq 0,\quad\forall j,i,r.
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
\sigma_i^2 &=\sum_{j=1}^{J}\sum_{k=1}^{J}\lambda_j\lambda_k\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{ik})+\theta^2\mathrm{var}(\tilde{x}_{io})\\
&-2\theta\sum_{j=1}^{J}\lambda_j\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{io}),\quad i=1,\dots,I,
\end{aligned}
$$

$$
\begin{aligned}
\sigma_r^2 &=\sum_{j=1}^{J}\sum_{k=1}^{J}\lambda_j\lambda_k\mathrm{cov}(\tilde{y}_{rj},\tilde{y}_{rk})+\mathrm{var}(\tilde{y}_{ro})\\
&-2\sum_{j=1}^{J}\lambda_j\mathrm{cov}(\tilde{y}_{rj},\tilde{y}_{ro}),\quad r=1,\dots,R.
\end{aligned}
$$

By applying the single factor assumption of random variables commonly used in economics and finance, model (4) can be converted into the following linear form (See [52,53]):

$$
\begin{aligned}
&\theta_o^*=Min\,\theta\\
&s.t.\\
&\sum_{j=1}^{J}\lambda_j x_{ij}-\phi^{-1}(\alpha)\sigma(p_i^++p_i^-)\leq\theta x_{io}, i=1,\dots,I,\\
&\sum_{j=1}^{J}\lambda_j a_{ij}-\theta a_{io}=p_i^+-p_i^-,\ i=1,\dots,I,\\
&\sum_{j=1}^{J}\lambda_j y_{rj}+\phi^{-1}(\alpha)\sigma(q_r^++q_r^-)\geq y_{ro}, r=1,\dots,R,\\
&\sum_{j=1}^{J}\lambda_j b_{rj}-b_{ro}=q_r^+-q_r^-,\ r=1,\dots,R,\\
&\sum_{j=1}^{J}\lambda_j=1,\\
&\lambda_j,\ q_i^+,\ q_i^-,\ \ q_r^+,q_r^-\geq 0,\ \forall j,i,r.
\end{aligned}
\tag{5}
$$

In model (5), p_i^+, p_i^-, q_r^+ and q_r^- are deviation variables that appeared in the linearization process. Moreover, a_{ij} and b_{rj} are the standard deviations of $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$, respectively (The detailed procedure is presented in Appendix A).

Theorem 1. *For any predetermined level of $\alpha\leq 0.5$, the stochastic efficiency score calculated from model (5) ranges between 0 and 1 (See [54]).*

Definition 2. The unit under evaluation, DMU_o, is stochastically efficient under confidence level α, if and only is $\theta_o^*=1$.

4. Stochastic sustainability assessment model

In this section, we present a stochastic model for assessing sustainability based on three aspects: economic, environmental, and social, while considering undesirable outputs. Let us assume there are J DMUs, each depicted as shown in Fig. 1.

To introduce the SBM stochastic model, we start by defining the chance-constraint production possibility set. Suppose DMU_j consumes I random inputs $(\tilde{x}_{ij})$ to produce R random desirable output $(\tilde{\upsilon}_{rj})$, and N random undesirable output $(\tilde{w}_{nj})$. The stochastic production possibility set is defined as follows:

$T=\{(\tilde{x},\tilde{\upsilon},\tilde{w})|$

Economic constraints:

$$
\begin{aligned}
&P\left\{\sum_{j=1}^{J}\lambda_j^{(EC)}\tilde{x}_{ij}^{(EC)}-s_i^{(EC)}\geq\tilde{x}_{io}^{(EC)}\right\}\geq 1-\alpha, i\in I^{(EC)},\\
&P\left\{\sum_{j=1}^{J}\lambda_j^{(EC)}\tilde{\upsilon}_{rj}^{(EC)}-s_r^{(EC)}\geq\tilde{\upsilon}_{ro}^{(EC)}\right\}\geq 1-\alpha,\ \ r\in R^{(EC)},\\
&P\left\{\sum_{j=1}^{J}\lambda_j^{(EC)}\tilde{w}_{nj}^{(EC)}+s_n^{(EC)}\leq\tilde{w}_{no}^{(EC)}\right\}\geq 1-\alpha,\ n\in N^{(EC)},\\
&\sum_{j=1}^{J}\lambda_j^{(EC)}=1, \lambda_j^{(EC)}, s_i^{(EC)}, s_r^{(EC)}, s_n^{(EC)}\geq 0,\forall j,i,r,n,
\end{aligned}
$$

Environmental constraints:

$$
P\left\{\sum_{j=1}^{J}\lambda_j^{(EN)}\tilde{x}_{ij}^{(EN)}-s_i^{(EN)}\geq\tilde{x}_{io}^{(EN)}\right\}\geq 1-\alpha, i\in I^{(EN)},
$$

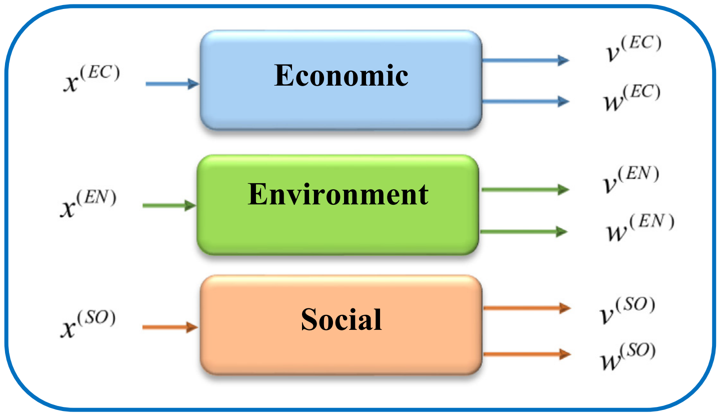

Fig. 1. Structure of $DMU_j : j = 1, \dots, J$.

$$P\left\{\sum_{j=1}^{J} \lambda_j^{(EN)} \widetilde{v}_{rj}^{(EN)} - s_r^{(EN)} \geq \widetilde{v}_{ro}^{(EN)}\right\} \geq 1-\alpha,\ \ r \in R^{(EN)}, \tag{6}$$

$$P\left\{\sum_{j=1}^{J} \lambda_j^{(EN)} \widetilde{w}_{nj}^{(EN)} + s_n^{(EN)} \leq \widetilde{w}_{no}^{(EN)}\right\} \geq 1-\alpha,\ n \in N^{(EN)},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} = 1, \lambda_j^{(EN)}, s_i^{(EN)}, s_r^{(EN)}, s_n^{(EN)} \geq 0, \forall j, i, r, n,$$

Social constraints:

$$P\left\{\sum_{j=1}^{J} \lambda_j^{(SO)} \widetilde{x}_{ij}^{(SO)} - s_i^{(SO)} \geq \widetilde{x}_{io}^{(SO)}\right\} \geq 1-\alpha, i \in I^{(SO)},$$

$$P\left\{\sum_{j=1}^{J} \lambda_j^{(SO)} \widetilde{v}_{rj}^{(SO)} - s_r^{(SO)} \geq \widetilde{v}_{ro}^{(SO)}\right\} \geq 1-\alpha,\ \ r \in R^{(SO)},$$

$$P\left\{\sum_{j=1}^{J} \lambda_j^{(SO)} \widetilde{w}_{nj}^{(SO)} + s_n^{(SO)} \leq \widetilde{w}_{no}^{(SO)}\right\} \geq 1-\alpha,\ \ n \in N^{(SO)},$$

$$\sum_{j=1}^{J} \lambda_j^{(SO)} = 1, \lambda_j^{(SO)}, s_i^{(EC)}, s_r^{(EN)}, s_n^{(SO)} \geq 0, \forall j, i, r, n.\}$$

The production possibility set in (6) is a probabilistic mathematical programming problem. In this model, $\widetilde{x}_{ij}$, $\widetilde{v}_{rj}$ and $\widetilde{w}_{nj}$ are the input/output parameters and $\lambda_j^{(EC)}, \lambda_j^{(EN)}, \lambda_j^{(SO)}$ represent intensity variables for the economic, environmental, and social dimensions. Moreover, α takes values between 0 and 1, indicating the confidence level.

The production possibility set (6) was established under the assumption of managerial disposability, indicating that as inputs increase, desirable outputs increase while undesirable outputs decrease simultaneously. To taking three different dimensions (economic, environmental, and social) into consideration, three different groups of constraints are defined.

Now, let x_{ij}, v_{rj}, and w_{nj} represent the mean values of inputs, desirable outputs and undesirable outputs of DMU_j, respectively. According to the central limit theorem, technology set (6) can be transformed into the deterministic non-linear form, and following Cooper et al. [29], it can be converted into the linear form as follows (Here, we did not provide the details of linearization procedure. To see the mathematical details of the transformation, interested readers can refer to Cooper et al. [29]):

$T = \{(x, v, w)|$

Economic constraints:

$$\sum_{j=1}^{J} \lambda_j^{(EC)} x_{ij}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EC)} + q_i^{-(EC)}) - s_i^{(EC)} = x_{io}^{(EC)},\ i \in I^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} a_{ij}^{(EC)} - a_{io}^{(EC)} = q_i^{+(EC)} - q_i^{-(EC)},\ i \in I^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} v_{rj}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EC)} + q_r^{-(EC)}) - s_r^{(EC)} = v_{ro}^{(EC)},\ r \in R^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} b_{rj}^{(EC)} - b_{ro}^{(EC)} = q_r^{(EC)} - q_r^{-(EC)}, r \in R^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} w_{nj}^{(EC)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EC)} + q_n^{-(EC)}) + s_n^{(EC)} = w_{no}^{(EC)}, n \in N^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} c_{nj}^{(EC)} - c_{no}^{(EC)} = q_n^{+(EC)} + q_n^{-(EC)}, n \in N^{EC},$$

$$\sum_{j=1}^{J} \lambda_j^{(EC)} = 1,$$

$$\lambda_j^{(EC)}, s_i^{(EC)}, s_r^{(EC)}, s_n^{(EC)}, q_i^{+(EC)}, q_i^{-(EC)}, q_r^{+(EC)}, q_r^{-(EC)},$$
$$q_n^{+(EC)}, q_n^{-(EC)} \geq 0, \forall j, i, r, n,$$

Environmental constraints:

$$\sum_{j=1}^{J} \lambda_j^{(EN)} x_{ij}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EN)} + q_i^{-(EN)}) - s_i^{(EN)} = x_{io}^{(EN)},\ i \in I^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} a_{ij}^{(EN)} - a_{io}^{(EN)} = q_i^{+(EN)} - q_i^{-(EN)},\ i \in I^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} v_{rj}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EN)} + q_r^{-(EN)}) - s_r^{(EN)} = v_{ro}^{(EN)},\ r \in R^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} b_{rj}^{(EN)} - b_{ro}^{(EN)} = q_r^{+(EN)} - q_r^{-(EN)}, r \in R^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} w_{nj}^{(EN)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EN)} + q_n^{-(EN)}) + s_n^{(EN)} = w_{no}^{(EN)}, n \in N^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} \sigma(w_{nj}^{(EN)}) - \sigma(w_{no}^{(EN)}) = q_n^{+(EN)} + q_n^{-(EN)}, n \in T^{EN},$$

$$\sum_{j=1}^{J} \lambda_j^{(EN)} = 1,$$

$$\lambda_j^{(EN)}, s_i^{(EN)}, s_r^{(EN)}, s_n^{(EN)}, q_i^{+(EN)}, q_i^{-(EN)}, q_r^{+(EN)}, q_r^{-(EN)},$$
$$q_n^{+(EN)}, q_n^{-(EN)} \geq 0, \forall j, i, r, n,$$

Social constraints:

$$\sum_{j=1}^{J} \lambda_j^{(SO)} x_{ij}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_i^{+(SO)} + q_i^{-(SO)}) - s_i^{(SO)} = x_{io}^{(SO)},\ i \in I^{Social}, \tag{7}$$

$$\sum_{j=1}^{J} \lambda_j^{(SO)} a_{ij}^{(SO)} - a_{io}^{(SO)} = q_i^{+(SO)} - q_i^{-(SO)},\ i \in I^{Social},$$

$$\sum_{j=1}^{J} \lambda_j^{(SO)} v_{rj}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_r^{+(SO)} + q_r^{-(SO)}) - s_r^{(SO)} = v_{ro}^{(SO)},\ r \in R^{Social},$$

$$\sum_{j=1}^{J} \lambda_j^{(SO)} b_{rj}^{(SO)} - b_{ro}^{(SO)} = q_r^{+(SO)} - q_r^{-(SO)}, r \in R^{Social},$$

$$\sum_{j=1}^{J}\lambda_j^{(SO)}w_{nj}^{(SO)}-\phi^{-1}(\alpha)\sigma(q_n^{+(SO)}+q_n^{-(SO)})+s_n^{(SO)}=w_{no}^{(SO)},n\in N^{Social},$$
$$\sum_{j=1}^{J}\lambda_j^{(SO)}c_{nj}^{(SO)}-c_{no}^{(SO)}=q_n^{+(SO)}+q_n^{-(SO)},n\in N^{Social},$$
$$\sum_{j=1}^{J}\lambda_j^{(SO)}=1,$$
$$\lambda_j^{(SO)},s_i^{(SO)},s_r^{(SO)},s_n^{(SO)},q_i^{+(SO)},q_i^{-(SO)},q_r^{+(SO)},q_r^{-(SO)},$$
$$q_n^{+(SO)},q_n^{-(SO)}\geq 0,\forall j,i,r,n.$$

In the technology set (7), ϕ^{-1} is a known parameter and represents the inverse of the standard normal distribution function, σ represents the standard normal deviation. q_r^+ and q_r^+ are deviation variables that appeared in the linearization process. Moreover, a_{ij}, b_{rj} and c_{nj} are the standard deviations of $\tilde{x}_{ij}$, $\tilde{v}_{rj}$ and $\tilde{w}_{nj}$, respectively.

Considering the technology set (7), the stochastic SBM efficiency for each of the three aspects — economic, environmental, and social — can be calculated separately by using the following three models (the detailed constraints of these models are provided in the Appendix B):

$$\delta_o^{*Economic}=\frac{1-\frac{1}{N^{(EC)}}\left(\sum_{n=1}^{N^{(EC)}}\frac{s_n^{(EC)}}{w_{no}^{(EC)}}\right)}{1+\frac{1}{I^{(EC)}+R^{(EC)}}\left(\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(EC)}}{x_{io}^{(EC)}}+\sum_{r=1}^{R^{(EC)}}\frac{s_r^{(EC)}}{v_{ro}^{(EC)}}\right)}\tag{8}$$

$s.t.$

the economic constraints of Technology (7).

$$\delta_o^{*Environment}=\frac{1-\frac{1}{N^{(EN)}}\left(\sum_{n=1}^{N^{(EN)}}\frac{s_n^{(EN)}}{w_{no}^{(EN)}}\right)}{1+\frac{1}{I^{(EN)}+R^{(EN)}}\left(\sum_{i=1}^{I^{(EN)}}\frac{s_i^{(EN)}}{x_{io}^{(EN)}}+\sum_{r=1}^{R^{(EN)}}\frac{s_r^{(EN)}}{v_{ro}^{(EN)}}\right)}\tag{9}$$

$s.t.$

the environment constraints of Technology (7).

$$\delta_o^{*Social}=\frac{1-\frac{1}{N^{(SO)}}\left(\sum_{n=1}^{N^{(SO)}}\frac{s_n^{(SO)}}{w_{no}^{(SO)}}\right)}{1+\frac{1}{I^{(SO)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(SO)}}\frac{s_i^{(SO)}}{x_{io}^{(SO)}}+\sum_{r=1}^{R^{(SO)}}\frac{s_r^{(SO)}}{v_{ro}^{(SO)}}\right)}\tag{10}$$

$s.t.$

the social constraints of Technology (7).

Models (8)–(10) are feasible across all $\alpha\in[0,1]$, as error levels and evaluate stochastic efficiency measures from economic, environmental, and social perspectives, respectively. In these models, the intensity variables $\lambda_j^{(EC)}$, $\lambda_j^{(EN)}$ and $\lambda_j^{(SO)}$ assign proportional weights to the DMU under evaluation, while the slack variables ($s_i^{(EC)}$, $s_r^{(EC)}$, $s_n^{(EC)}$, $s_i^{(EN)}$, $s_r^{(EN)}$, $s_n^{(EN)}$, $s_i^{(SO)}$, $s_r^{(SO)}$, $s_n^{(SO)}$) quantify inefficiencies originating from stochastic constraints. Under the managerial disposability assumption, efficiency is enhanced by increasing desirable inputs/outputs or reducing undesirable outputs. The optimal values $\delta_o^{*Economic}$, $\delta_o^{*Environment}$, and $\delta_o^{*Social}$ represent the stochastic efficiency scores for the economic, environmental, and social dimensions at confidence level α, respectively. Consequently, the evaluated unit DMU_o is classified as stochastically efficient in a given dimension at level α **if and only if** its corresponding optimal value equals 1. This condition is equivalent to all associated slack variables being zero.

Models (8), (9), and (10) are linear fractional programming problems that can be transformed into a linear form using the manner of Charnes and Cooper [55] linear transformation.

Given our primary goal of evaluating stochastic sustainability from economic, environmental, and social perspectives, all three dimensions are considered simultaneously according to technology (7). The stochastic sustainability model, based on the SBM model, is proposed as (the detailed constraints of the model are provided in the Appendix B) (see the Eq. (11) in Box I): the economic, environmental, and social constraints defined in Technology (7).

In the above model, the intensity variables $\lambda_j^{(EC)}$, $\lambda_j^{(EN)}$, and $\lambda_j^{(SO)}$ allocate proportional weights to the DMU under evaluation, whereas the slack variables $s_i^{(EC)},s_r^{(EC)},s_n^{(EC)},s_i^{(EC)},s_r^{(EC)},s_n^{(EC)},s_i^{(EN)},s_r^{(EN)}$, and $s_n^{(EN)}$ measure inefficiencies arising from the stochastic constraints. Model (11) is a linear fractional programming problem and proceeding with the usual manner of Charnes and Cooper [55], it can easily be transformed into a linear format. To do so, make the following changes of variables (see the equation in Box II): $\beta\lambda_j^{(EC)}=\gamma_j^{(EC)}$, $\beta\lambda_j^{(EN)}=\gamma_j^{(EN)}$, $\beta\lambda_j^{(SO)}=\gamma_j^{(SO)}$, $\beta q_i^{+(EC)}=p_i^{+(EC)}$, $\beta q_r^{+(EC)}=p_r^{+(EC)}$, $\beta q_n^{+(EC)}=p_n^{+(EC)}$, $\beta q_i^{+(EN)}=p_i^{+(EN)}$, $\beta q_r^{+(EN)}=p_r^{+(EN)}$, $\beta q_n^{+(EN)}=p_n^{+(EN)}$, $\beta q_i^{+(SO)}=p_i^{+(SO)}$, $\beta q_r^{+(SO)}=p_r^{+(SO)}$, $\beta q_n^{+(SO)}=p_n^{+(SO)}$, $\beta s_i^{(EC)}=\eta_i^{(EC)}$, $\beta s_r^{(EC)}=\eta_r^{(EC)}$, $\beta s_n^{(EC)}=\eta_n^{(EC)}$, $\beta s_i^{(EN)}=\eta_i^{(EN)}$, $\beta s_r^{(EN)}=\eta_r^{(EN)}$, $\beta s_n^{(EN)}=\eta_n^{(EN)}$, $\beta s_i^{(SO)}=\eta_i^{(SO)}$, $\beta s_r^{(SO)}=\eta_r^{(SO)}$, and $\beta s_n^{(SO)}=\eta_n^{(SO)}$. Therefore, we have

$$\delta_o^*=Min\ \beta-\frac{1}{N^{(EC)}+N^{(EN)}+N^{(SO)}}\times\left(\sum_{n=1}^{T^{(EC)}}\frac{\eta_n^{(EC)}}{w_{no}^{(EC)}}+\sum_{n=1}^{T^{(EC)}}\frac{\eta_n^{(EN)}}{w_{no}^{(EN)}}+\sum_{n=1}^{T^{(EC)}}\frac{\eta_n^{(SO)}}{w_{no}^{(SO)}}\right)$$

$s.t.$

Economic constraints:

$$\sum_{j=1}^{J}\gamma_j^{(EC)}x_{ij}^{(EC)}+\phi^{-1}(\alpha)\sigma(p_i^{+(EC)}+p_i^{-(EC)})-\eta_i^{(EC)}=\beta x_{io}^{(EC)},\ i\in I^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}a_{ij}^{(EC)}-\beta a_{io}^{(EC)}=p_i^{+(EC)}-p_i^{-(EC)},\ i\in I^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}v_{rj}^{(EC)}+\phi^{-1}(\alpha)\sigma(p_r^{+(EC)}+p_r^{-(EC)})-\eta_r^{(EC)}=v_{ro}^{(EC)},\ r\in R^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}b_{rj}^{(EC)}-\beta b_{ro}^{(EC)}=p_r^{(EC)}-p_r^{-(EC)},r\in R^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}w_{nj}^{(EC)}-\phi^{-1}(\alpha)\sigma(p_n^{+(EC)}+p_n^{-(EC)})+\eta_n^{(EC)}=w_{no}^{(EC)},n\in N^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}c_{nj}^{(EC)}-c_{no}^{(EC)}=p_n^{+(EC)}+p_n^{-(EC)},n\in N^{EC},$$
$$\sum_{j=1}^{J}\gamma_j^{(EC)}=\beta,$$
$$\gamma_j^{(EC)},\eta_i^{(EC)},\eta_r^{(EC)},\eta_n^{(EC)},p_i^{+(EC)},p_i^{-(EC)},p_r^{+(EC)},p_r^{-(EC)},$$
$$p_n^{+(EC)},p_n^{-(EC)}\geq 0,\forall j,i,r,n,$$

Environmental constraints:

$$\sum_{j=1}^{J}\gamma_j^{(EN)}x_{ij}^{(EN)}+\phi^{-1}(\alpha)\sigma(p_i^{+(EN)}+p_i^{-(EN)})-\eta_i^{(EN)}=\beta x_{io}^{(EN)},\ i\in I^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}a_{ij}^{(EN)}-\beta a_{io}^{(EN)}=p_i^{+(EN)}-p_i^{-(EN)},\ i\in I^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}v_{rj}^{(EN)}+\phi^{-1}(\alpha)\sigma(p_r^{+(EN)}+p_r^{-(EN)})-\eta_r^{(EN)}=\beta v_{ro}^{(EN)},\ r\in R^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}b_{rj}^{(EN)}-\beta b_{ro}^{(EN)}=p_r^{+(EN)}-p_r^{-(EN)},r\in R^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}w_{nj}^{(EN)}-\phi^{-1}(\alpha)\sigma(p_n^{+(EN)}+p_n^{-(EN)})+\eta_n^{(EN)}=\beta w_{no}^{(EN)},n\in N^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}c_{nj}^{(EN)}-\beta c_{no}^{(EN)}=p_n^{+(EN)}+p_n^{-(EN)},n\in N^{EN},$$
$$\sum_{j=1}^{J}\gamma_j^{(EN)}=\beta,$$
$$\gamma_j^{(EN)},\eta_i^{(EN)},\eta_r^{(EN)},\eta_n^{(EN)},p_i^{+(EN)},p_i^{-(EN)},p_r^{+(EN)},p_r^{-(EN)},$$
$$p_n^{+(EN)},p_n^{-(EN)}\geq 0,\forall j,i,r,n,$$

$$\delta_o^* = Min \frac{1-\frac{1}{N^{(EC)}+N^{(EN)}+N^{(SO)}}\left(\sum_{n=1}^{N^{(EC)}}\frac{s_n^{(EC)}}{w_{no}^{(EC)}}+\sum_{n=1}^{N^{(EC)}}\frac{s_n^{(EN)}}{w_{no}^{(EN)}}+\sum_{n=1}^{N^{(EC)}}\frac{s_n^{(SO)}}{w_{no}^{(SO)}}\right)}{1+\frac{1}{I^{(EC)}+I^{(EN)}+I^{(SO)}+R^{(EC)}+R^{(EN)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(EC)}}{x_{io}^{(EC)}}+\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(EN)}}{x_{io}^{(EN)}}+\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(SO)}}{x_{io}^{(SO)}}+\sum_{r=1}^{R^{(EC)}}\frac{s_r^{(EC)}}{v_{ro}^{(EC)}}+\sum_{r=1}^{R^{(EN)}}\frac{s_r^{(EN)}}{v_{ro}^{(EN)}}+\sum_{r=1}^{R^{(SO)}}\frac{s_r^{(SO)}}{v_{ro}^{(SO)}}\right)} \quad (11)$$

s.t.

Box I.

$$\frac{1}{1+\frac{1}{I^{(EC)}+I^{(EN)}+I^{(SO)}+R^{(EC)}+R^{(EN)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(EC)}}{x_{io}^{(EC)}}+\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(EN)}}{x_{io}^{(EN)}}+\sum_{i=1}^{I^{(EC)}}\frac{s_i^{(SO)}}{x_{io}^{(SO)}}+\sum_{r=1}^{R^{(EC)}}\frac{s_r^{(EC)}}{v_{ro}^{(EC)}}+\sum_{r=1}^{R^{(EN)}}\frac{s_r^{(EN)}}{v_{ro}^{(EN)}}+\sum_{r=1}^{R^{(SO)}}\frac{s_r^{(SO)}}{v_{ro}^{(SO)}}\right)} = \beta(>0),$$

Box II.

Social constraints

$$\begin{aligned}
&\sum_{j=1}^{J}\gamma_j^{(SO)}x_{ij}^{(SO)}+\phi^{-1}(\alpha)\sigma(p_i^{+(SO)}+p_i^{-(SO)})-\eta_i^{(SO)}=\beta x_{io}^{(SO)},\ i\in I^{SO},\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}a_{ij}^{(SO)}-\beta a_{io}^{(SO)}=p_i^{+(SO)}-p_i^{-(SO)},\ i\in I^{SO},\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}v_{rj}^{(SO)}+\phi^{-1}(\alpha)\sigma(p_r^{+(SO)}+p_r^{-(SO)})-\eta_r^{(SO)}=\beta v_{ro}^{(SO)},\ r\in R^{SO},\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}b_{rj}^{(SO)}-\beta b_{ro}^{(SO)}=p_r^{+(SO)}-p_r^{-(SO)}, r\in R^{SO}, \qquad (12)\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}w_{nj}^{(SO)}-\phi^{-1}(\alpha)\sigma(p_n^{+(SO)}+p_n^{-(SO)})+\eta_n^{(SO)}=\beta w_{no}^{(SO)}, n\in N^{SO},\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}c_{nj}^{(SO)}-\beta c_{no}^{(SO)}=p_n^{+(SO)}+p_n^{-(SO)}, n\in N^{SO},\\
&\sum_{j=1}^{J}\gamma_j^{(SO)}=\beta,\\
&\gamma_j^{(SO)},\eta_i^{(SO)},\eta_r^{(SO)},\eta_n^{(SO)},p_i^{+(SO)},p_i^{-(SO)},p_r^{+(SO)},p_r^{-(SO)},\\
&\quad p_n^{+(SO)},p_n^{-(SO)}\ge 0,\forall j,i,r,n,
\end{aligned}$$

Generic constraints:

$$\beta+\frac{1}{I^{(EC)}+I^{(EN)}+I^{(SO)}+R^{(EC)}+R^{(EN)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(EC)}}\frac{\eta_i^{(EC)}}{x_{io}^{(EC)}}+\sum_{i=1}^{I^{(EC)}}\frac{\eta_i^{(EN)}}{x_{io}^{(EN)}}+\sum_{i=1}^{I^{(EC)}}\frac{\eta_i^{(SO)}}{x_{io}^{(SO)}}+\sum_{r=1}^{R^{(EC)}}\frac{\eta_r^{(EC)}}{v_{ro}^{(EC)}}+\sum_{r=1}^{R^{(EN)}}\frac{\eta_r^{(EN)}}{v_{ro}^{(EN)}}+\sum_{r=1}^{R^{(SO)}}\frac{\eta_r^{(SO)}}{v_{ro}^{(SO)}}\right)=1,$$

$\beta > 0.$

In model (12), the intensity variables $\gamma_j^{(EN)}, \gamma_j^{(SO)}$, and $\gamma_j^{(EC)}$ determine the proportional contributions of the DMUs, while the slack variables $\eta_i^{(EC)}, \eta_r^{(EC)}, \eta_n^{(EC)}, \eta_i^{(EN)}, \eta_r^{(EN)}, \eta_n^{(EN)} \eta_i^{(SO)}, \eta_r^{(SO)}$, and $\eta_n^{(SO)}$ capture inefficiencies induced by stochastic constraints.

Definition 3. DMU_o is stochastic sustainable if and only if $\delta_o^* = 1$, indicating that all optimal slack variables are zero ($\eta_i^{*(EC)} = \eta_r^{*(EC)} = \eta_n^{*(EC)} = \eta_i^{*(EN)} = \eta_r^{*(EN)} = \eta_n^{*(EN)} = \eta_i^{*(SO)} = \eta_r^{*(SO)} = \eta_n^{*(SO)} = 0$).

According to Model (12), stochastic sustainability for three aspects: economic, environmental, and social, can be obtained as follows:

$$\delta_o^{*Economic} = \frac{1-\frac{1}{N^{(EC)}}\left(\sum_{n=1}^{N^{(EC)}}\frac{s_n^{*(EC)}}{w_{no}^{(EC)}}\right)}{1+\frac{1}{I^{(EC)}+R^{(EC)}}\left(\sum_{i=1}^{I^{(EC)}}\frac{s_i^{*(EC)}}{x_{io}^{(EC)}}+\sum_{r=1}^{R^{(EC)}}\frac{s_r^{*(EC)}}{v_{ro}^{(EC)}}\right)} \quad (13)$$

$$\delta_o^{*Environment} = \frac{1-\frac{1}{N^{(EN)}}\left(\sum_{n=1}^{N^{(EN)}}\frac{s_n^{*(EN)}}{w_{no}^{(EN)}}\right)}{1+\frac{1}{I^{(EN)}+R^{(EN)}}\left(\sum_{i=1}^{I^{(EN)}}\frac{s_i^{*(EN)}}{x_{io}^{(EN)}}+\sum_{r=1}^{R^{(EN)}}\frac{s_r^{*(EN)}}{v_{ro}^{(EN)}}\right)} \quad (14)$$

$$\delta_o^{*Social} = \frac{1-\frac{1}{N^{(SO)}}\left(\sum_{n=1}^{N^{(SO)}}\frac{s_n^{*(SO)}}{w_{no}^{(SO)}}\right)}{1+\frac{1}{I^{(SO)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(SO)}}\frac{s_i^{*(SO)}}{x_{io}^{(SO)}}+\sum_{r=1}^{R^{(SO)}}\frac{s_r^{*(SO)}}{v_{ro}^{(SO)}}\right)} \quad (15)$$

Given that β is positive, $s_i^{*(EC)}, s_r^{*(EC)}, s_n^{*(EC)}, s_i^{*(EN)}, s_r^{*(EN)}, s_n^{*(EN)}, s_i^{*(SO)}$, $s_r^{*(SO)}$, and $s_n^{*(SO)}$ can be obtained from the optimal values of Model (12) as $s_i^{*(EC)} = \frac{\eta_i^{*(EC)}}{\beta^*}$, $s_r^{*(EC)} = \frac{\eta_r^{*(EC)}}{\beta^*}$, $s_n^{*(EC)} = \frac{\eta_n^{*(EC)}}{\beta^*}$, $s_i^{*(EN)} = \frac{\eta_i^{*(EN)}}{\beta^*}$, $s_r^{*(EN)} = \frac{\eta_r^{*(EN)}}{\beta^*}$, $s_n^{*(EN)} = \frac{\eta_n^{*(EN)}}{\beta^*}$, $s_i^{*(SO)} = \frac{\eta_i^{*(SO)}}{\beta^*}$, $s_r^{*(SO)} = \frac{\eta_r^{*(SO)}}{\beta^*}$, $s_n^{*(SO)} = \frac{\eta_n^{*(SO)}}{\beta^*}$.

Definition 4. DMU_o is stochastically economic sustainable if and only if $\delta_o^{*Economic} = 1$.

Definition 5. DMU_o is stochastically environmentally sustainable if and only if $\delta_o^{*Environment} = 1$.

Definition 6. DMU_o is stochastically social sustainable if and only if $\delta_o^{*Social} = 1$.

Based on the above definitions, DMU_o is stochastically sustainable if and only if it is sustainable from all three aspects.

5. An illustrative application

Assessing the sustainability of countries is crucial for policymakers to identify areas for improvement, monitor progress over time, and make informed decisions to promote sustainable development. Given the significance of this issue, this section aims to investigate the sustainability of 59 countries spanning four continents — Africa, Europe, North America, and Asia — over a 12-year period from 2010 to 2022. The analysis focuses on evaluating country-level performance; accordingly, each country is treated as a DMU. The initial data were sourced from the *World Bank* (https://www.worldbank.org/), and after necessary modifications, we obtained data for 59 countries across these four continents. For illustrative purposes, all input and output variables are treated as stochastic. We first checked the Normality of the data set. Toward this end, we performed "Goodness of fit" test to check if the data follows Normal distribution. After the test, we concluded that there is no evidence to reject the Normal distribution assumption of the data. In this case, the parameters of the distribution (the means and standard deviations) are calculated across the 12-year period for each country.

Incorporating stochasticity, particularly through the use of variance, enhances the modeling process by capturing the inherent uncertainty in the data. This approach allows for a clearer distinction between countries that demonstrate consistent performance across time and those whose efficiency is more volatile and susceptible to fluctuations, which might be overlooked in deterministic models that rely solely on mean values. In the subsequent subsections, we will introduce inputs and outputs, followed by the presentation of results.

5.1. Description of variables and data

The selection of indicators for economic, environmental, and social dimensions was guided by two main criteria: (i) availability and reliability of data from reputable sources such as the World Bank, and (ii) alignment with sustainability concepts as evidenced in previous literature (see Section 2). Indicators were chosen to cover different aspects of each dimension — inputs, desirable outputs, and undesirable outputs — while ensuring consistency and comparability across countries. Inputs and outputs for each dimension — economic, environmental, and social — are introduced as follows: (Please note that descriptions are based on data from the World Bank.)

◆ **Economic dimension**

- **Inputs**
  - *Imports of goods and services* represent the value of all goods and other market services received from the rest of the world.
- **Desirable Outputs**
  - *GDP* is the total monetary or market value of all the finished goods and services produced within a country's borders in a specific period.
  - *GNI* is the sum of value added by all resident producers plus any product taxes (less subsidies) not included in the valuation of output plus net receipts of primary income (compensation of employees and property income) from abroad.
  - *Exports of goods and services* represent the value of all goods and other market services provided to the rest of the world.
- **Undesirable Output**
  - *Inflation:* Inflation as measured by the consumer price index reflects the annual percentage change in the cost to the average consumer of acquiring a basket of goods and services that may be fixed or changed at specified intervals, such as yearly.

◆ **Environmental dimension**

- **Inputs**
  - *Energy use* refers to use of primary energy before transformation to other end-use fuels, which is equal to indigenous production plus imports and stock changes, minus exports and fuels supplied to ships and aircraft engaged in international transport.
- **Desirable Outputs**
  - *Agriculture, forestry, and fishing, value added* encompass activities such as forestry, hunting, and fishing, as well as the cultivation of crops and livestock production.
- **Undesirable Output**
  - *PM2.5 Air pollution* is defined as the percentage of the population exposed to ambient concentrations of PM2.5 that exceed the WHO guideline value.

◆ **Social dimension**

- **Inputs**
  - *Prevalence of moderate or severe food insecurity in the population:* The percentage of people in the population who live in households classified as moderately or severely food insecure.
  - *Cost of business start-up procedures:* The cost to register a business is normalized by presenting it as a percentage of gross national income (GNI) per capita.
- **Desirable Outputs**
  - *Life expectancy at birth* indicates the number of years a newborn infant would live if prevailing patterns of mortality at the time of its birth were to stay the same throughout its life.
  - *Current health expenditure* estimates of current health expenditures include healthcare goods and services consumed during each year. This indicator does not include capital health expenditures such as buildings, machinery, IT and stocks of vaccines for emergency or outbreaks.
  - *Human capital index:* The HCI calculates the contributions of health and education to worker productivity. The final index score ranges from zero to one and measures productivity as the future potential of a child born today relative to the benchmark of full health and complete education.
- **Undesirable Output**
  - *Unemployment* refers to the share of the labor force that is without work but available for and seeking employment. Definitions of labor force and unemployment differ by country.

The selected indicators are macro-level variables that are inherently influenced by economic cycles, policy interventions, institutional changes, environmental conditions, and external shocks. Consequently, these variables exhibit intertemporal variability and measurement uncertainty, making a stochastic representation more appropriate than a purely deterministic treatment.

It is important to note that we incorporated the social input of "Prevalence of moderate or severe food insecurity in the population" under the assumption of natural disposability, indicating that the mentioned input was decreased. The statistical summary, including the mean and standard deviation of variables for each dimension, is provided in Tables 1 and 2.

5.2. Results of stochastic sustainability assessment for countries

To assess the stochastic sustainability of 59 countries, we considered five confidence levels (0.1, 0.2, 0.3, 0.4, and 0.5). Cooper et al. [56] suggest that, in general, the tolerance level of chance constraints should satisfy $0 < \alpha \leq 0.5$, for which technical efficiency scores lie between zero and one. When the data involve high levels of uncertainty and $0.5 \leq \alpha < 1$, the efficiency frontier shifts toward the observed data, leading many observations to attain unity or near-unity efficiency scores. Under such conditions, the results of the chance-constrained program may be questioned. For this reason, most studies in stochastic settings consider confidence levels within the interval $0 < \alpha \leq 0.5$. The detailed results for stochastic sustainability across all three aspects —

Table 1
Statistical summary of mean of variables in economic, environmental, and social dimensions.

Dimension	Type of variables	Variables	Min	Mean	Max	S. D.	Q1	Median	Q3
Economic	Input	Imports of goods and services	2.02E+01	4.87E+01	1.57E+02	2.59E+01	2.02E+01	4.12E+01	5.89E+01
	Desirable outputs	GNI	6.55E+09	3.28E+11	3.90E+12	6.33E+11	6.55E+09	7.20E+10	3.09E+11
		GDP	6.40E+09	3.28E+11	3.76E+12	6.21E+11	6.40E+09	7.35E+10	2.91E+11
		Exports of goods and services	9.76E+00	4.70E+01	1.89E+02	3.13E+01	9.76E+00	3.94E+01	5.83E+01
	Undesirable output	Inflation	2.26E−01	4.70E+00	3.40E+01	5.30E+00	2.26E−01	2.92E+00	4.81E+00
Environmental	Input	Energy use	1.35E+02	2.56E+03	1.77E+04	2.84E+03	1.35E+02	1.94E+03	3.28E+03
	Desirable output	Agriculture, forestry, and fishing, value added	2.42E−01	8.88E+00	3.75E+01	9.21E+00	2.42E−01	4.73E+00	1.23E+01
	Undesirable output	PM2.5 Air pollution	1.84E−01	8.83E+01	1.00E+02	2.75E+01	1.84E−01	1.00E+02	1.00E+02
Social	Inputs	Prevalence of moderate or severe food insecurity in the population	2.46E+00	2.46E+01	8.69E+01	2.33E+01	2.46E+00	1.21E+01	4.26E+01
		Cost of business start-up procedures	4.00E−01	9.40E+01	3.54E+03	4.55E+02	4.00E−01	1.59E+01	3.65E+01
	Desirable outputs	Life expectancy at birth	5.76E+01	7.34E+01	8.32E+01	7.57E+00	5.76E+01	7.47E+01	8.07E+01
		Current health expenditure	2.38E+00	6.70E+00	1.14E+01	2.49E+00	2.38E+00	6.70E+00	8.83E+00
		Human capital index	3.17E−01	6.07E−01	8.11E−01	1.43E−01	3.17E−01	6.23E−01	7.43E−01
	Undesirable output	Unemployment	5.53E−01	6.90E+00	2.22E+01	4.54E+00	5.53E−01	5.57E+00	9.23E+00

Table 2
Statistical summary of standard deviation of variables in economic, environmental, and social dimensions.

Dimension	Type of variables	Variables	Min	Mean	Max	S. D.	Q1	Median	Q3
Economic	Input	Imports of goods and services	1.37E+00	5.30E+00	1.44E+01	2.77E+00	1.37E+00	4.67E+00	6.38E+00
	Desirable outputs	GNI	1.08E+09	3.12E+10	2.55E+11	5.08E+10	1.08E+09	1.46E+10	2.93E+10
		GDP	1.07E+09	3.43E+10	3.34E+11	5.79E+10	1.07E+09	1.36E+10	3.29E+10
		Exports of goods and services	1.55E+00	4.88E+00	1.51E+01	2.63E+00	1.55E+00	4.63E+00	5.88E+00
	Undesirable output	Inflation	9.64E−01	3.98E+00	5.92E+01	7.55E+00	9.64E−01	2.25E+00	3.66E+00
Environmental	Input	Energy use	2.51E+00	1.22E+02	7.57E+02	1.45E+02	2.51E+00	7.76E+01	1.50E+02
	Desirable output	Agriculture, forestry, and fishing, value added	2.77E−02	9.57E−01	5.66E+00	1.24E+00	2.77E−02	4.01E−01	1.11E+00
	Undesirable output	PM2.5 Air pollution	0.00E+00	2.00E+00	1.75E+01	4.16E+00	0.00E+00	1.93E−02	1.57E+00
Social	Inputs	Prevalence of moderate or severe food insecurity in the population	8.16E−02	2.11E+00	8.36E+00	1.94E+00	8.16E−02	1.43E+00	2.73E+00
		Cost of business start-up procedures	1.50E−01	8.93E+01	3.58E+03	4.63E+02	1.50E−01	5.82E+00	1.81E+01
	Desirable outputs	Life expectancy at birth	3.48E−01	9.31E−01	4.81E+00	6.40E−01	3.48E−01	7.11E−01	1.13E+00
		Current health expenditure	1.12E−01	5.03E−01	1.36E+00	2.55E−01	1.12E−01	5.02E−01	6.18E−01
		Human capital index	0.00E+00	1.63E+00	5.75E+00	1.42E+00	0.00E+00	1.12E+00	2.31E+00
	Undesirable output	Unemployment	1.37E+00	5.30E+00	1.44E+01	2.77E+00	1.37E+00	4.67E+00	6.38E+00

economic, environmental, and social — are presented in the Appendix C, Tables 3, 4, 5, and 6.

Among the continents of Europe, Africa, and Asia, Sweden, Ethiopia, and Kuwait exhibit the highest levels of stochastic sustainability across all confidence levels, respectively. Notably, Ethiopia and Kuwait demonstrate stochastic environmental sustainability, while Sweden showcases stochastic social sustainability.

Ethiopia's high environmental efficiency contrasts with its lower economic performance, reflecting that resource-dependent economies may achieve environmental gains primarily due to limited industrial activity rather than proactive environmental management. Sweden's strong social efficiency aligns with well-developed welfare systems, emphasizing the role of institutional capacity in supporting social outcomes.

In North America, Ecuador achieves maximum stochastic sustainability at confidence levels of 0.1 and 0.2, El Salvador at 0.3, and Guatemala at 0.4 and 0.5. However, none of these countries exhibit stochastic sustainability across any aspect. The African continent shows the lowest values for stochastic sustainability, with Algeria recording minimum values at confidence levels 0.1 and 0.2, and Angola spanning confidence levels 0.3 to 0.5. In Europe, Serbia holds the minimum value at 0.1 confidence level, while Spain demonstrates minimum values across confidence levels ranging from 0.2 to 0.5. Turning to North America, Mexico shows the minimum value for stochastic sustainability at confidence level 0.1, whereas the Dominican Republic records minimum values across confidence levels 0.2 to 0.5. Furthermore, in Asia, Uzbekistan and Lebanon display the minimum values of stochastic sustainability, with two (0.1 and 0.2) and three (0.3, 0.4, and 0.5) confidence levels, respectively.

These patterns reveal structural vulnerabilities: North American countries, despite relatively high HDI, show lower resilience under stochastic evaluation, likely due to environmental pressures or social

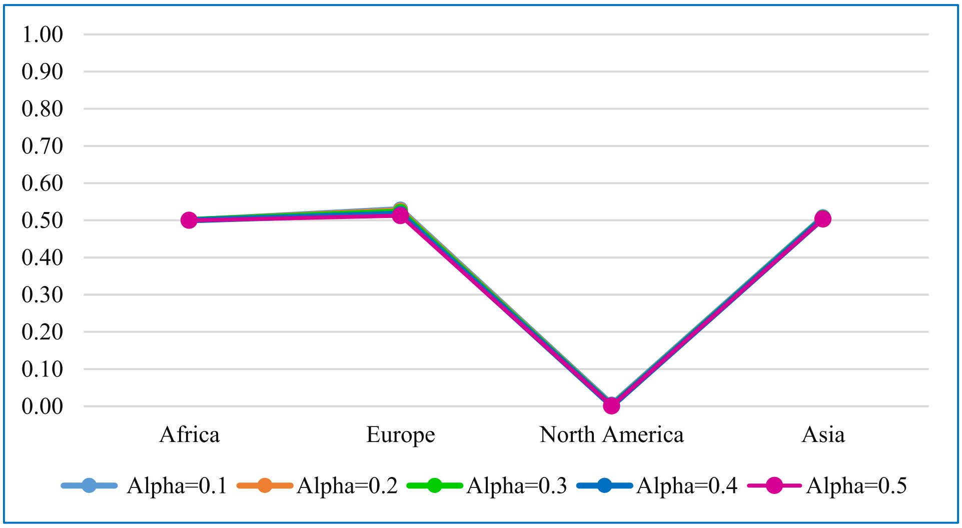

Fig. 2. Average stochastic sustainability.

disparities. African countries with lower HDI, like Algeria and Angola, display sporadic resilience in specific dimensions, indicating that conventional development metrics alone do not fully capture sustainability under uncertainty. Asia presents mixed outcomes, with countries such as Kuwait and Cambodia showing strong performance in environmental and social dimensions respectively, aligned with intermediate HDI levels.

Across the three dimensions, the social dimension had the fewest sustainable countries. Only Europe and Asia had stochastic social sustainable countries among the four continents. Specifically, Finland and Sweden from Europe, with five confidence levels, and Cambodia from Asia, with a confidence level of 0.1, are stochastic social sustainable. In contrast, the environmental dimension exhibited the highest number of sustainable countries. Ethiopia, an African nation, achieved stochastic sustainability at five confidence levels, while Niger and Benin attained stochastic sustainability at four confidence levels. In Europe, Finland, Iceland, and Norway demonstrated stochastic sustainability across all five confidence levels, while Estonia achieved it across four confidence levels. Additionally, Cambodia and Kuwait displayed stochastic sustainability with all five confidence levels. Notably, no country in North America demonstrated stochastic sustainability.

Regarding the economic dimension, Lebanon was the only country in Asia that demonstrated stochastic sustainability at just two levels. In Europe, Bosnia and Herzegovina and Greece achieved stochastically sustainability with three confidence levels, while Germany, Luxembourg, Malta, Moldova and Switzerland achieved it with all confidence levels. Unlike the two aforementioned continents, Africa and North America lacked any instances of stochastic sustainability.

5.3. Results of stochastic sustainability assessment for continents

In this section, the results of the stochastic sustainability assessment for four continents — Africa, Europe, North America, and Asia — are presented. The evaluation was conducted across five confidence levels (0.1, 0.2, 0.3, 0.4, and 0.5). Fig. 2 illustrates the average stochastic sustainability across all confidence levels for the four continents. According to this figure, Europe exhibits the highest level of stochastic sustainability, reflecting its strong performance across economic, social, and environmental dimensions. In contrast, North America shows the lowest level of sustainability, indicating significant challenges in achieving balanced development across these dimensions. These results highlight the varying levels of progress among continents in achieving sustainable development goals.

Fig. 3 illustrates the economic dimension of stochastic sustainability across the four continents. Europe consistently achieves the highest economic sustainability scores across all confidence levels, reflecting its strong economic performance, stability, and effective management of key economic indicators. Asia follows closely, demonstrating relatively high economic sustainability, which indicates effective economic policies and resource management. Africa shows gradual improvement across confidence levels, suggesting progress in economic policies and resource management, though it still lags behind Europe and Asia. In contrast, North America exhibits the lowest scores, highlighting significant economic challenges such as inflation, reliance on raw material exports, and limited economic diversification. These results underscore the importance of robust economic policies and targeted interventions to enhance sustainability, particularly in regions like North America and Africa, where economic performance falls short of global standards.

Fig. 4 displays regional environmental sustainability performance. Asia leads with strong green policies and renewable energy investments. Africa, despite developmental challenges, shows good results due to limited industrialization and traditional sustainable agriculture. Europe demonstrates moderate performance, indicating room for improved policy implementation. Meanwhile, North America ranks lowest, hindered by unsustainable consumption patterns and fossil fuel dependence.

Fig. 5 presents the social dimension of stochastic sustainability. Africa achieves the highest scores, reflecting robust community-based social structures and equitable development initiatives. Asia and Europe follow closely, with Europe's advanced social welfare systems nearly matching Asia's rapid improvements in quality-of-life indicators—though both regions face persistent inequities. North America trails significantly, its scores undermined by systemic disparities in healthcare access, income inequality, and social mobility. These contrasts reveal how institutional frameworks and cultural priorities shape social sustainability, urging targeted policy interventions in underperforming regions.

To further contextualize these results, continent-level stochastic sustainability outcomes are compared with trends in the Human Development Index (HDI) over the period 2010–2022 (source: UNDP, Human Development Report, https://hdr.undp.org/data-center/human-development-index). Europe and North America exhibit relatively high HDI growth compared to the global average, but their stochastic sustainability rankings differ substantially, particularly under higher confidence levels. This divergence indicates that improvements in human development do not necessarily translate into resilience under uncertainty.

Conversely, Africa shows moderate HDI growth relative to other continents but continues to lag in economic and social stochastic

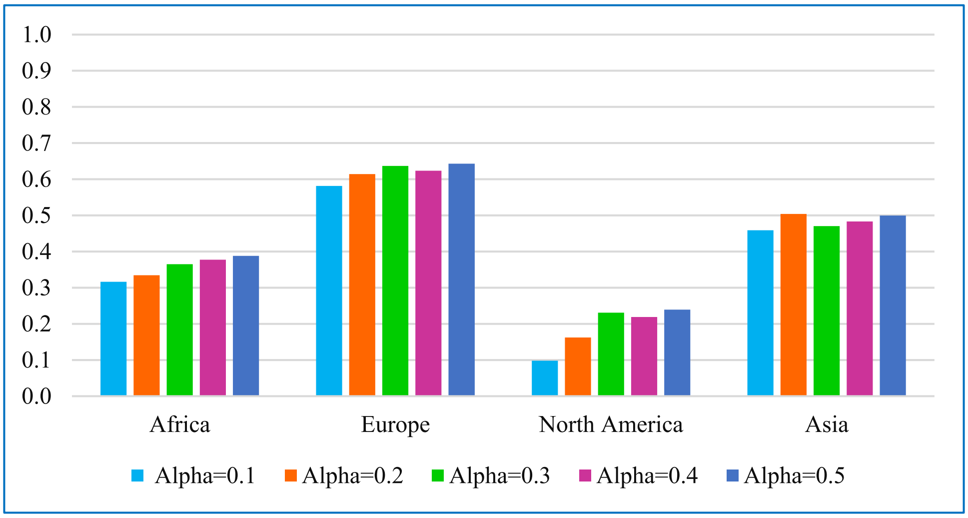

Fig. 3. Average stochastic sustainability of the economic dimension.

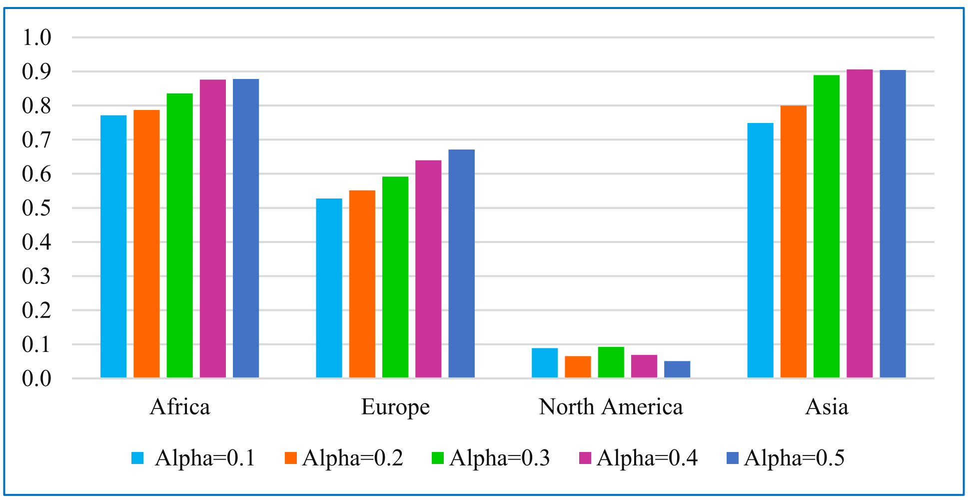

Fig. 4. Average stochastic sustainability of the environmental dimension.

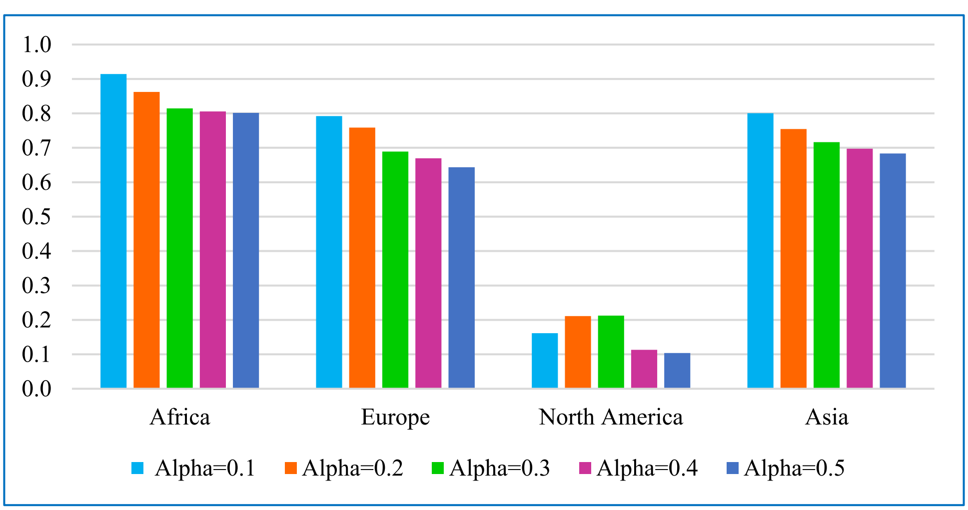

Fig. 5. Average stochastic sustainability of the social dimension.

sustainability, suggesting that development gains alone are insufficient to ensure balanced sustainability performance. Asia demonstrates intermediate HDI growth and generally consistent sustainability across dimensions, highlighting the potential alignment between human development progress and stochastic resilience. Overall, this comparison confirms that the proposed stochastic DEA framework captures structural aspects of sustainability that are not reflected in conventional development indices.

5.4. Policy implications

Our findings carry significant policy implications for sustainable development, environmental indicators, and economic growth, aligning closely with several Sustainable Development Goals (SDGs). From a theoretical and modeling perspective, the results underscore the importance of confidence levels in determining efficiency and highlight the necessity of considering all dimensions of sustainability — economic, environmental, and social — when assessing performance. Focusing on a single dimension fails to provide a comprehensive understanding of a region's sustainability, as the interplay between these dimensions is critical for achieving balanced development.

The study reveals that certain regions have achieved notable success in specific dimensions of sustainability. For instance, some regions in Africa and Asia have demonstrated strong environmental sustainability, while others in Europe have excelled in social sustainability. These regions can serve as benchmarks for others striving to improve their sustainability performance. However, the analysis also identifies a recurring challenge: regions that perform well economically often struggle with environmental sustainability, and vice versa. This misalignment between economic development and environmental protection, particularly in more developed regions, underscores the need for integrated policies that address these trade-offs.

Among the three dimensions, social sustainability emerges as the most challenging, with fewer regions achieving significant progress in this area. Key social factors such as food insecurity, barriers to business start-ups, life expectancy, healthcare expenditure, human capital development, and unemployment rates require urgent attention, especially in regions where social sustainability lags. In contrast, environmental sustainability shows the highest number of successful cases, with several regions effectively managing natural resources and reducing pollution. However, the study highlights that economic sustainability often comes at the expense of environmental health, indicating that economic growth and environmental preservation have not yet been harmonized in many regions.

These findings emphasize the importance of adopting a multidimensional approach to sustainability. Policymakers must design strategies that simultaneously address economic, environmental, and social challenges, ensuring that progress in one dimension does not undermine efforts in another. By learning from regions that have achieved success in specific dimensions and addressing the gaps in others, it is possible to move closer to the global goals of sustainable development. This requires not only targeted interventions but also a commitment to long-term, integrated planning that aligns economic growth with environmental stewardship and social equity.

6. Concluding remarks

Sustainability assessment has become a central focus for researchers in recent years. Surveys indicate that the majority of studies concentrate on sustainability from a singular perspective, with only a few conducting comprehensive analyses across economic, social, and environmental dimensions. Recognizing the stochastic and uncertain nature of data in various applications, this study introduces a model for evaluating sustainability within a stochastic environment using the DEA technique.

The proposed model explores sustainability from economic, social, and environmental standpoints, incorporating the assumption of managerial disposability to address undesirable factors. Applied to assess the sustainability of 59 countries across Africa, Europe, North America, and Asia, the results revealed that among the four continents, Europe exhibited the highest levels of stochastic sustainability, while North America demonstrated the lowest. Furthermore, Europe, Asia, and Africa showcased the highest levels of sustainability in the economic, environmental, and social dimensions, respectively. Notably, North America lacked stochastic sustainability across all mentioned dimensions, while Africa lacked it in the social and economic dimensions.

While this study contributes to literature by providing a comprehensive stochastic DEA model for sustainability assessment, it is not without limitations. First, the selection of variables was constrained by data availability, which may have influenced the breadth of the analysis. Second, the inclusion of diverse countries, while offering a global perspective, introduces heterogeneity that could affect the comparability of results. In particular, differences in the number and characteristics of countries selected from each continent may have influenced the DEA outcomes. Since DEA assumes homogeneity among DMUs, such regional disparities, especially between countries from Africa or Asia, could pose a challenge to this assumption. Due to data limitations, we conducted our analysis based on available data; however, future research may benefit from continent-specific models or clustering of countries with similar development levels to ensure more homogeneous comparisons. Additionally, the use of alternative indicators or more granular data could enhance the robustness of the findings.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Let the stochastic inputs and outputs be expressed as $\widetilde{x}_{ij} = x_{ij} + a_{ij}\widetilde{\epsilon}_{ij}$ and $\widetilde{y}_{rj} = y_{rj} + b_{rj}\widetilde{\tau}_{rj}$, respectively. Here, x_{ij} and y_{rj} denote the expected input and output levels, while a_{ij} and b_{rj} represent the standard deviations of $\widetilde{x}_{ij}$ and $\widetilde{y}_{rj}$, respectively. Furthermore, suppose that all the inputs and outputs are uncorrelated, i.e. for each $j \neq k$, $\text{cov}(\widetilde{x}_{ij}, \widetilde{x}_{ik}) = 0$ and $\text{cov}(\widetilde{y}_{rj}, \widetilde{y}_{rk}) = 0$. Therefore, the stochastic inputs and outputs are assumed to follow normal distributions, such that $\widetilde{x}_{ij} \sim N(x_{ij}, \sigma^2 a_{ij}^2)$ and $\widetilde{y}_{rj} \sim N(y_{rj}, \sigma^2 b_{rj}^2)$.

For each $j = 1, \ldots, J$, let $\widetilde{\epsilon}_i = \widetilde{\epsilon}_{ij}$, $\widetilde{\tau}_r = \widetilde{\tau}_{rj}$ and let $\sigma = 1$. Now, consider the ith input chance constraint in model (3) as follows:

$$P\left\{\sum_{j=1}^{J} \lambda_j \widetilde{x}_{ij} \le \theta \widetilde{x}_{io}\right\} \ge 1 - \alpha, \ i = 1, \ldots, I.$$

By substituting $\widetilde{h}_i = \sum_{j=1}^{j} \lambda_j \widetilde{x}_{ij} - \theta \widetilde{x}_{io}$, and under the assumptions of normality, the specified error structure, and the absence of correlation among inputs and outputs, we have:

$$\widetilde{h}_i = \sum_{j=1}^{J} \lambda_j x_{ij} - \theta x_{io} + \widetilde{\epsilon}_i\left(\sum_{j=1}^{J} \lambda_j a_{ij} - \theta a_{io}\right) \text{ where}$$

$$\widetilde{h}_i \sim N\left(\sum_{j=1}^{J} \lambda_j \widetilde{x}_{ij} - \theta \widetilde{x}_{io},\ \ \sigma^2\left(\sum_{j=1}^{J} \lambda_j a_{ij} - \theta a_{io}\right)^2\right).$$

Therefore, the deterministic form of the input constraint becomes $\sum_{j=1}^{J}\lambda_j x_{ij} - \phi^{-1}(\alpha)\sigma\left|\sum_{j=1}^{J}\lambda_j a_{ij} - \theta a_{io}\right| \le \theta x_{io}$. Similarly, the deterministic form of the output chance constraint of model (3) can be expressed as $\sum_{j=1}^{J}\lambda_j y_{rj} - \phi^{-1}(\alpha)\sigma\left|\sum_{j=1}^{J}\lambda_j b_{rj} - b_{ro}\right| \le y_{ro}$. Consequently, model (3) can be expressed in its deterministic form as

$$
\begin{aligned}
& Min\,\theta \\
& s.t. \\
& \sum_{j=1}^{J}\lambda_j x_{ij} - \phi^{-1}(\alpha)\sigma\left|\sum_{j=1}^{J}\lambda_j a_{ij} - \theta a_{io}\right| \le \theta x_{io}, i=1,\dots,I, \\
& \sum_{j=1}^{J}\lambda_j y_{rj} + \phi^{-1}(\alpha)\sigma\left|\sum_{j=1}^{J}\lambda_j b_{rj} - b_{ro}\right| \ge y_{ro}, r=1,\dots,R, \\
& \sum_{j=1}^{J}\lambda_j = 1, \\
& \lambda_j \ge 0,\ \forall j,i,r.
\end{aligned} \tag{16}
$$

Model (16) is nonlinear due to the presence of absolute terms and is linearized through the following transformations:

$$\left|\sum_{j=1}^{J}\lambda_j a_{ij} - \theta a_{io}\right| = p_i^+ + p_i^-(\forall i),$$

$$\sum_{j=1}^{J}\lambda_j a_{ij} - \theta a_{io} = p_i^+ - p_i^-(\forall i),$$

$$p_i^+ p_i^- = 0(\forall i),$$

$$\left|\sum_{j=1}^{J}\lambda_j b_{rj} - b_{ro}\right| = q_r^+ + q_r^-(\forall r),$$

$$\sum_{j=1}^{J}\lambda_j b_{rj} - b_{ro} = q_r^+ - q_r^-(\forall r),$$

$$q_r^+ q_r^- = 0(\forall r),$$

$$p_i^+, p_i^-, q_r^+, q_r^- \ge 0(\forall i, r).$$

The constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$ make problem (?) nonlinear. Dropping these constraints yields a linear programming problem. If an optimal solution exists, it occurs at an extreme point where at least one of p_i^+ or p_i^-, and one of q_r^+ or q_r^-, is zero. Thus, solving the linear program via the simplex method identifies such an extreme point, allowing the constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$ to be implicitly satisfied. Consequently, model (16) can be rewritten as follows (see the equation in Box III):

$$
\begin{aligned}
& \theta_o^* = Min\,\theta \\
& s.t. \\
& \sum_{j=1}^{J}\lambda_j x_{ij} - \phi^{-1}(\alpha)\sigma(p_i^+ + p_i^-) \le \theta x_{io}, i=1,\dots,I, \\
& \sum_{j=1}^{J}\lambda_j\, a_{ij} - \theta a_{io} = p_i^+ - p_i^-,\ i=1,\dots,I, \\
& \sum_{j=1}^{J}\lambda_j y_{rj} + \phi^{-1}(\alpha)\sigma(q_r^+ + q_r^-) \ge y_{ro}, r=1,\dots,R, \\
& \sum_{j=1}^{J}\lambda_j\, b_{rj} - b_{ro} = q_r^+ - q_r^-,\ r=1,\dots,R, \\
& \sum_{j=1}^{J}\lambda_j = 1, \\
& \lambda_j,\ q_i^+,\ q_i^-,\ \ q_r^+, q_r^- \ge 0, \forall j,i,r.
\end{aligned} \tag{17}
$$

Appendix B

$$
\begin{aligned}
& \delta_o^{*Economic} = \frac{1 - \frac{1}{N^{(EC)}}\left(\sum_{n=1}^{N^{(EC)}} \frac{s_n^{(EC)}}{w_{no}^{(EC)}}\right)}{1 + \frac{1}{I^{(EC)}+R^{(EC)}}\left(\sum_{i=1}^{I^{(EC)}} \frac{s_i^{(EC)}}{x_{io}^{(EC)}} + \sum_{r=1}^{R^{(EC)}} \frac{s_r^{(EC)}}{v_{ro}^{(EC)}}\right)} \\
& s.t. \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} x_{ij}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EC)} + q_i^{-(EC)}) - s_i^{(EC)} = x_{io}^{(EC)},\ i \in I^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} a_{ij}^{(EC)} - a_{io}^{(EC)} = q_i^{+(EC)} - q_i^{-(EC)},\ i \in I^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} v_{rj}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EC)} + q_r^{-(EC)}) - s_r^{(EC)} = v_{ro}^{(EC)},\ r \in R^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} b_{rj}^{(EC)} - b_{ro}^{(EC)} = q_r^{(EC)} - q_r^{-(EC)}, r \in R^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} w_{nj}^{(EC)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EC)} + q_n^{-(EC)}) + s_n^{(EC)} = w_{no}^{(EC)}, n \in N^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} c_{nj}^{(EC)} - c_{no}^{(EC)} = q_n^{+(EC)} + q_n^{-(EC)}, n \in N^{EC}, \\
& \sum_{j=1}^{J}\lambda_j^{(EC)} = 1, \\
& \lambda_j^{(EC)}, s_i^{(EC)}, s_r^{(EC)}, s_n^{(EC)}, q_i^{+(EC)}, q_i^{-(EC)}, q_r^{+(EC)}, q_r^{-(EC)}, \\
& \quad q_n^{+(EC)}, q_n^{-(EC)} \ge 0, \forall j,i,r,n.
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
& \delta_o^{*Environment} = \frac{1 - \frac{1}{N^{(EN)}}\left(\sum_{n=1}^{N^{(EN)}} \frac{s_n^{(EN)}}{w_{no}^{(EN)}}\right)}{1 + \frac{1}{I^{(EN)}+R^{(EN)}}\left(\sum_{i=1}^{I^{(EN)}} \frac{s_i^{(EN)}}{x_{io}^{(EN)}} + \sum_{r=1}^{R^{(EN)}} \frac{s_r^{(EN)}}{v_{ro}^{(EN)}}\right)} \\
& s.t. \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} x_{ij}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EN)} + q_i^{-(EN)}) - s_i^{(EN)} = x_{io}^{(EN)},\ i \in I^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} a_{ij}^{(EN)} - a_{io}^{(EN)} = q_i^{+(EN)} - q_i^{-(EN)},\ i \in I^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} v_{rj}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EN)} + q_r^{-(EN)}) - s_r^{(EN)} = v_{ro}^{(EN)},\ r \in R^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} b_{rj}^{(EN)} - b_{ro}^{(EN)} = q_r^{+(EN)} - q_r^{-(EN)}, r \in R^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} w_{nj}^{(EN)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EN)} + q_n^{-(EN)}) + s_n^{(EN)} = w_{no}^{(EN)}, n \in N^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} \sigma(w_{nj}^{(EN)}) - \sigma(w_{no}^{(EN)}) = q_n^{+(EN)} + q_n^{-(EN)}, n \in T^{EN}, \\
& \sum_{j=1}^{J}\lambda_j^{(EN)} = 1, \\
& \lambda_j^{(EN)}, s_i^{(EN)}, s_r^{(EN)}, s_n^{(EN)}, q_i^{+(EN)}, q_i^{-(EN)}, q_r^{+(EN)}, q_r^{-(EN)}, \\
& \quad q_n^{+(EN)}, q_n^{-(EN)} \ge 0, \forall j,i,r,n.
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
& \delta_o^{*Social} = \frac{1 - \frac{1}{N^{(SO)}}\left(\sum_{n=1}^{N^{(SO)}} \frac{s_n^{(SO)}}{w_{no}^{(SO)}}\right)}{1 + \frac{1}{I^{(SO)}+R^{(SO)}}\left(\sum_{i=1}^{I^{(SO)}} \frac{s_i^{(SO)}}{x_{io}^{(SO)}} + \sum_{r=1}^{R^{(SO)}} \frac{s_r^{(SO)}}{v_{ro}^{(SO)}}\right)} \\
& s.t \\
& \sum_{j=1}^{J}\lambda_j^{(SO)} x_{ij}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_i^{+(SO)} + q_i^{-(SO)}) - s_i^{(SO)} = x_{io}^{(SO)},\ i \in I^{Social},
\end{aligned}
$$

$$\delta_o^* = Min \frac{1 - \frac{1}{N^{(EC)}+N^{(EN)}+N^{(SO)}} \left(\sum_{n=1}^{N^{(EC)}} \frac{s_n^{(EC)}}{w_{no}^{(EC)}} + \sum_{n=1}^{N^{(EC)}} \frac{s_n^{(EN)}}{w_{no}^{(EN)}} + \sum_{n=1}^{N^{(EC)}} \frac{s_n^{(SO)}}{w_{no}^{(SO)}} \right)}{1 + \frac{1}{I^{(EC)}+I^{(EN)}+I^{(SO)}+R^{(EC)}+R^{(EN)}+R^{(SO)}} \left(\sum_{i=1}^{I^{(EC)}} \frac{s_i^{(EC)}}{x_{io}^{(EC)}} + \sum_{i=1}^{I^{(EC)}} \frac{s_i^{(EN)}}{x_{io}^{(EN)}} + \sum_{i=1}^{I^{(EC)}} \frac{s_i^{(SO)}}{x_{io}^{(SO)}} + \sum_{r=1}^{R^{(EC)}} \frac{s_r^{(EC)}}{v_{ro}^{(EC)}} + \sum_{r=1}^{R^{(EN)}} \frac{s_r^{(EN)}}{v_{ro}^{(EN)}} + \sum_{r=1}^{R^{(SO)}} \frac{s_r^{(SO)}}{v_{ro}^{(SO)}} \right)}$$

Box III.

$$\sum_{j=1}^{J} \lambda_j^{(SO)} a_{ij}^{(SO)} - a_{io}^{(SO)} = q_i^{+(SO)} - q_i^{-(SO)},\ i \in I^{Social},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} v_{rj}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_r^{+(SO)} + q_r^{-(SO)}) - s_r^{(SO)} = v_{ro}^{(SO)},\ r \in R^{Social},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} b_{rj}^{(SO)} - b_{ro}^{(SO)} = q_r^{+(SO)} - q_r^{-(SO)}, r \in R^{Social},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} w_{nj}^{(SO)} - \phi^{-1}(\alpha)\sigma(q_n^{+(SO)} + q_n^{-(SO)}) + s_n^{(SO)} = w_{no}^{(SO)}, n \in N^{Social},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} c_{nj}^{(SO)} - c_{no}^{(SO)} = q_n^{+(SO)} + q_n^{-(SO)}, n \in N^{Social},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} = 1,$$
$$\lambda_j^{(SO)}, s_i^{(SO)}, s_r^{(SO)}, s_n^{(SO)}, q_i^{+(SO)}, q_i^{-(SO)}, q_r^{+(SO)}, q_r^{-(SO)},$$
$$q_n^{+(SO)}, q_n^{-(SO)} \geq 0, \forall j, i, r, n.$$

s.t. (20)

Economic constraints:

$$\sum_{j=1}^{J} \lambda_j^{(EC)} x_{ij}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EC)} + q_i^{-(EC)}) - s_i^{(EC)} = x_{io}^{(EC)},\ i \in I^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} a_{ij}^{(EC)} - a_{io}^{(EC)} = q_i^{+(EC)} - q_i^{-(EC)},\ i \in I^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} v_{rj}^{(EC)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EC)} + q_r^{-(EC)}) - s_r^{(EC)} = v_{ro}^{(EC)},\ r \in R^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} b_{rj}^{(EC)} - b_{ro}^{(EC)} = q_r^{(EC)} - q_r^{-(EC)}, r \in R^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} w_{nj}^{(EC)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EC)} + q_n^{-(EC)}) + s_n^{(EC)} = w_{no}^{(EC)}, n \in N^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} c_{nj}^{(EC)} - c_{no}^{(EC)} = q_n^{+(EC)} + q_n^{-(EC)}, n \in N^{EC},$$
$$\sum_{j=1}^{J} \lambda_j^{(EC)} = 1,$$
$$\lambda_j^{(EC)}, s_i^{(EC)}, s_r^{(EC)}, s_n^{(EC)}, q_i^{+(EC)}, q_i^{-(EC)}, q_r^{+(EC)}, q_r^{-(EC)},$$
$$q_n^{+(EC)}, q_n^{-(EC)} \geq 0, \forall j, i, r, n,$$

Environmental constraints:

$$\sum_{j=1}^{J} \lambda_j^{(EN)} x_{ij}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_i^{+(EN)} + q_i^{-(EN)}) - s_i^{(EN)} = x_{io}^{(EN)},\ i \in I^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} a_{ij}^{(EN)} - a_{io}^{(EN)} = q_i^{+(EN)} - q_i^{-(EN)},\ i \in I^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} v_{rj}^{(EN)} + \phi^{-1}(\alpha)\sigma(q_r^{+(EN)} + q_r^{-(EN)}) - s_r^{(EN)} = v_{ro}^{(EN)},\ r \in R^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} b_{rj}^{(EN)} - b_{ro}^{(EN)} = q_r^{+(EN)} - q_r^{-(EN)}, r \in R^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} w_{nj}^{(EN)} - \phi^{-1}(\alpha)\sigma(q_n^{+(EN)} + q_n^{-(EN)}) + s_n^{(EN)} = w_{no}^{(EN)}, n \in N^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} \sigma(w_{nj}^{(EN)}) - \sigma(w_{no}^{(EN)}) = q_n^{+(EN)} + q_n^{-(EN)}, n \in N^{EN},$$
$$\sum_{j=1}^{J} \lambda_j^{(EN)} = 1,$$
$$\lambda_j^{(EN)}, s_i^{(EN)}, s_r^{(EN)}, s_n^{(EN)}, q_i^{+(EN)}, q_i^{-(EN)}, q_r^{+(EN)}, q_r^{-(EN)},$$
$$q_n^{+(EN)}, q_n^{-(EN)} \geq 0, \forall j, i, r, n,$$

Social constraints:

$$\sum_{j=1}^{J} \lambda_j^{(SO)} x_{ij}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_i^{+(SO)} + q_i^{-(SO)}) - s_i^{(SO)} = x_{io}^{(SO)},\ i \in I^{SO},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} a_{ij}^{(SO)} - a_{io}^{(SO)} = q_i^{+(SO)} - q_i^{-(SO)},\ i \in I^{SO},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} v_{rj}^{(SO)} + \phi^{-1}(\alpha)\sigma(q_r^{+(SO)} + q_r^{-(SO)}) - s_r^{(SO)} = v_{ro}^{(SO)},\ r \in R^{SO},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} b_{rj}^{(SO)} - b_{ro}^{(SO)} = q_r^{+(SO)} - q_r^{-(SO)}, r \in R^{SO}, \quad (21)$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} w_{nj}^{(SO)} - \phi^{-1}(\alpha)\sigma(q_n^{+(SO)} + q_n^{-(SO)}) + s_n^{(SO)} = w_{no}^{(SO)}, n \in N^{SO},$$
$$\sum_{j=1}^{J} \lambda_j^{(SO)} c_{nj}^{(SO)} - c_{no}^{(SO)} = q_n^{+(SO)} + q_n^{-(SO)}, n \in N^{SO},$$

Table 3
Results of stochastic sustainability.

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Africa	Algeria	0.4875	0.4867	0.4981	0.4979	0.4979
	Angola	0.4902	0.4945	0.4963	0.4966	0.4967
	Benin	0.5038	0.503	0.5023	0.5016	0.501
	Cameroon	0.5031	0.501	0.5005	0.5002	0.5006
	Congo, Dem. Rep.	0.5025	0.5024	0.5014	0.4991	0.4974
	Congo, Rep.	0.4992	0.499	0.4989	0.4987	0.4986
	Cote d'Ivoire	0.505	0.5046	0.5039	0.503	0.5013
	Ethiopia	0.5061	0.5064	0.5047	0.5033	0.502
	Ghana	0.4982	0.5012	0.5004	0.5	0.4999
	Niger	0.5034	0.5031	0.5029	0.5025	0.5021
	Senegal	0.5023	0.5019	0.5015	0.5008	0.5003
	Togo	0.5005	0.5004	0.5004	0.5002	0.5001

(continued on next page)

Table 3 (*continued*).

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Europe	Austria	0.5482	0.5417	0.5368	0.5325	0.5291
	Belgium	0.5271	0.5153	0.5135	0.511	0.5084
	Bosnia and Herzegovina	0.501	0.5171	0.5153	0.4985	0.4987
	Croatia	0.5045	0.5021	0.5047	0.503	0.5014
	Czechia	0.5421	0.5367	0.5306	0.5246	0.5158
	Denmark	0.5394	0.5376	0.5357	0.536	0.5188
	Estonia	0.5112	0.5107	0.509	0.5071	0.5036
	Finland	0.5944	0.592	0.5893	0.5844	0.5808
	Germany	0.5571	0.5543	0.5524	0.551	0.5494
	Greece	0.5178	0.5113	0.5022	0.4913	0.4836
	Hungary	0.5123	0.508	0.5144	0.5083	0.5024
	Iceland	0.504	0.5037	0.5034	0.5032	0.5031
	Latvia	0.4983	0.4988	0.4975	0.4927	0.4896
	Italy	0.5027	0.5041	0.5024	0.5011	0.4992
	Lithuania	0.5008	0.501	0.501	0.5006	0.5003
	Luxembourg	0.5046	0.5023	0.5006	0.497	0.4924
	Malta	0.5234	0.5204	0.5184	0.5168	0.5155
	Moldova	0.5256	0.5224	0.5109	0.5072	0.5051
	Netherlands	0.5774	0.5746	0.5717	0.5658	0.5428
	Norway	0.6137	0.6016	0.5902	0.5793	0.566
	Poland	0.5427	0.5427	0.5323	0.5136	0.4948
	Portugal	0.5223	0.5179	0.5089	0.4944	0.4908
	Romania	0.501	0.5007	0.5002	0.5	0.4999
	Serbia	0.4974	0.4989	0.4998	0.499	0.4989
	Slovak Republic	0.52	0.5087	0.5131	0.5096	0.5034
	Slovenia	0.5168	0.5122	0.5063	0.5022	0.5003
	Spain	0.5023	0.4909	0.4793	0.4607	0.446
	Sweden	0.6263	0.6182	0.6102	0.5973	0.5887
	Switzerland	0.57	0.5604	0.5542	0.5498	0.5449
	Ukraine	0.4992	0.5001	0.5	0.4999	0.4998
North America	Dominican Republic	0.4935	0.4933	0.4931	0.4963	0.4958
	Ecuador	0.5057	0.5039	0.4998	0.4994	0.4993
	El Salvador	0.5013	0.5015	0.5009	0.5001	0.4986
	Guatemala	0.5012	0.5021	0.501	0.5004	0.5
	Haiti	0.4985	0.4989	0.4984	0.4989	0.4987
	Honduras	0.4984	0.4983	0.4993	0.499	0.4989
	Mexico	0.4828	0.4987	0.4995	0.4996	0.4996
Asia	Cambodia	0.5078	0.5036	0.5031	0.5028	0.5026
	Indonesia	0.5125	0.5111	0.5237	0.5197	0.5087
	Kazakhstan	0.5084	0.507	0.5059	0.5044	0.503
	Kuwait	0.53	0.527	0.5302	0.5259	0.5225
	Lebanon	0.5246	0.5139	0.4983	0.4935	0.4896
	Mongolia	0.4996	0.4997	0.4995	0.4991	0.4988
	Myanmar	0.5114	0.5108	0.51	0.5094	0.5087
	Philippines	0.5025	0.4986	0.4993	0.4994	0.4994
	Russian Federation	0.5001	0.5016	0.5004	0.5001	0.4999
	Uzbekistan	0.4987	0.4979	0.5017	0.5014	0.5008

Table 4
Results of stochastic economic sustainability.

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Africa	Algeria	0.3539	0.3524	0.4834	0.5365	0.5759
	Angola	0.3686	0.4287	0.5177	0.5756	0.6177
	Benin	0.2944	0.2942	0.2936	0.2925	0.2918
	Cameroon	0.3007	0.3016	0.3017	0.3017	0.3166
	Congo, Dem. Rep.	0.3175	0.3179	0.3098	0.2926	0.2903
	Congo, Rep.	0.296	0.2952	0.2947	0.2942	0.2941
	Cote d'Ivoire	0.3265	0.3261	0.3192	0.319	0.3068
	Ethiopia	0.2948	0.2969	0.293	0.2898	0.2868
	Ghana	0.3401	0.4991	0.6638	0.7403	0.7875
	Niger	0.2914	0.2912	0.2909	0.2904	0.2908
	Senegal	0.3264	0.3257	0.3229	0.3116	0.3099
	Togo	0.2878	0.2877	0.2876	0.2874	0.2872

(*continued on next page*)

$$\sum_{j=1}^{J} \lambda_j^{(SO)} = 1,$$

$$\lambda_j^{(SO)}, s_i^{(SO)}, s_r^{(SO)}, s_n^{(SO)}, q_i^{+(SO)}, q_i^{-(SO)}, q_r^{+(SO)}, q_r^{-(SO)},$$

$$q_n^{+(SO)}, q_n^{-(SO)} \geq 0, \forall j, i, r, n.$$

Appendix C

See Tables 3–6.

Table 4 (*continued*).

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Europe	Austria	0.5562	0.5581	0.5587	0.5568	0.5572
	Belgium	0.751	0.9013	0.8975	0.8813	0.868
	Bosnia and Herzegovina	1	1	1	0.3029	0.2971
	Croatia	0.3478	0.3448	0.3841	0.3824	0.3826
	Czechia	0.4636	0.4612	0.4487	0.4753	0.4912
	Denmark	0.5499	0.5612	0.5679	0.5891	0.7358
	Estonia	0.2999	0.3	0.2977	0.2936	0.2904
	Finland	0.3696	0.3672	0.3647	0.36	0.3566
	Germany	1	1	1	1	1
	Greece	1	1	1	0.7813	0.7665
	Hungary	0.379	0.3772	0.4222	0.4388	0.4447
	Iceland	0.2892	0.2889	0.2887	0.2886	0.2883
	Latvia	0.8153	0.8237	0.8451	0.8345	0.8281
	Italy	0.3726	0.4547	0.5671	0.5574	0.8435
	Lithuania	0.2907	0.291	0.3465	0.3754	0.4002
	Luxembourg	0.3368	0.3354	0.3251	0.3089	0.2949
	Malta	1	1	1	1	1
	Moldova	1	1	1	1	1
	Netherlands	0.7339	0.7337	0.7269	0.7057	0.7236
	Norway	0.4149	0.4031	0.4145	0.4392	0.4661
	Poland	0.5613	0.5668	0.6064	0.6822	0.7777
	Portugal	0.4714	0.4891	0.5819	0.7853	0.8037
	Romania	0.546	0.9836	0.9944	0.9891	0.9911
	Serbia	0.341	0.4015	0.5674	0.8493	0.913
	Slovak Republic	0.4252	0.3873	0.4121	0.4229	0.4203
	Slovenia	0.3408	0.3307	0.3275	0.3097	0.3067
	Spain	0.6919	0.6983	0.6962	0.6441	0.6014
	Sweden	0.4893	0.4851	0.4763	0.4551	0.4418
	Switzerland	1	1	1	1	1
	Ukraine	0.6028	0.8766	0.9872	0.9912	0.993
North America	Dominican Republic	0.3152	0.315	0.3143	0.3731	0.3848
	Ecuador	0.3655	0.505	0.8814	0.9186	0.9368
	El Salvador	0.3095	0.3563	0.4408	0.5231	0.7729
	Guatemala	0.3061	0.3293	0.391	0.435	0.4694
	Haiti	0.2992	0.2977	0.2965	0.3459	0.3612
	Honduras	0.3059	0.3056	0.3728	0.3983	0.4276
	Mexico	0.718	0.9329	0.9768	0.9851	0.9889
Asia	Cambodia	0.2906	0.2906	0.2901	0.2897	0.2892
	Indonesia	0.4737	0.4818	0.546	0.5792	0.684
	Kazakhstan	0.2872	0.2873	0.2872	0.2855	0.284
	Kuwait	0.3339	0.3278	0.3573	0.3734	0.3873
	Lebanon	1	1	0.31	0.295	0.2866
	Mongolia	0.2897	0.2884	0.2869	0.2863	0.2859
	Myanmar	0.2874	0.288	0.2873	0.2867	0.2862
	Philippines	0.5171	0.8548	0.9427	0.9617	0.9705
	Russian Federation	0.8182	0.9299	0.9739	0.9819	0.9858
	Uzbekistan	0.2926	0.2933	0.4241	0.4935	0.5379

Table 5
Results of stochastic environmental sustainability.

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Africa	Algeria	0.4828	0.4863	0.6171	0.6699	0.7073
	Angola	0.4273	0.4949	0.6185	0.6731	0.7121
	Benin	0.9999	1	1	1	1
	Cameroon	0.9231	0.8971	0.8815	0.8699	0.6697
	Congo, Dem. Rep.	0.686	0.676	0.7543	0.9894	0.9881
	Congo, Rep.	0.9074	0.9086	0.9094	0.9101	0.9043
	Cote d'Ivoire	0.6669	0.6654	0.7386	0.7302	0.89
	Ethiopia	1	1	1	1	1
	Ghana	0.5705	0.722	0.8987	0.9754	0.9801
	Niger	1	1	1	1	0.9807
	Senegal	0.5938	0.5964	0.6098	0.6975	0.7058
	Togo	0.9993	0.9992	0.9994	0.9994	0.9994

(*continued on next page*)

Table 5 (*continued*).

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Europe	Austria	0.2885	0.2846	0.2828	0.2812	0.2814
	Belgium	0.2228	0.1982	0.1967	0.2051	0.213
	Bosnia and Herzegovina	0.1857	0.2311	0.3755	0.8204	0.8716
	Croatia	0.5965	0.605	0.6798	0.7463	0.8482
	Czechia	0.435	0.4319	0.4338	0.5313	0.5536
	Denmark	0.3298	0.311	0.3098	0.3342	0.2885
	Estonia	1	1	1	1	0.9733
	Finland	1	1	1	1	1
	Germany	0.204	0.2016	0.2	0.1985	0.1974
	Greece	0.2327	0.323	0.4722	0.5806	0.6952
	Hungary	0.4736	0.4787	0.5556	0.5895	0.6827
	Iceland	1	1	1	1	1
	Latvia	0.2908	0.3098	0.3748	0.4219	0.461
	Italy	0.7415	0.8092	0.9076	0.9725	0.9723
	Lithuania	0.9897	0.9899	0.9924	0.993	0.9934
	Luxembourg	0.6485	0.6526	0.6997	0.794	0.8767
	Malta	0.1836	0.181	0.1798	0.1788	0.178
	Moldova	0.2677	0.3257	0.4854	0.581	0.6698
	Netherlands	0.2624	0.2593	0.2579	0.2561	0.2241
	Norway	1	1	1	1	1
	Poland	0.378	0.3856	0.4491	0.4915	0.6466
	Portugal	0.6577	0.6764	0.7696	0.7799	0.8145
	Romania	0.6924	0.9234	0.9852	0.9907	0.9928
	Serbia	0.7828	0.8282	0.9096	0.9318	0.9816
	Slovak Republic	0.4358	0.5001	0.5753	0.6422	0.8169
	Slovenia	0.6139	0.6732	0.687	0.8801	0.8909
	Spain	0.2737	0.2613	0.2656	0.2705	0.2763
	Sweden	0.5433	0.5182	0.51	0.5249	0.5338
	Switzerland	0.213	0.2071	0.2033	0.1996	0.1972
	Ukraine	0.8877	0.9797	0.9952	0.997	0.9979
North America	Dominican Republic	0.6697	0.6677	0.6688	0.7252	0.8164
	Ecuador	0.6887	0.7981	0.93	0.9521	0.9628
	El Salvador	0.8586	0.8816	0.9198	0.9322	0.9299
	Guatemala	0.8326	0.9099	0.9639	0.9679	0.9705
	Haiti	0.7952	0.8111	0.8098	0.842	0.8505
	Honduras	0.726	0.7279	0.7848	0.9401	0.9449
	Mexico	0.5078	0.8546	0.9609	0.9745	0.9807
Asia	Cambodia	1	1	1	1	1
	Indonesia	0.381	0.3563	0.3989	0.4201	0.4008
	Kazakhstan	0.8455	0.8397	0.8376	0.8346	0.832
	Kuwait	1	1	1	1	1
	Lebanon	0.1734	0.1945	0.7614	0.8592	0.8588
	Mongolia	0.9293	0.9545	0.9765	0.9808	0.9803
	Myanmar	0.9941	0.9937	0.9934	0.9931	0.9914
	Philippines	0.3438	0.7384	0.958	0.9867	0.9897
	Russian Federation	0.8285	0.939	0.9834	0.9893	0.9918
	Uzbekistan	0.9944	0.9832	0.9854	0.9978	0.9983

Table 6
Results of stochastic social sustainability.

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Africa	Algeria	0.8052	0.8055	0.4673	0.4153	0.3883
	Angola	0.8228	0.582	0.4353	0.3915	0.3687
	Benin	0.9561	0.9541	0.9564	0.9606	0.9624
	Cameroon	0.9456	0.9338	0.9341	0.9351	0.9434
	Congo, Dem. Rep.	0.9249	0.928	0.9265	0.9238	0.9218
	Congo, Rep.	0.9568	0.9593	0.961	0.9625	0.9643
	Cote d'Ivoire	0.9199	0.921	0.9139	0.9152	0.9111
	Ethiopia	0.8907	0.9031	0.9039	0.9049	0.9059
	Ghana	0.8217	0.4348	0.3406	0.3186	0.3103
	Niger	0.978	0.978	0.9785	0.9804	0.9811
	Senegal	0.9614	0.9607	0.9649	0.9667	0.9681
	Togo	0.9897	0.9901	0.9901	0.9908	0.9908

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Table 6 (*continued*).

Continent	Country name	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$
Europe	Austria	0.9716	0.951	0.9311	0.9164	0.8884
	Belgium	0.8916	0.8697	0.8754	0.7868	0.7242
	Bosnia and Herzegovina	0.6079	0.5234	0.3906	0.9658	0.9759
	Croatia	0.891	0.8885	0.6469	0.6128	0.5763
	Czechia	0.799	0.7956	0.8039	0.5843	0.5205
	Denmark	0.846	0.8656	0.8315	0.6981	0.5807
	Estonia	0.9704	0.9637	0.9695	0.9719	0.9749
	Finland	1	1	1	1	1
	Germany	0.9714	0.9724	0.9731	0.9751	0.9748
	Greece	0.5127	0.4106	0.3406	0.3413	0.3197
	Hungary	0.8818	0.8725	0.6398	0.5621	0.5041
	Iceland	0.9775	0.9797	0.98	0.9802	0.9809
	Latvia	0.4701	0.4453	0.396	0.3723	0.3572
	Italy	0.6506	0.4754	0.3779	0.3742	0.3029
	Lithuania	0.9561	0.9604	0.6237	0.5457	0.4996
	Luxembourg	0.9112	0.9062	0.9186	0.9247	0.9426
	Malta	0.949	0.9579	0.9568	0.9568	0.9559
	Moldova	0.5362	0.4447	0.3477	0.3244	0.3113
	Netherlands	0.9412	0.9427	0.944	0.9448	0.9395
	Norway	0.8523	0.8231	0.7185	0.6182	0.5426
	Poland	0.7341	0.6989	0.5217	0.4218	0.3418
	Portugal	0.556	0.5137	0.4006	0.3201	0.3107
	Romania	0.4162	0.2921	0.2873	0.2872	0.2868
	Serbia	0.7023	0.5263	0.3727	0.3038	0.2939
	Slovak Republic	0.8496	0.8547	0.6744	0.5865	0.5208
	Slovenia	0.9742	0.9714	0.9531	0.9477	0.9485
	Spain	0.6059	0.5794	0.5276	0.491	0.4625
	Sweden	1	1	1	1	1
	Switzerland	0.9826	0.9793	0.977	0.9817	0.9769
	Ukraine	0.3598	0.2989	0.287	0.2865	0.2863
North America	Dominican Republic	0.9007	0.9031	0.9059	0.6034	0.5486
	Ecuador	0.6987	0.4276	0.3007	0.2952	0.2926
	El Salvador	0.8909	0.6327	0.4622	0.393	0.3146
	Guatemala	0.9023	0.7133	0.5185	0.4555	0.4221
	Haiti	0.9638	0.9788	0.9814	0.6543	0.6019
	Honduras	0.9427	0.9437	0.5917	0.5079	0.4664
	Mexico	0.3513	0.3005	0.2898	0.2883	0.2876
Asia	Cambodia	1	0.9763	0.9749	0.9749	0.9751
	Indonesia	0.6688	0.6855	0.5698	0.5162	0.4455
	Kazakhstan	0.9687	0.9687	0.9689	0.9687	0.9685
	Kuwait	0.8291	0.8319	0.6796	0.6131	0.5694
	Lebanon	0.7705	0.6323	0.9484	0.9433	0.9487
	Mongolia	0.9814	0.9891	0.9888	0.9885	0.9883
	Myanmar	0.9898	0.9886	0.9878	0.9873	0.9865
	Philippines	0.6219	0.3145	0.2912	0.2883	0.2876
	Russian Federation	0.3207	0.2991	0.29	0.2884	0.2877
	Uzbekistan	0.8557	0.8579	0.465	0.4032	0.3776

Data availability

Data will be made available on request.

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