Revista Minelor – Mining Revue
ISSN-L 1220-2053 / ISSN 2247-8590
vol. 31, issue 2, pp. 25-30, selected paper from the First International EnviroMining Virtual Conference 2025

DIFFERENTIAL EVOLUTION FOR MAXIMUM CUBOID EXTRACTION IN SUSTAINABLE STONE CUTTING: APPLICATION TO A GRANITE ROCK EXAMPLE

Margarita ANTONIOU[1], Drago TORKAR[2], Zoran LEVNAJIĆ[3], Vida VUKAŠINOVIĆ[4]*

[1]Computer Systems Department, Jožef Stefan Institute, Jožef Stefan International Postgraduate School, Ljubljana, Slovenia, margarita.antoniou@ijs.si
[2]Computer Systems Department, Jožef Stefan Institute, Complex Systems and Data Science Lab, Faculty of Information Studies in Novo Mesto, Ljubljana, Slovenia, drago.torkar@ijs.si
[3]Complex Systems and Data Science Lab, Faculty of Information Studies in Novo Mesto, Novo Mesto, Slovenia, zoran.levnajic@fis.unm.si
[4]Computer Systems Department, Jožef Stefan Institute, Complex Systems and Data Science Lab, Faculty of Information Studies in Novo Mesto, Ljubljana, Slovenia, vida.vukasinovic@ijs.si

DOI: 10.2478/minrv-2025-0014
received – 17.01.2025 / revised – 31.01.2025
accepted – 18.02.2025 / published – 18.06.2025

Abstract: *Maximizing the extraction of usable volume in the form of a cuboid from polyhedra is a complex optimization challenge. It has immediate application in mining and stone-cutting, with significant implications for sustainable resource utilization. This study addresses the problem by employing Differential Evolution (DE), a population-based evolutionary algorithm, to identify the axis-independent maximum cuboid within polyhedra. Using a granite rock as a test case, the algorithm iteratively refines solutions through mutation, crossover, and selection operators. The algorithm's behaviour and performance is evaluated via 20 independent runs. The results are presented through convergence plots and snapshots illustrating the evolution of the solutions. Our findings demonstrate the potential of DE in solving these complex geometric optimization problems and contributing to sustainable mining practices.*
Keywords: *differential evolution, cuboid extraction, stone cutting, sustainable mining, granite rock, geometric optimization.*

1. Introduction

The mining industry faces significant challenges in optimizing resource extraction while minimizing environmental impact and operational costs. In the context of irregular rock or stone formations, which can be readily modelled as polyhedra, it becomes critical to extract the largest usable volume. Due to commercial constraints, this usable volume must be in the form of a cuboid. This allows us to formulate the problem in terms of mathematical (geometric) optimization: How to extract the cuboid of the biggest possible volume from a given arbitrary polyhedron? Efficient solutions to this problem directly influence sustainability and efficiency in mining operations.

Recent advances in 3D scanning technology have enabled the creation of detailed geological models, providing mining engineers with accurate representations of rock formations to develop optimal extraction strategies [1]. These models improve planning processes, minimize waste, and enhance recovery rates, but optimizing extraction geometries remains a computationally challenging task.

The problem of identifying the maximum volume cuboid within a polyhedron has been studied already, with methods such as those proposed by Mondal and Bhattacharya [2] and Dumitrescu and Jiang [3]. However, these approaches are limited to axis-parallel configurations and convex polyhedra, making them less applicable to the irregular and non-convex shapes encountered in real-world mining scenarios.

Meanwhile, Differential Evolution (DE) [4] has proven an effective optimization strategy for solving high-dimensional, non-linear, and constrained optimization problems. In this study, DE is applied to extract

* Corresponding author: Margarita Antoniou, PhD, Researcher, Computer Systems Department, Jožef Stefan Institute, Ljubljana, Slovenia, margarita.antoniou@ijs.si

vol. 31, issue 2 / 2025
pp. 25-30

the maximum non-axis-aligned cuboid from polyhedra, addressing alignment and feasibility constraints to ensure practical applicability. The method's performance is demonstrated using a 3D granite rock model, where the polyhedron is convexified to simplify the optimization process. The results are evaluated to highlight the potential of DE in addressing these geometric challenges.

The rest of the paper is structured as follows: Section 2 describes the proposed DE-based method. Section 3 presents the results applied to the 3D granite rock model. Section 4 concludes the paper and outlines future work.

2. Methodology

In this section, we briefly explain the problem formulation and the Differential Evolution (DE) steps employed in the proposed method for optimizing the extraction of maximal cuboids from polyhedra.

2.1. Problem Formulation

The objective is to maximize the volume of a cuboid, $V = \ell_x \times \ell_y \times \ell_z$, where ℓ_x, ℓ_y, and ℓ_z are the cuboid dimensions, and its orientation is defined by Euler angles (α,β,γ). These parameters allow for non-axis-aligned cuboids to better fit within the polyhedron.

The cuboid's vertices **v**, transformed by the rotation matrix $\mathbf{R}(\alpha,\beta,\gamma)$ and translation vector **t**, must satisfy:

$$p_i = R(\alpha,\beta,\gamma) \cdot v_i + t, \quad p_i \in Convex\ Hull, \quad \forall i = 1, \dots, 8, \tag{1}$$

ensuring all vertices remain inside the polyhedron. A penalty is applied in the fitness function to discourage constraint violations. This formulation balances volume maximization and feasibility within the convex polyhedron.

2.2. DE Algorithm Steps

The proposed DE algorithm takes as input an arbitrary polyhedron (convex or otherwise). It follows the workflow depicted in Figure 1 and consists of the following steps:

Step 1: Convexification: The input 3D rock geometry, represented as a polygon mesh, is transformed into its convex hull [5]. This simplifies the optimization process by eliminating concavities, ensuring efficient and well-defined geometric constraints.

Step 2: Initialization: A population of candidate solutions is generated, where each solution is defined by the cuboid dimensions (ℓ_x,ℓ_y,ℓ_z), center coordinates (cx, cy, cz), and Euler angles (α,β,γ). These solutions serve as the starting points for optimization.

Step 3: Mutation and Crossover: New candidate solutions are created through mutation, where a weighted difference between randomly selected solutions x_{r2} and x_{r3} is added to a third solution x_{r1}:

$$v_i = x_{r1} + F \cdot (x_{r2} - x_{r3}) \tag{2}$$

where F is the mutation factor. Crossover combines features of parent solutions and mutants to maintain diversity and avoid premature convergence.

Step 4: Selection: Candidate solutions are evaluated based on their fitness, defined by the cuboid volume and penalized for constraint violations. The best solutions are retained for the next generation, ensuring the algorithm focuses on promising candidates.

Step 5: Termination: The process repeats until a predefined maximum number of generations, (G_{max}), is reached. Convergence can also be monitored based on the improvement in cuboid volume over successive generations.

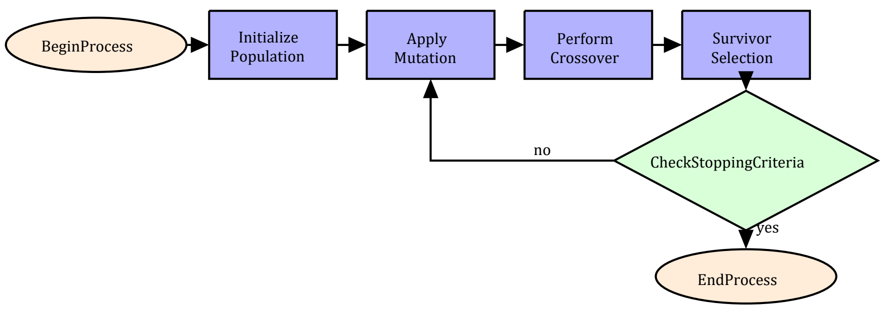

Fig.1. Flowchart of the Differential Evolution Algorithm

3. Results

The Differential Evolution (DE) algorithm was applied to a real-world polyhedron - a granite rock formation from actual mining operations. The dataset was retrieved from https://www.cgtrader.com/free-3d-models/exterior/landscape/granite-stone-02. It has 1900 triangular polygons and real-life dimensions of approximately 3 meters in length and 1 meter in height. This model serves as a test case to illustrate the application of our novel approach in optimizing cuboid extraction from complex geological formations (Figure 2). Since this is a stochastic algorithm, we conducted 20 independent runs to evaluate its performance.

Fig.2. Granite stone example

Since the optimization process operates on the convexified representation of the original geometry, it is important to evaluate how well the resulting cuboid aligns with the original polyhedron. A metric how much of material is used in the percentage of how much material is used compared to the total material is a material usage efficiency. Convexification simplifies the optimization by eliminating concavities in the polyhedron but may introduce discrepancies between the original and convexified shapes. To assess this, we quantify how much of the maximum cuboid extracted from the convexified polyhedron remains inside the original polyhedron by proposing the *Convex Retention Ratio (CRR)*. It is defined as:

$$CRR = \frac{V_{inside}}{V_{cuboid}} \times 100, \tag{3}$$

where V_{inside} represents the volume of the cuboid contained within the original polyhedron and V_{cuboid} is the total volume of the cuboid. A higher CRR indicates that a larger proportion of the cuboid is retained within the original boundaries, providing insight into the effect of convexification on the extraction process.

Table 1 summarizes the results of 20 independent runs, reporting the maximum cuboid volume, the efficiency of material usage, and the convex retention ratio (CRR). Material usage efficiency (%) refers to a metric that shows how much of material is used compared to the total material. On average, the algorithm achieved a maximum cuboid volume of 2.42 units^3, with a material usage efficiency of 20.77% and an average CRR of 91.42%. The high CRR values, with most runs exceeding 90%, indicate that the convexification process introduces minimal discrepancy between the convexified polyhedron and the original geometry. The material usage efficiency of around 21% highlights the impact of the irregular shape of the polyhedron on the extraction process, which naturally limits the proportion of usable volume that can be converted into a cuboid.

vol. 31, issue 2 / 2025
pp. 25-30

Table 1. Optimized Cuboid Volumes, Material Usage, and Convex Retention Ratio (CRR)

Run	Maximum Cuboid volume (units^3)	Material Usage Efficiency (%)	CRR (%)
1	2.28	19.82	86.37
2	2.41	20.92	84.20
3	2.52	21.93	97.24
4	2.29	19.87	98.05
5	2.43	21.09	91.09
6	2.46	21.35	83.32
7	2.62	22.76	94.47
8	2.45	21.31	98.64
9	2.22	19.28	96.78
10	2.03	17.67	80.86
11	2.23	19.37	93.54
12	2.59	22.55	86.10
13	2.53	21.95	98.22
14	2.43	21.13	82.72
15	2.47	21.48	85.11
16	2.43	21.14	91.80
17	2.21	19.21	98.37
18	2.26	19.62	97.65
19	2.53	21.99	94.07
20	2.46	21.35	86.81
Average	**2.42**	**20.77**	**91.42**

The performance of the DE algorithm is analyzed through convergence plots. Figure 3 shows the median optimized volume over generations across 20 runs. The plot demonstrates steady improvements in the cuboid volume as the algorithm progresses. The shaded region indicates the standard deviation among the runs, providing a visual representation of the algorithm's consistency over generations.

Snapshots of the cuboid identification process in specific generations (1, 51, and 100) are shown in Figure 4, where the cuboids found are shown in green. These visuals provide insight into how the cuboid evolves within the convexified granite rock as the algorithm refines its solution, adjusting both the size and positioning.

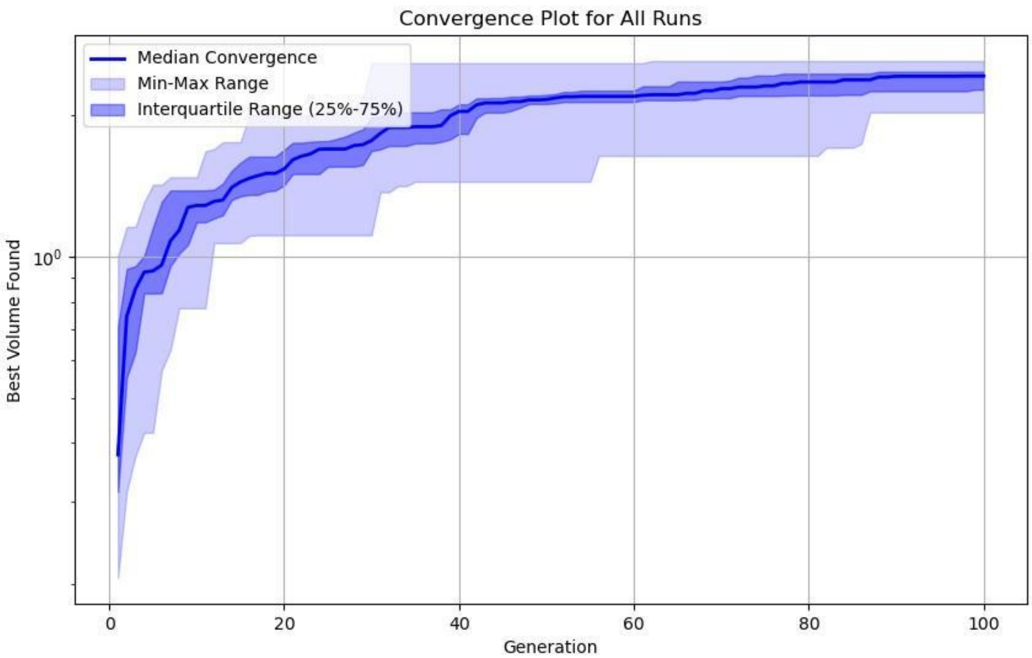

Fig.3. Convergence plot showing the median volume over generations with standard deviation shaded

vol. 31, issue 2 / 2025
pp. 25-30

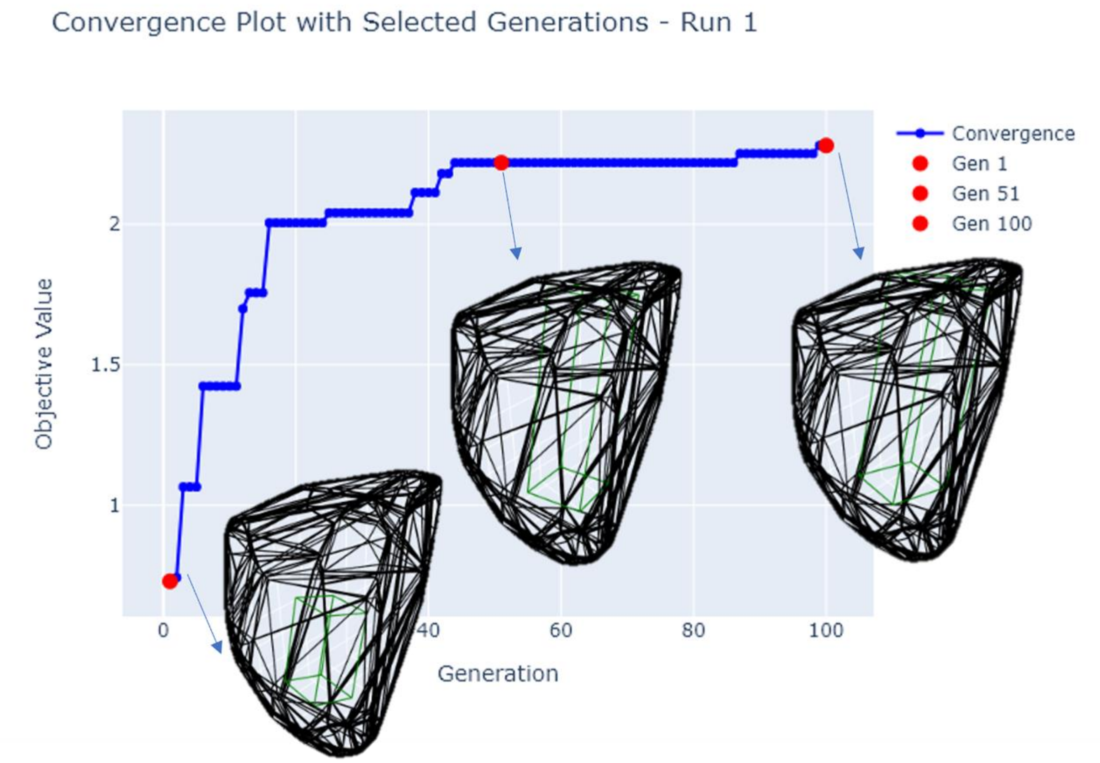

Fig.4. Snapshots of the cuboid (green) identification process at generations 1, 51 and 100 on the convergence plot for run 1, in the convexified granite rock

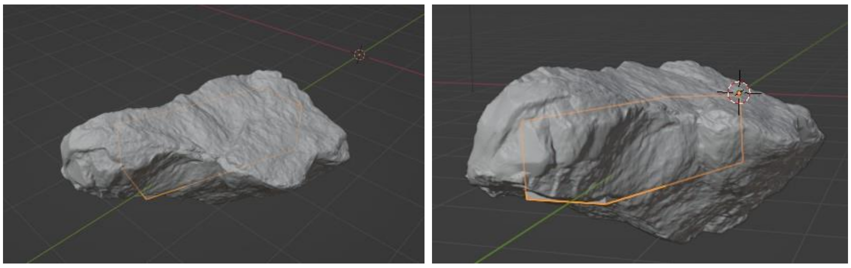

Fig.5. View of the maximum cuboid found in the original granite rock for run 1

Finally, Figure 5 shows the optimized cuboid extracted in Run 1 within the original granite rock polyhedron. While a small error exists, as reflected by the Convex Retention Ratio (CRR), this is an expected outcome given the inherent inaccuracies introduced during the 3D modeling process, the convexification of the polyhedron, and the optimization workflow. Despite these approximations, the visualization highlights the practical applicability of the algorithm, showcasing how the optimized cuboid closely aligns with the original geometry.

4. Conclusion and Future Work

This paper presented a method for maximizing the extraction of usable volume in the form of non-axis-aligned cuboids from polyhedra, using Differential Evolution (DE). By applying the method to a granite rock case study, the results demonstrated the effectiveness of DE in handling complex geometric optimization problems while adhering to geometric constraints. The iterative refinement process through mutation, crossover, and selection successfully optimized the cuboid dimensions and orientation, illustrating the method's applicability to real-world mining and stone-cutting scenarios.

Future work will include incorporating the original shape into the final checks to ensure closer alignment with the initial geometry. Further research will also focus on enhancing the performance of the DE algorithm and exploring its applicability in more complex geometric problems. In addition, incorporating user-defined constraints may further improve flexibility for practical applications in the mining industry. Finally, the ultimate goal of this endeavor is to package these algorithms into a user-friendly software that could realistically be of use in quarries and actual mining operations.

vol. 31, issue 2 / 2025
pp. 25-30

Acknowledgements

This work was supported by the Ministry of Education, Science, and Sport of the Republic of Slovenia via ERA-MIN financing framework, project AI-COSTQO, contract number C3330-22-252011.

References

[1] **Singh S. K., Banerjee B. P., Raval S.**, 2023
A review of laser scanning for geological and geotechnical applications in underground mining, International Journal of Mining Science and Technology, 33(2), 133-154.

[2] **Mondal S., Biswas A., Sarkar A.**, 2021
Finding the largest empty cuboid inside a 3d digital object, Multimedia Tools and Applications, 80, 1–21, https://doi.org/10.1007/S11042-020-10490-8.

[3] **Dumitrescu A., Jiang M.**, 2009
On the largest empty axis-parallel box amidst n points.

[4] **Storn R., Price K.**, 1997
Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces, Journal of Global Optimization, 11(4), 341–359.

[5] **Barber C.B., Dobkin D.P., Huhdanpaa H.**, 1996
The quickhull algorithm for convex hulls, ACM Transactions on Mathematical Software (TOMS), 22(4), 469–483.