Revising the Full One-Loop Gauge Prefactor in Electroweak Vacuum Stability

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We revisit the decay rate of the electroweak vacuum in the standard model with the full one-loop prefactor. We focus on the gauge degrees of freedom and derive the degeneracy factors appearing in the functional determinant using group theoretical arguments. Our treatment shows that the transverse modes were previously overcounted, so we revise the calculation of that part of the prefactor. The new result modifies the gauge fields' contribution by 6% and slightly decreases the previously predicted lifetime of the electroweak vacuum, which remains much longer than the age of the Universe. Our discussion of the transverse mode degeneracy applies to any calculation of functional determinants involving gauge fields in four dimensions.

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Introduction—The standard model of particle physics (SM) is the cornerstone of our understanding of elementary particle interactions. The only fundamental scalar field in the SM is the Higgs doublet H, responsible for the spontaneous breaking of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ electroweak (EW) gauge symmetry. With the current central values of SM parameters, the potential $V(h)$ of the physical Higgs h, once extrapolated to the regime of an extremely intense field, turns negative for $\langle h \rangle \gtrsim 10^{10}$ GeV $\gg v \approx 246$ GeV. This makes the EW vacuum at $\langle h \rangle = v$ metastable due to quantum tunneling.

The tunneling rate per unit volume γ can be computed using the methods of [1,2], and expressed as $\gamma = \mathcal{A} e^{-S}$, where S is the action of the bounce in Euclidean spacetime and $\mathcal{A}$ is the prefactor with mass dimension four. The bounce $\bar{h}(\rho)$ is an $O(4)$-symmetric instanton solution in terms of the Euclidean radius $\rho^2 = t^2 + |\mathbf{x}|^2$ that connects a point close to the absolute vacuum at $\rho = 0$ to the unstable vacuum at $\rho = \infty$. It is computed by approximating the Higgs potential as $\lambda(H^\dagger H)^2$, with λ negative, thus neglecting the quadratic term and treating $v \sim 0$. This is justified because the Higgs field travels over large field values until the potential becomes negative. In this approximation the potential is classically scale invariant and the bounce is given by the Fubini-Lipatov instanton $\bar{h}(\rho) = \sqrt{8/|\lambda|} \times [R/(\rho^2 + R^2)]$, whose action is $S = 8\pi^2/(3|\lambda|)$. The free parameter R signals the classical symmetry under dilatations, which is broken by quantum corrections. This effectively fixes $R^{-1} \sim 10^{17}$ GeV, which is the scale where the beta function of the running quartic coupling λ vanishes, assuming only SM degrees of freedom.

To obtain $\mathcal{A}$ one has to compute the functional determinants corresponding to one-loop diagrams, where the fields running in the loop are the scalar, fermion, and gauge boson fluctuations that couple to the Higgs bounce $\bar{h}(\rho)$. Collecting the dominant contributions we can write

$$\mathcal{A} = V_G \mathcal{A}^{(h)} \mathcal{A}^{(t)} \mathcal{A}^{(Z,\varphi_Z)} \mathcal{A}^{(W^\pm,\varphi^\pm)}. \tag{1}$$

Here, $V_G = 2\pi^2$ is the volume of the SU(2) group broken by the bounce, the superscripts denote the species: t is the top quark, Z and $W^\pm$ the gauge bosons and $\varphi_Z, \varphi^\pm$ the would-be Nambu-Goldstone bosons (NGBs). We ignore the dilatational zero mode in (1) for the moment and come back to it later.

The stability of the EW vacuum has been investigated by several authors [3–15]. The calculation of the full one-loop prefactor was first done in [6] before the Higgs discovery and updated in [16–18] with a careful treatment of dilatation [16–18] and gauge zero modes [19,20].

In this Letter we revisit the calculation of the gauge prefactor. We find that the transverse mode degeneracy was not properly taken into account. Once corrected, the central value of the SM rate γ increases only slightly, from 10^{-877} to 10^{-871} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$. The SM vacuum lifetime remains longer than the current age of the Universe and there is no occasion for anxiety [1].

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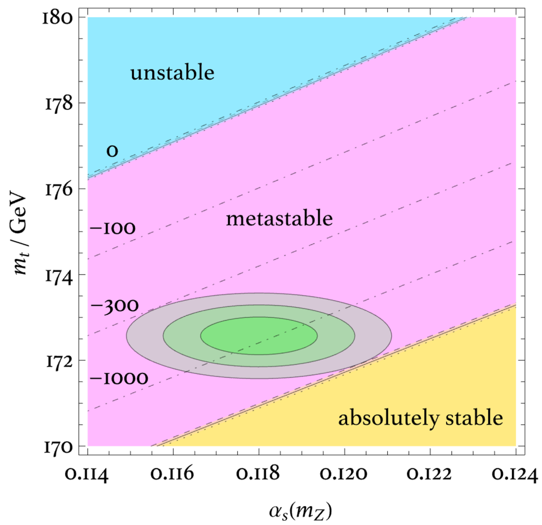

FIG. 1. Contours of γ as a function of $\alpha_s(m_Z)$ and the top mass m_t. The black dot-dashed contours correspond to $\gamma = 1, 10^{-100}, 10^{-300}$, and 10^{-1000} $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$ for the Higgs mass $m_h = 125.20$ GeV. In the blue region γ becomes larger than H_0^4, with H_0 the current Hubble parameter, while in the yellow region the EW vacuum is stable. The boundaries of these regions are given by the plain lines for $m_h = 125.20$ GeV, the dotted lines for $m_h = 125.09$ GeV, the dashed line for $m_h = 125.31$ GeV. The green region, for which $\gamma = 10^{-871}$ $\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}$ at the center, shows the experimentally measured values of $\alpha_s(m_Z)$ and the top mass with their 1σ (inside), 2σ (middle), and 3σ (outside) uncertainties in quadrature.

We first introduce the fluctuation operator in the gauge sector. Then we explain how to build suitable bases for the scalar and gauge fields, given the Euclidean spherical 4D symmetry of the bounce, and discuss the counting of degeneracy factors. Next, we obtain the analytic expression of our correction to the vacuum decay rate in the SM and give the numerical full one-loop vacuum decay rate with the current central values of the SM couplings. Our final result is summarized in Fig. 1.

Gauge fluctuation operator—In the presence of the bounce, gauge fields and NGBs mix. The fluctuation operator does not mix Z_μ and $W^\pm_\mu$ in the SM, so we unify the notation as $A_\mu = Z_\mu, W^\pm_\mu$ and $\varphi = \varphi_Z, \varphi^\pm$. The prefactor, coming from A_μ and φ, is then

$$\mathcal{A}^{(A,\varphi)} = J_G \left(\frac{\det' S''^{(A,\varphi)}}{\det \hat{S}''^{(A,\varphi)}} \right)^{-\frac{1}{2}}, \tag{2}$$

where the prime indicates the subtraction of the zero mode, associated with the spontaneous breaking of gauge symmetry, and J_G is the group space integral Jacobian. The fluctuation operator in the (A^μ, φ) basis is given by the 5×5 matrix

$$S''^{(A,\varphi)} = \begin{pmatrix} (-\partial^2 + \frac{1}{4} g^2 \bar{h}^2)\delta_{\mu\nu} & \frac{1}{2} g \bar{h}' \hat{x}_\nu - \frac{1}{2} g \bar{h} \partial_\nu \\ g \bar{h}' \hat{x}_\mu + \frac{1}{2} g \bar{h} \partial_\mu & -\partial^2 + \lambda \bar{h}^2 \end{pmatrix}, \tag{3}$$

while $\hat{S}''^{(A,\varphi)}$ is the same operator, but with $\bar{h} = 0$. Here, $\bar{h}' = \partial_\rho \bar{h}$, g is the gauge coupling for $W^\pm$ or Z, and $\hat{x}_\mu$ is a unit vector, such that $x_\mu = \rho \hat{x}_\mu$. We work in the Fermi gauge with $\xi = 1$, which is defined through the gauge fixing term $\mathcal{L}_{\mathrm{GF}} = (\partial_\mu A^\mu)^2/2$. Given the $O(4)$ symmetry of the bounce, the $4D$ Laplacian is conveniently written in spherical coordinates as $\partial^2 = \partial_\rho^2 + 3\rho^{-1}\partial_\rho - L^2/\rho^2$, where $L^2 = L^{\mu\nu} L_{\mu\nu}/2$, and $L_{\mu\nu} = -i(x_\mu \partial_\nu - x_\nu \partial_\mu)$ is the orbital angular momentum operator.

The fluctuation operator commutes with rotations, and therefore with all the components of the total angular momentum operator, $[J_{\mu\nu}, S''^{(A,\varphi)}] = 0$. This implies that only modes with the same total angular momentum quantum numbers mix under the action of (3). In the following we decompose φ and A_μ into $J_{\mu\nu}$ bases. In the calculation of functional determinants, it is the *total*, not the orbital, angular momentum operator that dictates the counting of degeneracies.

Scalar field basis—The scalar field transforms under Euclidean rotations like $\varphi(x) \to \varphi(R^{-1}x)$, where R is a finite rotation of coordinates. For the scalar, the total and orbital angular momenta coincide, $J_{\mu\nu} = L_{\mu\nu}$. The space of scalar fields carries an infinite dimensional unitary representation of the compact group SO(4) that, thanks to the Peter-Weyl theorem, admits an orthogonal decomposition into irreducible finite dimensional representations (irreps). SO(4) is locally isomorphic to $\mathrm{SU}(2)_A \otimes \mathrm{SU}(2)_B$. Following textbook conventions, in $\mathrm{SU}(2)_A$ we define the total angular momentum operator in the $\hat{3}$ direction as J_{A3} and the Casimir operator as J_A^2. The latter has eigenvalues $j_A(j_A+1)$ with j_A a half-integer. The same holds for $\mathrm{SU}(2)_B$ and we label the irreps with (j_A, j_B), with dimension of $(2j_A+1)(2j_B+1)$.

The representation space of the scalar field splits into $\oplus_{j=0}^\infty (j/2, j/2)$, where the label $j = j_A + j_B$ is an integer. Each $(j/2, j/2)$ multiplet is an eigenstate of $J^{\mu\nu} J_{\mu\nu}/2 = J^2 = 2(J_A^2 + J_B^2)$ with eigenvalue $j(j+2)$ and degeneracy factor $d_j^{(\varphi)} \equiv \dim(j/2, j/2) = (j+1)^2$.

An explicit basis for such irreps can be formed with hyperspherical harmonics $Y_{jm_Am_B}$ that satisfy $L^2 Y_{jm_Am_B} = j(j+2) Y_{jm_Am_B}$. Here, m_A and m_B run in integer steps between $-j/2$ and $j/2$, giving the multiplicity $d_j^{(\varphi)}$. The NGB φ is written as

$$\varphi(x) = \sum_{j=0}^{\infty} \sum_{m_A, m_B} \varphi_{jm_Am_B}(\rho) Y_{jm_Am_B}(\hat{x}). \tag{4}$$

Vector field basis—For the spin-1 field, the total angular momentum is $J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$, where $[S_{\mu\nu}]_{\rho\sigma} = -i(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})$ are the generators acting on the spin component. Equivalently, the vector field transforms like $A_\mu(x) \to R_{\mu\nu}A_\nu(R^{-1}x)$, where $R_{\mu\nu}$ is the vector rotation matrix. The representation of $A_\mu(x)$ is isomorphic to the tensor product of a spin-1 component $(1/2, 1/2)$, with an orbital component, transforming as $\oplus_{l=0}^{\infty}(l/2, l/2)$. To understand the SO(4) irreps (j_A, j_B) of the vector field, we expand the tensor product:

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\bigoplus_{l=0}^{\infty}\left(\frac{l}{2},\frac{l}{2}\right)$$
$$=\left(\frac{1}{2},\frac{1}{2}\right)\oplus\bigoplus_{l=1}^{\infty}\left[\left(\frac{l+1}{2},\frac{l+1}{2}\right)\oplus\left(\frac{l-1}{2},\frac{l-1}{2}\right)\right. \quad (5)$$
$$\left.\oplus\left(\frac{l+1}{2},\frac{l-1}{2}\right)\oplus\left(\frac{l-1}{2},\frac{l+1}{2}\right)\right]$$
$$=(0,0)_{l=1}\oplus\bigoplus_{j=1}^{\infty}\left[\left(\frac{j}{2},\frac{j}{2}\right)_{l=j+1}\oplus\left(\frac{j}{2},\frac{j}{2}\right)_{l=j-1}\right]$$
$$\oplus\bigoplus_{j=1}^{\infty}\left[\left(\frac{j+1}{2},\frac{j-1}{2}\right)_{l=j}\oplus\left(\frac{j-1}{2},\frac{j+1}{2}\right)_{l=j}\right]. \quad (6)$$

To go from (5) to (6) we simply relabeled and rearranged the sum. In (6) we included a subscript for each (j_A, j_B) multiplet to track the orbital angular momentum quantum number l. We call the multiplets with $j_A = j_B$ the diagonal modes $\mathcal{D}_j^{\pm}$, corresponding to $(j/2, j/2)_{j\pm 1}$ (note $\mathcal{D}_0^-$ is zero); they have eigenvalue $j(j+2)$ under J^2, and degeneracy $d_j^{\mathcal{D}} \equiv \dim[(j/2), (j/2)] = (j+1)^2$. These are the same total angular momentum quantum numbers as the NGB scalars, hence in general the fluctuation operator matrix mixes these states. We identify the multiplets having $j_A = j_B \pm 1$ with the transverse modes [21], and call them $\mathcal{T}_j^{\pm}$ for $[(j\pm 1)/2, (j\mp 1)/2]_j$. Their J^2 eigenvalue is $(j+1)^2$, while the degeneracy factor is

$$d_j^{\mathcal{T}} \equiv \dim\left(\frac{j\pm 1}{2},\frac{j\mp 1}{2}\right) = j(j+2). \quad (7)$$

Figure 2 summarizes the group theoretical construction and there $\mathcal{D}_j^{\pm}$ correspond to the diagonal circles, where the two overlapping circles are distinguished by the eigenvalues of L^2. The $\mathcal{T}_j^{\pm}$ correspond to the lower and upper off-diagonal circles, respectively.

We decompose $A_\mu(x)$ as a sum of radial functions times a basis of vector fields,

$$A_\mu(x) = \sum_{j_A, j_B, l, m_A, m_B} A_l^{j_A j_B m_A m_B}(\rho) V_{l;\mu}^{j_A j_B m_A m_B}(\hat{x}), \quad (8)$$

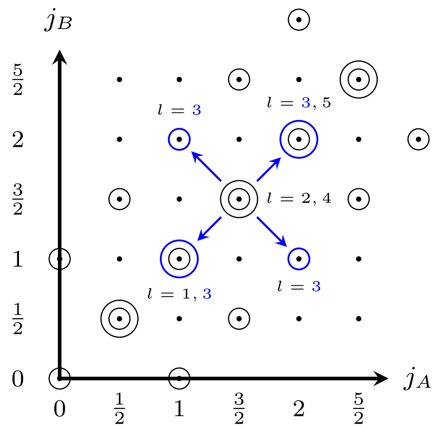

FIG. 2. Decomposition of a 4-vector field $A_\mu(\hat{x})$ under SO(4). The diagonal modes $\mathcal{D}_j^{\pm}$ with $j_A = j_B$ appear in double copies, except for (0,0). The off-diagonal multiplets with $j_A = j_B \pm 1$ correspond to the transverse modes $\mathcal{T}_j^{\pm}$, and appear as single copies. Blue circles with $l = 3$ exemplify how the L^2 eigenspace gets distributed within the (j_A, j_B) lattice.

where the real field A_μ is expanded with a complex basis, without affecting the degeneracy, as follows:

$$V_{l;\mu}^{j_A j_B m_A m_B}(\hat{x}) = \sum_{m_{l_A}, m_{l_B}, m_{s_A}, m_{s_B}} C_{\frac{l}{2}m_{l_A}\frac{1}{2}m_{s_A}}^{j_A m_A}$$
$$\times C_{\frac{l}{2}m_{l_B}\frac{1}{2}m_{s_B}}^{j_B m_B} (\tilde{\sigma}_\mu)_{m_{s_A} m_{s_B}} Y_{l m_{l_A} m_{l_B}}(\hat{x}). \quad (9)$$

This choice has the virtue of containing familiar objects that are in direct correspondence with the group theory construction of Fig. 2. The $(\tilde{\sigma}_\mu)_{m_{s_A} m_{s_B}}$ matrices correspond to the $(1/2, 1/2)$ object that ensures the proper transformation under rotations, and are given by $\tilde{\sigma}_\mu = \varepsilon \cdot \sigma_\mu$, where ε is the two-dimensional Levi-Civita symbol, $\sigma_{1,2,3}$ are the usual Pauli matrices, and $\sigma_4 = \mathrm{diag}(i, i)$. The entries of $\tilde{\sigma}$ are ordered such that $m_s = 1/2$ comes before $m_s = -1/2$. $C_{l_A m_{l_A} s_A m_{s_A}}^{j_A, m_A}$ denote the $\mathrm{SU}(2)_A$ Clebsch-Gordan coefficients (and the same for B). These are nonzero only when $|2j_{A,B} - l| = 1$, $m_A = m_{l_A} + m_{s_A}$ and $m_B = m_{l_B} + m_{s_B}$. We shall suppress the m indices from here on for brevity; they label states with the same fluctuation operator and their summation only appears as the degeneracy factor. Then, the nonzero components of $V_{l;\mu}^{j_A j_B}$ are $V_{l=j\pm 1;\mu}^{(j/2)(j/2)}$ and $V_{l=j;\mu}^{[(j\pm 1)/2][(j\mp 1)/2]}$, corresponding to the basis of $\mathcal{D}_j^{\pm}$ and $\mathcal{T}_j^{\pm}$. These components are eigenfunctions of J^2, with the eigenvalues quoted earlier for $\mathcal{D}_j^{\pm}$ and $\mathcal{T}_j^{\pm}$. Each component is an eigenfunction of L^2 that acts only on $Y_{l m_{l_A} m_{l_B}}$.

Fluctuation operator decomposition—Let us decompose (3) using the scalar and gauge basis functions in (4) and (8). When (3) acts on (A_μ, φ), it splits into an infinite number of blocks with the same j_A, j_B, m_A and m_B: (A_1^{00}, φ_0) for

$j=0$, and $(A_{j+1}^{(j/2)(j/2)}, A_{j-1}^{(j/2)(j/2)}, \varphi_j)$, $(A_j^{[(j+1)/2][(j-1)/2]})$, $(A_j^{[(j-1)/2][(j+1)/2]})$ for $j>0$. The first two indicate 2×2 or 3×3 fluctuation matrices that mix the $\mathcal{D}_j^{\pm}$ modes with the NGB, and the last two are those for the $\mathcal{T}_j^{\pm}$ modes that do not mix.

With this decomposition, the prefactor in (2) factorizes into $\mathcal{A}^{(A,\varphi)} = \mathcal{A}^{(\mathcal{D},\varphi)}\mathcal{A}^{(\mathcal{T})}$. The first term $\mathcal{A}^{(\mathcal{D},\varphi)}$ was already computed correctly in the previous literature [6,16,17]. Therefore we only deal with the second one:

$$\mathcal{A}^{(\mathcal{T})} = \left[\prod_{j=1}^{\infty}\left(\frac{\det S_j^{\prime\prime(\mathcal{T})}}{\det \hat{S}_j^{\prime\prime(\mathcal{T})}}\right)^{2d_j^{\mathcal{T}}}\right]^{-\frac{1}{2}}. \tag{10}$$

The factor of 2 in the exponent $2d_j^{\mathcal{T}}$ comes from the two transverse modes $\mathcal{T}_j^{\pm}$ having the same fluctuation operator, given by $S_j^{\prime\prime(\mathcal{T})} = -\Delta_j + g^2\bar{h}^2/4$, where $\Delta_{j=l} = \partial_\rho^2 + 3\rho^{-1}\partial_\rho - l(l+2)\rho^{-2}$. There are no zero modes to worry about in (10), so there is no prime in the determinant at the numerator. (i) In previous calculations [6,16,17], the degeneracy factor for the transverse modes $\mathcal{T}_j^{\pm}$ was erroneously set equal to $(j+1)^2$. The correct one, as we have shown in (7), is $d_j^{\mathcal{T}} = j(j+2)$. This leads to a slightly different result for the prefactor $\mathcal{A}^{(\mathcal{T})}$. We will shortly revise that calculation. (ii) For the diagonal modes $\mathcal{D}_j^{\pm}$, our basis functions have one-to-one correspondence to the ones given in [6], which are then used in [16–18]. We differ for the transverse modes $\mathcal{T}_j^{\pm}$. In the Appendix we give an explicit demonstration of why the gauge boson basis in [6] has inconsistencies. In the Supplemental Material [22] we further elaborate and show that their basis functions are linearly dependent and do not span the entire space. Despite the degeneracy factor and completeness issues, the operator $S_j^{\prime\prime(\mathcal{T})}$ in [6,16,17] is correct. (iii) Reference [6] stated that, if one works in the background gauge with $\xi = 1$, i.e., $\mathcal{L}_{\rm GF} = (\partial_\mu A^\mu - g\bar{h}\varphi/2)^2/2$, the transverse and ghost modes mutually cancel. This is incorrect, given the corrected $d_j^{\mathcal{T}}$ in (7); one does need to include the ghost modes in this gauge.

Functional determinants for $\mathcal{T}^{\pm}$ modes—The infinite product in $\mathcal{A}^{(\mathcal{T})}$ is ultraviolet divergent. A commonly employed regularization method subtracts the $\mathcal{O}(\delta S'', \delta S''^2)$ terms from $\ln\mathcal{A}^{(A,\varphi)}$, and adds back their dimensionally regularized quantities [6]. Here, $\delta S'' = S''^{(A\varphi)} - \hat{S}''^{(A\varphi)}$. The added-back terms do not suffer from the mode counting subtlety since they are calculated in momentum space. We focus on the subtraction procedure and defer to existing literature for the rest.

The finite part with the correct degeneracy factor is

$$\ln\mathcal{A}_{\rm fin}^{(\mathcal{T})} = -\sum_{j=1}^{\infty} d_j^{\mathcal{T}}\left(\ln R_j - \ln R_j^{(1)} - \ln R_j^{(2)}\right), \tag{11}$$

and it was finite even with the incorrect $d_j^{\mathcal{T}}$. The analytic formula [16,18] for each term is

$$\ln R_j \equiv \ln\frac{\det S_j^{\prime\prime(\mathcal{T})}}{\det \hat{S}_j^{\prime\prime(\mathcal{T})}} = \ln\frac{\Gamma(j+1)\Gamma(j+2)}{\Gamma(j+\frac{3+\kappa}{2})\Gamma(j+\frac{3-\kappa}{2})}, \tag{12}$$

and

$$\ln R_j^{(1)} \equiv \left[\ln\frac{\det S_j^{\prime\prime(\mathcal{T})}}{\det \hat{S}_j^{\prime\prime(\mathcal{T})}}\right]_{\mathcal{O}(\delta S'')} = \frac{g^2}{2|\lambda|}\frac{1}{j+1}, \tag{13}$$

$$\ln R_j^{(2)} \equiv \left[\ln\frac{\det S_j^{\prime\prime(\mathcal{T})}}{\det \hat{S}_j^{\prime\prime(\mathcal{T})}}\right]_{\mathcal{O}(\delta S''^2)} = \frac{g^4}{4\lambda^2}\left(\frac{2j+3}{2(j+1)^2} - \psi'(j+1)\right), \tag{14}$$

with $\kappa = \sqrt{1-2g^2/|\lambda|}$ and $\psi(z)$ the digamma function.

As all the other fluctutations in the prefactor (1) were already computed correctly, we define the correction as

$$\delta\ln\mathcal{A} = \ln\mathcal{A}_{\rm prev}^{(A,\varphi)} - \ln\mathcal{A}^{(A,\varphi)}, \tag{15}$$

where $\ln\mathcal{A}_{\rm prev}^{(A,\varphi)}$ is the gauge prefactor in [16,18], and $\ln\mathcal{A}^{(A,\varphi)}$ the one computed with the correct degeneracy factor. We find

$$\begin{aligned}\delta\ln\mathcal{A} = {}& 1 - \frac{g^4}{16|\lambda|^2}(10-\pi^2) - \frac{g^2}{2|\lambda|}(2-\gamma) \\ & - \frac{5}{2}\ln 2\pi + \ln\left(\frac{4|\lambda|}{g^2}\cos\left(\frac{\pi}{2}\kappa\right)\right) \\ & - \frac{1-\kappa}{2}\ln\Gamma\left(\frac{3-\kappa}{2}\right) - \frac{1+\kappa}{2}\ln\Gamma\left(\frac{3+\kappa}{2}\right) \\ & + \psi^{(-2)}\left(\frac{3-\kappa}{2}\right) + \psi^{(-2)}\left(\frac{3+\kappa}{2}\right).\end{aligned} \tag{16}$$

Vacuum decay rate—To compute the final updated decay rate, we rely on the analytic formulas for all the remaining prefactors calculated in [16,18]. We follow [17,18] for the evaluation of the SM couplings at high energy. The gauge, top Yukawa, and Higgs quartic couplings at $\mu = m_t$ are determined using the fitting formulas in [10], at NNLO precision. The bottom and tau Yukawa couplings are determined by $y_b(m_b) = \sqrt{2}m_b(m_b)/v$ and $y_\tau(m_\tau) = \sqrt{2}m_\tau/v$. We ignore threshold corrections to bottom and tau masses; their effect on the vacuum decay rate is negligible. We run these couplings using the three-loop β functions given in [10]. The calculation of the rate involves an integral over the dilatation parameter R. The dilatation symmetry is broken by the running of the couplings and we include this effect by taking the renormalization scale to $\mu = 1/R$, following [17,18]. We also put a UV cutoff to the integral such that $1/R$ or the maximum value of the Higgs field do not exceed the Planck scale [18]. The rate is then calculated using the public code ELVAS [17,18], with the modification of the transverse mode degeneracy in the gauge sector.

We use the current values of the SM parameters and their 1σ errors from [23]: $m_h = 125.20 \pm 0.11$ GeV, $m_t = 172.57 \pm 0.29$ GeV, and $\alpha_s(m_Z) = 0.1180 \pm 0.0009$. Our final decay rate is

$$\log_{10} \frac{\gamma}{\mathrm{Gyr}^{-1}\,\mathrm{Gpc}^{-3}} = -871^{+35+175+209}_{-37-253-330}, \tag{17}$$

where the first, second, and third errors are calculated by changing the Higgs mass, the top mass, and the strong coupling by 1σ, respectively. The variations are summarized in Fig. 1.

Comparing to the most recent previous result [17], we find $\gamma/\gamma_{\mathrm{prev}} \approx 10^6$, using the central values of the SM parameters. The change of the central value is much smaller than the uncertainties in (17). However, it is still larger than the theoretical uncertainty ($\approx 10^3$), evaluated by setting the renormalization scale to $\mu = 2/R$ and $\mu = 1/(2R)$ instead of $\mu = 1/R$.

Summary—We revisited the total angular momentum decomposition of gauge fields in 4D and found an overcounting of degeneracies of the transverse modes gauge fluctuations in previous literature. We have recalculated the decay rate of the electroweak vacuum in the SM using the corrected counting, and found it increases a little. Even though the numerical difference compared to previous results is rather small, it is important to understand the conceptual issue with previous analyses and give a fully consistent picture of the metastability of the SM vacuum. The argument on the degeneracy factor applies to any calculation of gauge determinants in four dimensions.

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End Matter

Appendix: Previous vector field basis—In this Appendix we examine the basis for the vector field first introduced in Eq. (4.44) of [6] and then used in [16–18]. We show that the degeneracy factor of the transverse modes assumed in those works was incorrect. We write their basis as

$$A_\mu(x) = \sum_{j,m_A,m_B} \Big[A^{(B)}_{jm_Am_B}(\rho) B^{jm_Am_B}_\mu(\hat{x}) + A^{(L)}_{jm_Am_B}(\rho) L^{jm_Am_B}_\mu(\hat{x}) + A^{(T1)}_{jm_Am_B}(\rho) T^{1;jm_Am_B}_\mu(\hat{x}) + A^{(T2)}_{jm_Am_B}(\rho) T^{2;jm_Am_B}_\mu(\hat{x}) \Big]. \quad (A1)$$

Here, $A^{(B,L,T1,T2)}_{jm_Am_B}(\rho)$ are the radial parts, functions of the Euclidean radius ρ only, and

$$B^{jm_Am_B}_\mu(\hat{x}) = \hat{x}_\mu Y_{jm_Am_B}(\hat{x}),$$
$$L^{jm_Am_B}_\mu(\hat{x}) = \rho\partial_\mu Y_{jm_Am_B}(\hat{x}),$$
$$T^{i;jm_Am_B}_\mu(\hat{x}) = i\epsilon_{\mu\nu\rho\sigma} V^{(i)}_\nu L_{\rho\sigma} Y_{jm_Am_B}(\hat{x}), \quad (A2)$$

with $i = 1, 2$, are the angular basis elements, which are only functions of three angles in 4D. The $Y_{jm_Am_B}$ are eigenfunctions of the $L^2 = L^{\mu\nu}L_{\mu\nu}/2$ operator with $L^2 Y_{jm_Am_B} = j(j+2)Y_{jm_Am_B}$. Here, the label j is an integer, while m_A and m_B range between $-j/2$ and $j/2$ in integer steps, giving the multiplicity $(j+1)^2$. B is the "breathing mode" along the direction $\hat{x}_\mu$ and L the longitudinal one, along the momentum $\rho\partial_\mu$. The $T^{1,2}$ are the transverse modes, with $V^{(1)}_\mu$ and $V^{(2)}_\mu$ being two arbitrary independent vectors, orthogonal to both B and L. Finally, $\epsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita antisymmetric tensor.

In general, one can construct different bases for vectors and the number of eigenstates of L^2 with a fixed eigenvalue has to be the same, i.e., basis independent. The simplest way to count such states is to consider a basis made of four vectors of the form $\delta_{\mu i} Y_{jm_Am_B}$, with $i = 1, 2, 3, 4$. Each of them inherits the L^2 eigenvalue of $j(j+2)$ and each carries the degeneracy of $(j+1)^2$, totaling in $4(j+1)^2$ (for more details, see the last section of the Supplemental Material [22]).

Now let us consider the vector basis introduced in [6], count the number of eigenstates of L^2 with a fixed eigenvalue of $j(j+2)$ and check whether they add up to $4(j+1)^2$. The transverse modes $T^{i;jm_Am_B}_\mu$ themselves are already eigenfunctions of L^2 with the eigenvalue $j(j+2)$. Although *a priori* their degeneracy is not obvious, Ref. [6] assumes that the two of them account for $2(j+1)^2$ independent states. On the other hand, the modes $B^{jm_Am_B}_\mu$ and $L^{jm_Am_B}_\mu$ are not eigenfunctions of L^2, but the following linear combinations, already derived in [6], are

$$f^{jm_Am_B}_1 \propto (j+1)B^{(j+1)m_Am_B}_\mu + L^{(j+1)m_Am_B}_\mu,$$
$$f^{jm_Am_B}_2 \propto (j+1)B^{(j-1)m_Am_B}_\mu - L^{(j-1)m_Am_B}_\mu. \quad (A3)$$

Their eigenvalue of L^2 is $j(j+2)$ and the associated degeneracy factors are $(j+2)^2$ and j^2, respectively. Adding up all the degeneracies of the $f_{1,2}$ and $T^{1,2}$ basis, we have $(j+2)^2 + j^2 + 2(j+1)^2 = 4(j+1)^2 + 2$. Comparing to the expected result, we got two too many. What went wrong? It is the degeneracy assumed for the transverse modes. The correct combined degeneracy should be $2j(j+2)$, which is two units less than $2(j+1)^2$.

The point is that the linear transformation in (A2), defining $T^{i;jm_Am_B}_\mu$ out of $Y_{jm_Am_B}$, is not injective and we lose degrees of freedom from the $(j+1)^2$ of the hyperspherical harmonics. As a simple demonstration, let us consider the case with $j = 1$. We take the following real combination of $Y_{1m_Am_B}$,

$$Y_{1\alpha} = \left\{\frac{x}{\rho}, \frac{y}{\rho}, \frac{z}{\rho}, \frac{t}{\rho}\right\} = \frac{x_\alpha}{\rho} = \hat{x}_\alpha, \quad (A4)$$

which gives us four independent objects. Then one set of transverse modes with $V^{(1)}_\nu = (1, 0, 0, 0) = \delta_{\nu 1}$, is given by

$$T^{1;(\alpha)}_\mu = \epsilon_{\mu\nu\rho\sigma} V^{(1)}_\nu x_\rho \partial_\sigma Y_{1\alpha} = \epsilon_{\mu\nu\rho\sigma}\delta_{\nu 1} x_\rho \left(\frac{\delta_{\sigma\alpha}}{\rho} - \frac{x_\sigma x_\alpha}{\rho^3}\right) = \epsilon_{\mu 1\rho\alpha}\hat{x}_\rho. \quad (A5)$$

We see that $T^{1;(\alpha=1)}_\mu = 0$ and a degree of freedom is lost. The other transverse modes, taking $V^{(2)}_\nu = (0, 1, 0, 0) = \delta_{\nu 2}$, are $T^{2;(\alpha)}_\mu = \epsilon_{\mu 2\rho\alpha}\hat{x}_\rho$, so that $T^{2;(\alpha=2)}_\mu = 0$. It is then easy to check that, out of the remaining 6 transverse modes, only 5 are linearly independent, because $T^{1;(\alpha=2)}_\mu = -T^{2;(\alpha=1)}_\mu$.

We have thus shown that the degeneracy factor of the transverse modes defined in [6] is not $2(j+1)^2$, as assumed in that work and in subsequent literature. The functions $T^{1,2}$ actually cover a space with dimension $2j(j+2) - j$ for any $j > 0$, as proven for generic j in the Supplemental Material [22], and exemplified above for $j = 1$. This is neither the naïve $2(j+1)^2$, nor the correct $2j(j+2)$. This demonstrates that $T^{1,2}$ from (A2) are not linearly independent and do not form a complete basis for the transverse modes. Additional technical details of this issue are discussed in the Supplemental Material [22].