



OPEN A multi layered encryption framework using intuitionistic fuzzy graphs and graph theoretic domination for secure communication networks

A. Meenakshi¹, S. Dhanushiya¹, Leo Mrcic^{2,5}, Antonios Kalampakas³ & Sovan Samanta^{2,4,6}✉

Secure communication is essential in today's rapidly evolving digital environment, and strong encryption methods are required to protect private data from unwanted access. The aim of this study is to strengthen the security and complexity of encrypted communications by adopting a new form of cryptographic encryption technique based on the principles of an intuitionistic fuzzy graph. Key graph-theoretic measures, such as domination number, vertex categorization (alpha-strong, beta-strong, and gamma-strong), vertex order coloring, and chromatic number, play important roles in this process. Domination number finds the key vertices of the network, while vertex strength categorization and fuzzy graph coloring provide multiple encryption layers, hence the encoded message is highly resistant to decryption unless a proper key is used. The chromatic number offers further security through various patterns of vertex coloring. The comparative analysis shows the proposed approach to be superior compared to RSA, AES, ECC, and Blowfish due to its increased security, computational efficiency, and resilience to attacks. This framework can be applied to the protection of banking PINs, military access codes, government identification numbers, cryptographic keys, and medical records, so it is an extremely versatile solution for protecting sensitive data. This multi-step approach to encryption through the proposed technique ensures safe transfer and efficient encoding as it establishes a complicated framework.

Keywords Graph network, Vertex order coloring, α -strong vertex, β -strong vertex, γ -strong vertex, Domination number

Graph theory, originating with Euler's 1736 solution to the Königsberg bridge problem, has evolved to become a foundational tool in analyzing structures and connectivity in networks and has got diverse applications, ranging from engineering to computer science. Wilson¹ and Zhang and Chartrand² discussed the mathematical and theoretical aspects of graph theory, clarifying the basic concepts. Deo³ progressed from this basic level of discussion to introduce the application of graph theory in computer science and engineering, thus underlining its importance in technological fields. Fuzzy set theory has been used to develop intuitionistic fuzzy graphs (IFGs), which helps in modeling the uncertainty and vagueness prevailing in real life. It expanded classical graph theory into intuitionistic fuzzy domains in order to make complex system modeling possible⁴. Parvathi and Karunambigai⁵ have discussed the structural features of intuitionistic fuzzy graphs and have initiated the base of their theory. In Sahoo and Pal⁶ introduced insightistic fuzzy competition graphs that have been explained based upon its applications in biological and social networks. Rashmanlou et al.⁷⁻⁹ provided new ideas that directly arrived in intuitionistic fuzzy graphs (IFGs) as an advancement in the interaction of the two theories, namely fuzzy logic and graph theory^{10,11}. Gani et al. discussed the degree, order, and size in intuitionistic fuzzy graphs¹². The authors of the study have analyzed structural properties of IFGs. Some of the examples include network

¹Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, Tamilnadu, India. ²Department of Technical Sciences, Algebra Bernays University, Gradiscanska 24, 10000 Zagreb, Croatia. ³College of Engineering and Technology, American University of the Middle East, Egaila 54200, Kuwait. ⁴Research Center of Performance and Productivity Analysis, Istinye University, 34010 Istanbul, Turkey. ⁵Rudolfovo Science and Technology Centre Novo Mesto, Novo Mesto, Slovenia. ⁶Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk 721636, India. ✉email: ssamantavu@gmail.com

modeling for humans, image processing, and systems of decision making. Sadati, Rashmanlou, and Talebi published a research paper on domination in intuitionistic fuzzy incidence graphs¹³. The authors introduced the notion of strong edges and studied domination concerning some parameters of edges and vertices. Their work provided mathematical models that can be applied to real-life applications in the communication networks and the multi-agent systems. This study further, in this regard, extended the foundation built in domination theory on fuzzy graph-based systems. Rajeshkumar and Anto¹⁴ discussed several domination parameters in intuitionistic fuzzy graphs, by providing new insights into how these parameters can be used to optimize network structures. They discussed parameters like total domination and their role in optimizing decision-making processes in uncertain environments. Their work added much to the theoretical framework of domination in intuitionistic fuzzy networks, thus showing the significance of these parameters in complex systems. Yaqoob et al.¹⁵ simulated cellular network providers for optimization under uncertainty with the applications of intuitionistic fuzzy graphs. The pioneering work on these studies motivates further work in the context of network analysis and graph operations. Network security involves protecting data, communication channels, and network infrastructure from cyber threats such as unauthorized access, attacks, and data breaches. Fuzzy trees offer a flexible framework for modeling uncertain security parameters, allowing for more adaptive and intelligent security mechanisms^{16,17}. Kumaran et al.^{18,19} analyzed efficient domination in the context of fuzzy networks. The authors provided a framework that enhanced decision-making in uncertain scenarios through the integration of fuzzy logic and domination theory. They highlighted the flexibility of the framework in network analysis and multi-agent systems. Meenakshi et al.²⁰ presented new applications of these labelled structures in their investigation of total magic labelling in graph networks. It demonstrated the mathematical model of maximizing graph features, mainly those in the network with the need for a high level of efficiency and connectivity. An extension of the concept of efficient domination into cryptographic applications was done by Meenakshi and Babujee²¹. They studied the encryption approach using labelling techniques that had safe data transfer highlighted. Masud et al.²² proposed a new cryptographic method to encrypt and decrypt information to improve the security and efficiency of communication in the digital era. The method used sophisticated encryption algorithms and worked to ensure data confidentiality and integrity against outside attacks. It worked on new schemes of encryption and provided an in-depth performance analysis from the perspective of speed, security, scalability and cloud computing. Kaushik et al.²³ provided an overview of the existing cryptographic schemes to encrypt and decrypt information by giving an analysis of the pros and cons of such schemes. They discussed classical and sophisticated encryption methods, such as RSA, AES, and elliptic curve cryptography, and explained how they safeguard digital communications. They also discussed the recent advancements in cryptography, namely in quantum computing, blockchain technologies, and provided insights into the future of cryptographic research. A novel method of improving security protocols is given by the introduction of domination parameters into graph labelling. Meenakshi et al.²⁴ worked on effective dominance in intuitionistic fuzzy networks. Zuo et al.²⁵ proposed Global user graph enhanced network for next POI recommendation. The model improves the accuracy of recommendation by including spatial, temporal, sequential, and social dependencies. Cheng et al.²⁶ proposed an efficient data dependency capture model, which improves storage efficiency and retrieval performance. Wang et al.²⁷ presented FreqGAN, an infrared and visible image fusion model based on unified frequency adversarial learning. Their approach improves image quality through a combination of frequency-based features from both modalities with a resultant enhanced fusion performance. Zhou et al.²⁸ developed a binary optical computing method for resource conserving and high robustness image sensing. This approach reduces computation complexity enormously but preserves image quality acquisition at high levels. Also, one for All architecture comes with an overall generative model for classifying image emotion that utilizes deep learning-based generative models for maximizing accuracy in emotional content identification from images. Chu et al.²⁹ presented a deniable encryption technique for DNA storage in the modulation domain. Their encryption method improves data privacy and security with the ability for plausible deniability, which renders it challenging for attackers to distinguish between real and redundant data. Hu et al.³⁰ introduced headtrack, a real-time human-computer interaction system using wireless earphones. Their system monitors head motion to facilitate unobstructed interaction with digital devices, enhancing accessibility and hands-free operation. Sun et al.³¹ introduced a game-theoretic solution for multi priority data transmission in 5G vehicular networks. Their framework maximizes network resource allocation, achieving efficiency and fairness in data transmission among vehicles with varying priority levels. Guan et al.³² worked on domination in intuitionistic fuzzy directed graphs (IFDGs), namely in influential graph applications. The paper described the interaction of fuzzy logic and graph theory to model directed networks with uncertainty and vagueness embedded in them. They described domination types, such as vertex and edge domination, in fuzzy networks, citing their applicability to modeling real-world systems, such as social networks and information spread. The research proved the importance of effective dominance in guaranteeing efficient computation without undermining the integrity of the network. Meenakshi et al.^{33,34} proposed vertex order coloring techniques for effective network assessment. With the proposal of vertex order coloring techniques for effective network assessment by Meenakshi et al., we sought to build on this concept by proposing vertex order coloring in intuitionistic fuzzy vertex order coloring (IFVOC) for improved network assessment.

- The proposed work enhances encryption security using an intuitionistic fuzzy graph-based strategy.
- By integrating graph measures like domination number, vertex categorization, and coloring, it adds multiple encryption layers, making unauthorized decryption significantly harder.
- This approach strengthens data protection, particularly in sensitive domains like government, military, and banking.
- Additionally, it demonstrates the efficient application of intuitionistic fuzzy graphs in developing advanced multi-level encryption for secure communication.

Section “Preliminaries” contains the background information necessary for a better understanding of the concepts related to intuitionistic fuzzy graph (IFG) and coloring techniques. Section “Intuitionistic fuzzy based network” describes the methodology used in this research, including the design for the application of IFVOC. An example of the proposed research method is presented in Section “Example”. Section “Algorithm and flowchart” presents the pseudocode and flowchart of the proposed method. Section “Comparative study” gives the comparative analysis of various cryptographic methods followed by conclusion.

Preliminaries

Definition 2.1 ²⁹ A pair (\mathcal{U}, μ) with \mathcal{U} as a set and $\mu: \mathcal{U} \rightarrow [0, 1]$ as a membership function is called a fuzzy set \mathcal{F}_δ . The value $\mu(x)$ for each $x \in \mathcal{U}$ is the grade of membership function of the fuzzy set $\mathcal{A} = (\mathcal{U}, \mu)$, where the reference set \mathcal{U} is the universe of discourse.

Definition 2.2 ⁴ The form of an intuitionistic fuzzy set $\mathcal{I}_{\mathcal{F}_\delta}$ on universal set \mathcal{U} is $\{\mu, \sigma_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu), \mu_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu) \mid \mu \in \mathcal{U}\}$, where $\sigma_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu) \in [0, 1]$ is the degree of membership of μ in $\mathcal{I}_{\mathcal{F}_\delta}$, $\mu_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu) \in [0, 1]$ is the degree of non-membership of μ in $\mathcal{I}_{\mathcal{F}_\delta}$, and $\sigma_{\mathcal{I}_{\mathcal{F}_\delta}}, \mu_{\mathcal{I}_{\mathcal{F}_\delta}}$ satisfies the following condition for all $\mu \in \mathcal{U}$: $\sigma_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu) + \mu_{\mathcal{I}_{\mathcal{F}_\delta}}(\mu) \leq 1$.

Definition 2.3 ³³ Let $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$ be a fuzzy graph on the graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$ is a pair of functions $(\tilde{\sigma}, \tilde{\mu})$ where $\tilde{\sigma}: \mathcal{V} \rightarrow [0, 1]$ is a fuzzy subset of a non-empty set \mathcal{V} and $\tilde{\mu}: \mathcal{V} * \mathcal{V} \rightarrow [0, 1]$ is a symmetric fuzzy relation on $\tilde{\sigma}$ such that $\forall (i, j) \in \mathcal{V}, \tilde{\mu}(i, j) \leq \tilde{\sigma}(i) \wedge \tilde{\sigma}(j)$ is satisfied.

Definition 2.4 ³⁰ An intuitionistic fuzzy graph (IFG), $\mathcal{G}_{\mathcal{I}\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}_I, \tilde{\mu}_I)$ consists of the following components:

1. $\tilde{\sigma}_I 1. = (\tilde{\sigma}_{I1}, \tilde{\sigma}_{I2})$, where,

$\tilde{\sigma}_{I1}: \mathcal{V} \rightarrow [0, 1]$ represents the membership degree of each vertex $v \in \mathcal{V}$, and
 $\tilde{\sigma}_{I2}: \mathcal{V} \rightarrow [0, 1]$ represents the non-membership degree of each vertex $v \in \mathcal{V}$, with the condition $\tilde{\sigma}_{I1}(v) + \tilde{\sigma}_{I2}(v) \leq 1$.

2. $\tilde{\mu}_I 2. = (\tilde{\mu}_{I1}, \tilde{\mu}_{I2})$, where,

$\tilde{\mu}_{I1}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ represents the membership degree of edges, and.

$\tilde{\mu}_{I2}: \mathcal{V} \times \mathcal{V} \rightarrow [0, 1]$ represents the non-membership degree of edges, with the condition $\tilde{\mu}_{I1}(i, j) + \tilde{\mu}_{I2}(i, j) \leq 1$ for all $i, j \in \mathcal{V}$.

In an IFG, the edge membership function $\tilde{\mu}_{I1}(i, j)$ must satisfy $\tilde{\mu}_{I1}(i, j) \leq \min(\tilde{\sigma}_{I1}(i), \tilde{\sigma}_{I1}(j))$, and the edge non-membership function $\tilde{\mu}_{I2}(i, j)$ must satisfy $\tilde{\mu}_{I2}(i, j) \leq \max(\tilde{\sigma}_{I2}(i), \tilde{\sigma}_{I2}(j))$.

Definition 2.5 ⁵ The vertex cardinality of an IFG, $\mathcal{G}_{\mathcal{I}\mathcal{F}_g} = (\mathcal{U}, \mathcal{R})$, denoted by $|\mathcal{U}|$, is defined as:

$$|\mathcal{U}| = \sum_{u_i \in \mathcal{U}} \left[\frac{1 + \alpha(u_i) - \beta(u_i)}{2} \right]$$

where $\alpha(u_i)$ is the membership degree of the vertex u_i and $\beta(u_i)$ is the non-membership degree of the vertex u_i . The vertex cardinality represents the order of the graph $\mathcal{G}_{\mathcal{I}\mathcal{F}_g}$, which is denoted as $\mathcal{O}(\mathcal{G}_{\mathcal{I}\mathcal{F}_g})$.

Definition 2.6 ⁵ The edge cardinality of an intuitionistic fuzzy graph $\mathcal{G}_{\mathcal{I}\mathcal{F}_g} = (\mathcal{U}, \mathcal{R})$, denoted by $|\mathcal{R}|$, is given by:

$$|\mathcal{R}| = \sum_{u_i u_j \in \mathcal{R}} \left[\frac{1 + \gamma(u_i u_j) - \delta(u_i u_j)}{2} \right]$$

Where $\gamma(u_i u_j)$ is the membership degree of the edge $(u_i u_j)$ and $\delta(u_i u_j)$ is the non-membership degree of the edge $(u_i u_j)$. The edge cardinality is referred to as the size of the graph $\mathcal{G}_{\mathcal{I}\mathcal{F}_g}$, denoted as $\mathcal{S}(\mathcal{G}_{\mathcal{I}\mathcal{F}_g})$.

Definition 2.7 ³⁵ An edge $\mathcal{E} = (i, j)$ in a fuzzy graph $\mathcal{G}_{\mathcal{F}_g}$ is known as effective if $\tilde{\mu}(i, j) < \tilde{\sigma}(i) \wedge \tilde{\sigma}(j)$.

Definition 2.8 ³⁵ Given a graph, $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$ on the graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, the degree of a vertex v is represented by $d(v)$, which is the sum of the weights of all the vertices that are adjacent to v . Since $v \in \mathcal{V}$, the maximum degree is $\Delta(\mathcal{G}_{\mathcal{F}_g}) = \max\{d(v)\}$. The minimum degree of a vertex v is represented by $\delta(\mathcal{G}_{\mathcal{F}_g}) = \min\{d(v) : v \in \mathcal{V}\}$.

Definition 2.9 ³⁵ If v is the vertex of a graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, then $A(v) = \{u \in \mathcal{V} \mid \frac{1}{2}[\sigma(i) \wedge \sigma(j)] < \mu(i, j)\}$ are the adjacent vertices to the vertex v .

$$\begin{aligned}\mathcal{A}_\beta(v) &= \{u \in \mathcal{A}(v) | d(v) \geq d(u)\} \\ \mathcal{A}_\alpha(v) &= \{u \in \mathcal{A}(v) | d(v) \leq d(u)\}.\end{aligned}$$

Definition 2.10 ³⁵ In a fuzzy graph $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$, a vertex $v \in \mathcal{V}$ is said to be α -strong if for any $v \in \mathcal{A}(\mathcal{V})$ satisfies $d(v) \geq d(\mathcal{A}(v))$. The representation for it is $\alpha_{\beta_v}(v)$. The minimum α -strong vertex of a fuzzy graph is denoted as $\alpha_{\beta_v}(\mathcal{G}_{\mathcal{F}_g})$, and the maximum α -strong vertex is denoted by $\Delta\alpha_{\beta_v}(\mathcal{G}_{\mathcal{F}_g})$.

Definition 2.11 ³⁵ In a fuzzy graph $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$, a vertex $v \in \mathcal{V}$ is said to be γ -strong if for any $v \in \mathcal{A}(\mathcal{V})$ satisfies $d(v) \leq d(\mathcal{A}(v))$. The representation for it is $\gamma_{\alpha_v}(v)$. The minimum γ -strong vertex of a fuzzy graph is denoted as $\gamma_{\alpha_v}(\mathcal{G}_{\mathcal{F}_g})$, and the maximum γ -strong vertex is denoted by $\Delta\gamma_{\alpha_v}(\mathcal{G}_{\mathcal{F}_g})$.

Definition 2.12 ³⁵ A vertex that meets both the α -strong and γ -strong criteria is termed as β -strong Vertex, represented by (v) . The fuzzy graph's maximum β -strong vertex is indicated by $\Delta\beta_{\beta_v}(\mathcal{G}_{\mathcal{F}_g})$, while the minimum β -strong vertex is denoted by $\delta\beta_{\beta_v}(\mathcal{G}_{\mathcal{F}_g})$.

Definition 2.13 ³⁵ The color assignment of a crisp graph, also referred to as proper coloring, involves assigning colors to its vertices such that no two adjacent vertices share the same color. A color class consists of all the vertices that are assigned the same color and are independent from each other.

Definition 2.14 ³⁵ A k -fuzzy coloring of $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$ is a collection $\xi = \{\zeta_1, \zeta_2, \zeta_3, \dots, \zeta_k\}$ of fuzzy sets on a set \mathcal{V} .

- If (i) The union of all sets in ξ is equal to σ .
 (ii) The intersection of any two sets ζ_i and ζ_j is zero.
 (iii) For each strong edge (u, v) ($i.e. e(u, v) > 0$) in $\mathcal{G}_{\mathcal{F}_g}$, the minimum of $\zeta_i(u)$ and $\zeta_j(v)$ is 0 $\forall (1 \leq i \leq k)$.

The fuzzy chromatic number of $\mathcal{G}_{\mathcal{F}_g}$, denoted as $\chi_{\tau_0}(\mathcal{G}_{\mathcal{F}_g})$, is the smallest value k for which a k -fuzzy coloring exists.

Definition 2.15 ²⁴ An intuitionistic fuzzy vertex order coloring involves assigning an ascending sequence of k -colors to the vertices of a graph $\mathcal{G}_{\mathcal{F}_g}$, determined by their intuitionistic fuzzy strong degree. This degree considers both membership and non-certainty values. The sequence of colors follows the order, $\mathcal{S}_\alpha(\mathcal{V}) < \beta(\mathcal{V}) < \mathcal{W}_\gamma(\mathcal{V})$ where $\mathcal{S}_\alpha(\mathcal{V})$, $\beta(\mathcal{V})$, $\mathcal{W}_\gamma(\mathcal{V})$ represents the sets of strong vertices determined by intuitionistic fuzzy criteria.

A collection $\xi = \{\mathcal{S}_\alpha(\mathcal{V}), \beta(\mathcal{V}), \mathcal{W}_\gamma(\mathcal{V})\}$ of strong vertex sets on \mathcal{V} is defined as an intuitionistic fuzzy vertex order coloring for the graph $\mathcal{G}_{\mathcal{F}_g} = (\mathcal{V}, \tilde{\sigma}, \tilde{\mu})$. Here $\mu(i, j)$ and $\nu(i, j)$ indicate the membership and non-membership degrees of the edge (i, j) respectively. The following conditions must hold:

1. The union of the sets in ξ covers all vertices, i.e., $\bigcup \xi = \sigma$
2. The sets in ξ are pairwise disjoint, i.e., $\bigcap \xi = \emptyset$
3. For any edge (i, j) with $\mu(i, j) > 0$, if both i and j belong to $\mathcal{S}_\alpha(\mathcal{V})$ or $\beta(\mathcal{V})$, then their colors must be distinct, $c(i) \neq c(j)$
4. For any edge (i, j) with $\mu(i, j) > 0$, if $i \in \beta(\mathcal{V})$ and $j \in \mathcal{W}_\gamma(\mathcal{V})$, then their colors must be the same, $c(i) = c(j)$, and $\mu(i, j)$ is set to zero,
5. For any edge (i, j) with $\mu(i, j) = 0$, if i and j belong to the same set in ξ , their colors must also match, $c(i) = c(j)$.

The smallest number k for which such a coloring exists is referred to as the intuitionistic fuzzy chromatic number, denoted by $\chi_{\vartheta_0}(\mathcal{G}_{\mathcal{F}_g})$.

Definition 2.16 ²⁰ In a graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, a dominating set $\mathcal{D}^* \subseteq \mathcal{V}$ is a subset of vertices such that every vertex in \mathcal{V} is either in \mathcal{D}^* or adjacent to at least one vertex in \mathcal{D}^* . Formally, for every vertex $v \in \mathcal{V}$, either $v \in \mathcal{D}^*$ or there exists $u \in \mathcal{D}^*$ such that $(u, v) \in E(u, v)$. The domination number $\Upsilon(\mathcal{G}^*)$ is the minimum size of a dominating set in the graph \mathcal{G}^* .

Definition 2.17 ²⁰ In a graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$, a dominating set $\mathcal{D}^* \subseteq \mathcal{V}$ is a subset of vertices such that every vertex in \mathcal{V} is either in \mathcal{D}^* or adjacent to at least one vertex in \mathcal{D}^* . Formally, for every vertex $v \in \mathcal{V}$, either $v \in \mathcal{D}^*$ or there exists $u \in \mathcal{D}^*$ such that $(u, v) \in E(u, v)$. Furthermore, a total dominating set does not include isolated vertices. The total domination number, denoted $\Upsilon_\tau(\mathcal{G}^*)$, is the minimum size of a total dominating set in \mathcal{G}^* .

Definition 2.18 ²⁰ An efficient dominating set $\mathcal{D}^* \subseteq \mathcal{V}$ in a graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$ is a dominating set where each vertex in \mathcal{V} is adjacent to exactly one vertex in \mathcal{D}^* .

- i. \mathcal{D}^* is a dominating set.

- ii. For every $v \in \mathcal{V}$, there exists a unique $u \in \mathcal{D}^*$ such that $(u, v) \in E(u, v)$. Such a set ensures that no vertex is dominated by more than one vertex in \mathcal{D}^* , making the domination efficient.

Intuitionistic fuzzy based network

Networks have significant applications in numerous fields, including communication networks, biological networks, social networks, and transportation networks. In many real-world applications, the ambiguity of real networks necessitates sophisticated mathematical models to handle ambiguity and uncertainty. The evolution of classical networks to fuzzy and intuitionistic fuzzy networks significantly improved the description and analysis of uncertain systems. The advances continue to influence present applications, including secure network planning, artificial intelligence, and cryptographic security.

Formation of an intuitionistic graph-based network

Step 1: Construct the appropriate intuitionistic network $\mathcal{M}_{\mathcal{IN}}^*$ (Fig. 1), such that $\mathcal{V}\mathcal{V}\mathcal{V}\mathcal{V}(\text{mod } \mathbb{V}_n)$. Construct a partition of $\mathcal{V}\mathcal{V}\mathcal{V}\mathcal{V}(\text{mod } \mathbb{V}_n)$ into r subgroups $\mathbb{K}_{y-0}, \mathbb{K}_{y-1}, \mathbb{K}_{y-2}, \mathbb{K}_{y-3}, \dots, \mathbb{K}_{y-r}$ such that $\mathbb{K}_{y-0} \equiv \mathbb{R}1(\text{mod } \mathbb{V}_n)$ (where $\mathbb{R}1 = 0$), $\mathbb{K}_{y-1} \equiv \mathbb{R}2(\text{mod } \mathbb{V}_n)$ (where $\mathbb{R}2 = 1$), $\mathbb{K}_{y-2} \equiv \mathbb{R}3(\text{mod } \mathbb{V}_n)$ (where $\mathbb{R}3 = 2$), $\mathbb{K}_{y-3} \equiv \mathbb{R}4(\text{mod } \mathbb{V}_n)$ (where $\mathbb{R}4 = 3$), ..., $\mathbb{K}_{y-r} \equiv \mathbb{R}r(\text{mod } \mathbb{V}_n)$ (where $\mathbb{R}r = \mathbb{V}_n - 1$).

Step 2: Assign the \lceil efficient dominant nodes in the constructed network and frame the \lceil_i sub network, where $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ are the most effective dominating nodes. Let these nodes be the centers of the sub networks $\mathcal{M}_{\mathcal{IN}_1}^*, \mathcal{M}_{\mathcal{IN}_2}^*, \mathcal{M}_{\mathcal{IN}_3}^*, \dots, \mathcal{M}_{\mathcal{IN}_r}^*$ respectively. Let the neighboring nodes of $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ be $\zeta_{d_{11}}, \zeta_{d_{12}}, \dots, \zeta_{d_{1i}}, \zeta_{d_{21}}, \zeta_{d_{22}}, \dots, \zeta_{d_{2i}}, \dots, \zeta_{d_{r1}}, \zeta_{d_{r2}}, \dots, \zeta_{d_{ri}}$ respectively.

Step 3: By definition 2.4, The efficient dominating nodes $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ consists of

$$\{(\zeta_{d_{T_1}}, \zeta_{d_{F_1}}), (\zeta_{d_{T_2}}, \zeta_{d_{F_2}}), \dots, (\zeta_{d_{T_r}}, \zeta_{d_{F_r}})\}.$$

The neighboring vertices $\zeta_{d_{11}}, \zeta_{d_{12}}, \dots, \zeta_{d_{1i}}, \zeta_{d_{21}}, \zeta_{d_{22}}, \dots, \zeta_{d_{2i}}, \dots, \zeta_{d_{r1}}, \zeta_{d_{r2}}, \dots, \zeta_{d_{ri}}$ consists of

$$\{(\zeta_{d_{T_{11}}}, \zeta_{d_{F_{11}}}), (\zeta_{d_{T_{12}}}, \zeta_{d_{F_{12}}}), \dots, (\zeta_{d_{T_{1i}}}, \zeta_{d_{F_{1i}}}), (\zeta_{d_{T_{21}}}, \zeta_{d_{F_{21}}}), (\zeta_{d_{T_{22}}}, \zeta_{d_{F_{22}}}), \dots, (\zeta_{d_{T_{2i}}}, \zeta_{d_{F_{2i}}}), \dots, (\zeta_{d_{T_{r1}}}, \zeta_{d_{F_{r1}}}), (\zeta_{d_{T_{r2}}}, \zeta_{d_{F_{r2}}}), \dots, (\zeta_{d_{T_{ri}}}, \zeta_{d_{F_{ri}}})\};$$

$$(\zeta_{d_{T_{n1}}}, \zeta_{d_{F_{n1}}}), (\zeta_{d_{T_{n2}}}, \zeta_{d_{F_{n2}}}), \dots, (\zeta_{d_{T_{nm}}}, \zeta_{d_{F_{nm}}}).$$

$$\text{Step 4: Define } \mathcal{V}(\mathcal{G}_{\mathcal{IF}q}) = (\mathcal{V}, \mathcal{E}) \quad \text{where} \quad (\mathcal{G}_{\mathcal{IF}q}) = \{$$

$$\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}\} \cup \left\{ \zeta_{d_{1i}} \right\}_{i=1}^{\ell_1} \cup \left\{ \zeta_{d_{2i}} \right\}_{i=1}^{\ell_2} \cup \left\{ \zeta_{d_{3i}} \right\}_{i=1}^{\ell_3} \cup \left\{ \zeta_{d_{4i}} \right\}_{i=1}^{\ell_4}, \dots, \left\{ \zeta_{d_{ni}} \right\}_{i=1}^{\ell_n} \quad \text{and}$$

$$|\mathcal{V}(\mathcal{G}_{\mathcal{IF}q})| = \ell_1 + \ell_2 + \ell_3 + \ell_4 + \dots + \ell_n = \mathfrak{N}$$

$$\text{Let } \mathcal{E} = \{\zeta_{d_{1i}} \zeta_{d_{2j}}, \zeta_{d_{2j}} \zeta_{d_{3k}}, \zeta_{d_{3k}} \zeta_{d_{4l}}, \zeta_{d_{4l}} \zeta_{d_{5m}}, \dots, \zeta_{d_{6n}} \zeta_{d_{7p}}\} \quad \text{for only one}$$

$$= \mathfrak{N}C_2 = (\ell_1 + \ell_2 + \ell_3 + \ell_4 + \dots + \ell_n + n)C_2 - [\{\zeta_{d_i} \zeta_{d_{(i+1)j}}\} \quad \text{where } 1 \leq i \leq n-1; 1 \leq j \leq \ell_n,$$

$$m = 1 \text{ to } n$$

Step 5: Choose the certainty values of

$$\begin{aligned} \rho_{11}^* &= \varepsilon_\mu(\zeta_{d_1}, \zeta_{d_{11}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{11}}); \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{11}})\}; \\ \rho_{12}^* &= \varepsilon_\mu(\zeta_{d_1}, \zeta_{d_{12}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{12}}); \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{12}})\}; \dots; \\ \rho_{1i}^* &= \varepsilon_\mu(\zeta_{d_1}, \zeta_{d_{1i}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{T}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{1i}}); \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_1}) \wedge \mathcal{V}_{\sigma\mathcal{F}\mathcal{M}_{\mathcal{IN}_1}^*}(\zeta_{d_{1i}})\}. \end{aligned}$$

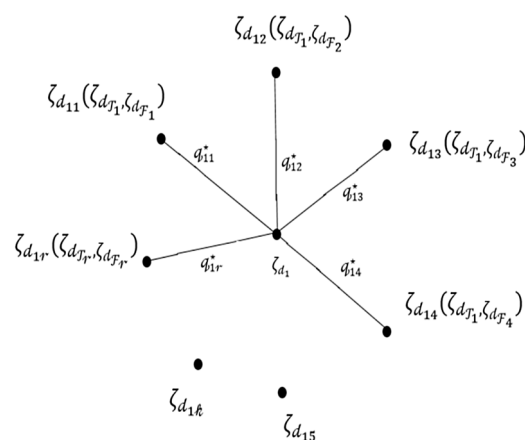


Fig. 1. Construction of sub network $\mathcal{M}_{\mathcal{IN}_1}^*$

Step 6: Similarly the certainty values of $\varphi_{21}^*, \varphi_{22}^*, \dots, \varphi_{2n}^*$ be $\zeta_{d_2} \zeta_{d_{21}}, \zeta_{d_2} \zeta_{d_{22}}, \dots, \zeta_{d_2} \zeta_{d_{2n}}$; of the second network $\mathcal{M}_{\mathcal{I}N_2}^*$. The certainty values of

$$\begin{aligned}\varphi_{21}^* &= \varepsilon_{\mu}(\zeta_{d_2}, \zeta_{d_{21}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{21}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{21}})\}; \\ \varphi_{22}^* &= \varepsilon_{\mu}(\zeta_{d_2}, \zeta_{d_{22}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{22}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{22}})\}; \dots; \\ \varphi_{2n}^* &= \varepsilon_{\mu}(\zeta_{d_2}, \zeta_{d_{2n}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{2n}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_2}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_2}^*(\zeta_{d_{2n}})\}.\end{aligned}$$

The certainty values for the edges of the sub network $\mathcal{M}_{\mathcal{I}N_r}^*$ is given by $\varphi_{r1}^*, \varphi_{r2}^*, \dots, \varphi_{rn}^*$. Where

$$\begin{aligned}\varphi_{r1}^* &= \varepsilon_{\mu}(\zeta_{d_r}, \zeta_{d_{r1}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{r1}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{r1}})\}; \\ \varphi_{r2}^* &= \varepsilon_{\mu}(\zeta_{d_r}, \zeta_{d_{r2}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{r2}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{r2}})\}; \dots; \\ \varphi_{rn}^* &= \varepsilon_{\mu}(\zeta_{d_r}, \zeta_{d_{rn}}) = \min\{\mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{T}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{rn}}); \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_r}) \wedge \mathcal{V}_{\sigma\mathcal{F}}\mathcal{M}_{\mathcal{I}N_r}^*(\zeta_{d_{rn}})\}\end{aligned}$$

respectively.

The secret key

A secret key can be used to authenticate users and verify that the data has been sent or received by a trusted party. We have chosen the efficient dominating node ($\mathcal{O}_{\mathcal{D}}^{\diamond}$) and the weight of alpha-strong vertices (\mathcal{W}_{α}^*) as the two key parameters for constructing the secret key to ensure specific critical aspects of network security and encryption efficiency.

The efficient dominating node represents a central element in the network, ensuring that every other node in its domination set is uniquely covered with minimal overlap. This minimizes redundancy while making the secret key adaptive to changes in network topology. The weight of alpha-strong vertices adds an additional layer of uniqueness by capturing the strength and connectivity of highly influential nodes in the network. This ensures that the key reflects both local and global network characteristics, making it highly specific and difficult to replicate. By applying two independent encryption algorithms, breaking one layer does not compromise the entire system, as the second layer adds an extra level of complexity. Even if an attacker discovers a vulnerability in one encryption method, they must also break the second method, significantly increasing the difficulty and time required for an attack.

Encryption framework

Input: The secret number, $\mathbb{S}_{\mathcal{Y}}^{\diamond} = \mathcal{Y}\mathcal{Y}\mathcal{Y}\mathcal{Y}$ such that $\mathbb{S}_{\mathcal{Y}}^{\diamond} \geq \mathbb{V}_n, \mathbb{V}_n \neq 0, \mathbb{S}_{\mathcal{Y}}^{\diamond}$ should be non-negative.

Output: The intuitionistic colored network ($\mathcal{O}_{\mathcal{M}_{\mathcal{I}N}}^*$) in its encrypted form.

Begin

Step 1: The original message (Secret number) $\mathbb{S}_{\mathcal{Y}}^{\diamond} = \mathcal{Y}\mathcal{Y}\mathcal{Y}\mathcal{Y}$ is subdivided into r subgroups such that $\mathbb{K}_{\mathcal{Y}-0}, \mathbb{K}_{\mathcal{Y}-1}, \mathbb{K}_{\mathcal{Y}-2}, \mathbb{K}_{\mathcal{Y}-3}, \dots, \mathbb{K}_{\mathcal{Y}-r}$ where $\mathbb{K}_{\mathcal{Y}-0} \equiv \mathbb{R}1 \pmod{\mathbb{V}_n}$ (where $\mathbb{R}1=0$), $\mathbb{K}_{\mathcal{Y}-1} \equiv \mathbb{R}2 \pmod{\mathbb{V}_n}$ (where $\mathbb{R}2=1$), $\mathbb{K}_{\mathcal{Y}-2} \equiv \mathbb{R}3 \pmod{\mathbb{V}_n}$ (where $\mathbb{R}3=2$), $\mathbb{K}_{\mathcal{Y}-3} \equiv \mathbb{R}4 \pmod{\mathbb{V}_n}$ (where $\mathbb{R}4=3$), ..., $\mathbb{K}_{\mathcal{Y}-r} \equiv \mathbb{R}r \pmod{\mathbb{V}_n}$ (where $\mathbb{R}r = \mathbb{V}_n - 1$).

Step 2: Assign the $\mathcal{O}_{\mathcal{D}}^{\diamond}$ efficient dominating nodes in the constructed network using the sub networks. The efficiently dominating nodes are $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ in the sub networks $\mathcal{M}_{\mathcal{I}N_1}^*, \mathcal{M}_{\mathcal{I}N_2}^*, \mathcal{M}_{\mathcal{I}N_3}^*, \dots, \mathcal{M}_{\mathcal{I}N_r}^*$ respectively and they are the hubs of the sub networks. $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ be $\zeta_{d_{11}}, \zeta_{d_{12}}, \dots, \zeta_{d_{1r_1}}, \zeta_{d_{21}}, \zeta_{d_{22}}, \dots, \zeta_{d_{2r_2}}, \dots, \zeta_{d_{r1}}, \zeta_{d_{r2}}, \dots, \zeta_{d_{rr_1}}, \zeta_{d_{rr_2}}, \dots, \zeta_{d_{rr_r}}$ are the neighbours of $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ respectively.

In an intuitionistic network, where truth membership ($\mathcal{V}_{\sigma\mathcal{T}}(i)$) and falsity membership ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) values are given equal importance, the design and analysis focus on balancing these measures to capture a nuanced representation of uncertainty and complementarity.

Step 3: The truth membership value ($\mathcal{V}_{\sigma\mathcal{T}}(i)$) is calculated in this step. Define $\mathcal{I}N'_1 = \frac{\mathbb{K}_{\mathcal{Y}-0}}{\mathbb{V}_n}, \mathcal{I}N'_2 = \frac{\mathbb{K}_{\mathcal{Y}-1}}{\mathbb{V}_n}, \dots, \mathcal{I}N'_r = \frac{\mathbb{K}_{\mathcal{Y}-r}}{\mathbb{V}_n}$.

$\mathcal{I}N_{11}^* = \mathcal{N}_{\mathcal{V}'_1} / \mathcal{I}N'_1$ where $\mathcal{N}_{\mathcal{V}'_1}$ denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{I}N'_1$.

$\mathcal{I}N_{12}^* = \mathcal{N}_{\mathcal{V}'_2} / \mathcal{I}N'_2$ where $\mathcal{N}_{\mathcal{V}'_2}$ denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{I}N'_2$, ...,

$\mathcal{I}N_{1r}^* = \mathcal{N}_{\mathcal{V}'_r} / \mathcal{I}N'_r$ where $\mathcal{N}_{\mathcal{V}'_r}$ denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{I}N'_r$. The calculated truth values are now splitted into parts as per the networks edge counts.

Step 4: Split $\zeta_{d_1} = \zeta_{d_{11}}, \zeta_{d_{12}}, \dots, \zeta_{d_{1r_1}}, \zeta_{d_2} = \zeta_{d_{21}}, \zeta_{d_{22}}, \dots, \zeta_{d_{2r_2}}, \dots, \zeta_{d_r} = \zeta_{d_{r1}}, \zeta_{d_{r2}}, \dots, \zeta_{d_{rr_r}}$,

$\zeta_{d_r} = \zeta_{d_{r1}}, \zeta_{d_{r2}}, \dots, \zeta_{d_{rr_r}}$. Assign certainty values for $\varphi_{11}^* = \{\mathcal{V}_{\sigma} \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_1}) \wedge \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_{11}})\};$

$\varphi_{12}^* = \{\mathcal{V}_{\sigma} \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_1}) \wedge \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_{12}})\}; \dots; \varphi_{1r}^* = \{\mathcal{V}_{\sigma} \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_1}) \wedge \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_{1r_1}})\}.$

Similarly assign values for $\varphi_{21}^* = \{\mathcal{V}_{\sigma} \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_2}) \wedge \mathcal{M}_{\mathcal{I}N_1}^*(\zeta_{d_{21}})\}; \varphi_{22}^* = \{\mathcal{V}_{\sigma}$

$\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*(\zeta_{d_2}) \wedge \mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*(\zeta_{d_{22}}); \dots; \mathcal{Z}_2^* = \{\mathcal{V}_\sigma \mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*(\zeta_{d_2}) \wedge \mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*(\zeta_{d_{2r}})\}$. Continuing this process until r th network, $\mathcal{Z}_1^* = \{\mathcal{V}_\sigma \mathcal{M}_{\mathcal{I}\mathcal{N}_r}^*(\zeta_{d_{11}}) \wedge \mathcal{M}_{\mathcal{I}\mathcal{N}_r}^*(\zeta_{d_{1n}})\}, \dots, \mathcal{Z}_n^* = \{\mathcal{V}_\sigma \mathcal{M}_{\mathcal{I}\mathcal{N}_r}^*(\zeta_{d_{r1}}) \wedge \mathcal{M}_{\mathcal{I}\mathcal{N}_r}^*(\zeta_{d_{rn}})\}$

Step 5: As previously discussed in the construction of an intuitionistic network, create the network using the minimum possible number of edges. It includes the edges, with the exception of $\mathcal{E}_{\mu_{del}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}})$.

$\mathcal{E}_{\mu_{del}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}) = \{\zeta_{d_1} \zeta_{d_{1j}}, 2 \leq j \leq \mathcal{A}_1; 1 \leq \mathcal{A}_1 \leq \mathcal{A}_1; \zeta_{d_2} \zeta_{d_{1j}}, 3 \leq j \leq \mathcal{A}_2, 1 \leq \mathcal{A}_2 \leq \mathcal{A}_2; \dots, \zeta_{d_{r+1}} \zeta_{d_{1j}}, (r+1) \leq j \leq \mathcal{A}_{r+1}, 1 \leq \mathcal{A}_{r+1} \leq \mathcal{A}_{r+1}\} \cup \{\zeta_{d_1} \zeta_{d_{1j}}, i = 1; 1 \leq \mathcal{A}_1 \leq \mathcal{A}_1\}$.

Step 6: Now the new network ($\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$) has to be constructed with the minimal number of edges using the sub networks $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*, \dots, \mathcal{M}_{\mathcal{I}\mathcal{N}_r}^*$ subject to certain conditions mentioned below.

Step 7: Now the minimum number of edges present in the new network is as follows.

$\mathcal{E}_{\min}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}) = \{\zeta_{d_1} \zeta_{d_{11}}, 1 \leq j \leq \mathcal{A}_1, \zeta_{d_2} \zeta_{d_{21}}, 1 \leq j \leq \mathcal{A}_2, \dots, \zeta_{d_r} \zeta_{d_{r1}}, 1 \leq j \leq \mathcal{A}_r\} \cup \{\zeta_{d_{11}} \zeta_{d_{21}}\} \cup \{\zeta_{d_{21}} \zeta_{d_{31}}\} \cup \{\zeta_{d_{31}} \zeta_{d_{41}}\} \cup \{\zeta_{d_{(-2)1}} \zeta_{d_{(-1)1}}\}$ for only one \mathcal{A} and \mathcal{A} . Therefore, there are $(\zeta_{d_r} - 1)$ edges present in the minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$.

The edges that connects the sub networks are given the following certainty values.

$\mathcal{E}_\mu(\zeta_{d_{11}} \zeta_{d_{21}}) = \{\mathcal{V}_{\sigma\tau}(\zeta_{d_{11}}) \wedge \mathcal{V}_{\sigma\tau}(\zeta_{d_{21}}); \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{11}}) \wedge \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{21}})\};$

$\mathcal{E}_\mu(\zeta_{d_{21}} \zeta_{d_{31}}) = \{\mathcal{V}_{\sigma\tau}(\zeta_{d_{21}}) \wedge \mathcal{V}_{\sigma\tau}(\zeta_{d_{31}}); \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{21}}) \wedge \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{31}})\}; \dots;$

$\mathcal{E}_\mu(\zeta_{d_{(-2)1}} \zeta_{d_{(-1)1}}) = \{\mathcal{V}_{\sigma\tau}(\zeta_{d_{(-2)1}}) \wedge \mathcal{V}_{\sigma\tau}(\zeta_{d_{(-1)1}}); \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{(-2)1}}) \wedge \mathcal{V}_{\sigma\mathcal{F}}(\zeta_{d_{(-1)1}})\}.$

Step 8: Using Definition 2.5 and 2.6 the certainty values for all the vertices and edges are calculated

Step 9: For the intuitionistic network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$ with \mathcal{V} vertices, find the m -coloring of the vertices. Let $\{\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_m\}$ be the vertices of $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$: $(\mathcal{V}, \mathcal{V}_\sigma, \mathcal{E}_\mu)$, and let $\{1, 2, \dots, m\}$ be the colors of the \mathcal{Z}^* vertices.

Step 10: The following condition is true since every edge in the network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$ is effective.

$\mathcal{E}_\mu(\zeta_i \zeta_j) = \frac{1}{2} [\mathcal{V}_\sigma(\zeta_i) \wedge \mathcal{V}_\sigma(\zeta_j)] \leq \mathcal{E}_\mu(\zeta_i \zeta_j).$

Step 11: The degree of each vertex is calculated in this step using $\Delta_{d,q}(v_i) = \sum (\Delta_{d,q}(\mathcal{A}(v_i)))$.

Step 12: Validate the characteristics of each vertex and classify them into $\alpha_{str}^*, \beta_{str}^*$ and γ_{str}^* vertices as stated in definitions 2.10, 2.11, 2.12.

Step 13: In the minimally connected network, assign colors 1 to \mathcal{A} from the set \mathcal{Z}^* to $\alpha_{str}^*(\mathcal{V})$.

Step 14: If any of the strong vertices are not adjacent, give them all the same color; if not, give each vertex a unique color that corresponds to their $\alpha_{str}^*(\mathcal{V})$ in the minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}$.

Step 15: Assign color values $\mathcal{A} + 1$ to \mathcal{A} to all $\beta_{str}^*(\mathcal{V})$. Assign the same value from $[\mathcal{A} + 1, m]$ to each $\gamma_{str}^*(\mathcal{V})$ vertex. If two γ_{str}^* vertices are next to each other, remove the edge connecting them. If not, give all of the vertices that are part of $\gamma_{str}^*(\mathcal{V})$ the same colors.

The built fuzzy network is now encrypted so that an intruder cannot decipher the message unless they understand the ideas of fuzzy vertex order coloring and efficient dominance. To decode the message, the decoder must utilize the secret key $(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*, \mathcal{W}_{\alpha_{str}^*}, \tilde{\chi}_o)$ on the encrypted network $(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}})$.

End

Significance of the encryption framework

The main aim of this encryption framework is to secure numerical integer through multiple layers of encryption. The initial step of encryption is to create the subnetworks. As per user's choice, the number of subnetworks can be increased or decreased based on the level of complexity required. The numerical integer is subdivided into smaller components based on modular arithmetic concepts, and each component is converted to a fuzzy value by sublevels of encryption, and these values are assigned as membership values in a subnetwork. Each subnetwork consists of an efficient dominating node. In this multi layered encryption framework, efficient dominating node plays a crucial role serving as the backbone of the encryption strategy. It is a core structural component that enhances both security and computational efficiency. The distance from the dominating node to its adjacent vertex is always 1, encrypted data stays organized and is only accessible through pre specified connections, and unauthorized decryption becomes very hard. It also prevents direct link formation between center nodes, strengthening encryption and it ensures optimized decryption by providing a structured key retrieval mechanism. The next level of encryption is vertex categorization namely α -strong, β -strong and γ -strong using intuitionistic vertex order coloring, which acts as the additional layer of security. Vertex order coloring is a graph coloring technique in which by vertices are colored based on their adjacency and degree to assign them priorities to be colored according to vertices of higher degree. Their strategic placement helps in preventing malicious interference. In decryption, identifying the efficient dominating nodes, extraction of chromatic numbers and determining the weight of alpha-strong vertices are used as keys for recovering the original encrypted structure. This controlled method ensures that only authorized decryption is possible and not brute-force attacks.

Framework for decryption

Intuitionistic Color-coded encrypted network $(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}})$ is the input.

Output: $\mathcal{S}_\mu = \mathcal{V}\mathcal{V}\mathcal{V}\mathcal{V}$

Key: The weight of the α strong vertex $(\mathcal{W}_{\alpha_{str}^*})$, efficient dominating set and the chromatic number.

Start

Step 1: Observe the Intuitionistic color—coded encrypted network and figure out the efficient dominating nodes using the efficient domination number as its key.

Step 2: Calculate the degree of each vertex $\Delta_{deg}(v_i) = \sum \Delta_{deg}(\mathcal{A}(v_i))$. Similarly, the degree of neighboring adjacent vertices of $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \zeta_{d_4}, \zeta_{d_5}, \zeta_{d_6}, \dots, \zeta_{d_r}$ are calculated.

Step 3: Based on the degree of the vertices in Table 4, they are categorized as α strong, β strong, and γ strong vertices. For every vertex v_i , if $\Delta_{deg}(v_i) \geq \Delta_{deg}(\mathcal{A}(v_i))$ then those v_i 's are termed as α strong vertices.

Step 4: Identify the network effective dominating nodes $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \dots, \zeta_{d_r}$ such that $\mathcal{N}_e[\zeta_{d_1}] \cap \mathcal{N}_e[\zeta_{d_2}] \cap \dots \cap \mathcal{N}_e[\zeta_{d_r}] = \emptyset$ where $\mathcal{N}_e[\zeta_{d_i}]$ represents the neighbors of the vertex ζ_{d_i} .

Step 5: $\mathbb{K}_{y-r} = \mathbb{V}_n(\sum_{j=1}^r \zeta_{d_j})$

Step 6: Consider the vertex properties and label them into three types namely α -strong, β -strong, and γ -strong, following the Definitions 2.10, 2.11, 2.12.

Step 7: Observe that the networks are connected with certain constraints, and figure out the minimally connected tree by deleting edges which are simply connected to look like a complex network.

Step 8: From the minimally connected tree, decrypt the subnetworks separately.

Step 9: The decrypted seven subnetworks are now observed with fuzzy membership values.

Step 10: Break the complex network into various subnetworks $\mathbb{K}_{y-0}, \mathbb{K}_{y-1}, \mathbb{K}_{y-2}, \dots, \mathbb{K}_{y-r}$.

Step 11: From each subnetwork the integer value has to be retrieved as follows.

End

Example

Construction of intuitionistic graph based network

The network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}^*}$, construction from the sub networks $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*, \dots, \mathcal{M}_{\mathcal{I}\mathcal{N}_7}^*$.

Step 1: Using suitable intuitionistic network $\mathbb{I}_{\mathcal{F}\mathcal{G}}$, construct 25,193 (mod 7). Divide 25,193 (mod 7) into the following 7 subgroups: $\mathbb{K}_{y-0}, \mathbb{K}_{y-1}, \mathbb{K}_{y-2}, \mathbb{K}_{y-3}, \mathbb{K}_{y-4}, \mathbb{K}_{y-5}, \mathbb{K}_{y-6}$, so that $\mathbb{K}_{y-0} = 7020 \equiv 6(\text{mod } 7)$, $\mathbb{K}_{y-1} = 5520 \equiv 4(\text{mod } 7)$, $\mathbb{K}_{y-2} = 2860 \equiv 4(\text{mod } 7)$, $\mathbb{K}_{y-3} = 3120 \equiv 5(\text{mod } 7)$, $\mathbb{K}_{y-4} = 3128 \equiv 6(\text{mod } 7)$, $\mathbb{K}_{y-5} = 1755 \equiv 5(\text{mod } 7)$, $\mathbb{K}_{y-6} = 1780 \equiv 2(\text{mod } 7)$.

Step 2: Assign the effective dominance nodes in the constructed network from the 7 sub networks. The nodes $\zeta_{d_1}, \zeta_{d_2}, \zeta_{d_3}, \zeta_{d_4}, \zeta_{d_5}, \zeta_{d_6}, \zeta_{d_7}$ are the efficient dominating ones. These nodes serves as the hubs of the sub networks $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_4}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_5}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_6}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_7}^*$. The neighbouring vertices of ζ_{d_1} , $\zeta_{d_2}, \zeta_{d_3}, \zeta_{d_4}, \zeta_{d_5}, \zeta_{d_6}, \zeta_{d_7}$ are $\zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \dots, \zeta_{d_{15}}, \zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \dots, \zeta_{d_{26}}, \zeta_{d_{31}}, \zeta_{d_{32}}, \zeta_{d_{33}}, \dots, \zeta_{d_{35}}, \zeta_{d_{41}}, \zeta_{d_{42}}, \zeta_{d_{43}}, \zeta_{d_{51}}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}, \zeta_{d_{61}}, \zeta_{d_{62}}, \zeta_{d_{63}}, \zeta_{d_{71}}, \zeta_{d_{72}}$ respectively.

Framework for encryption

Step 1: Create the initial $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*$ sub network (Fig. 2), whose ζ_{d_1} is the centre, and $\zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \dots, \zeta_{d_{15}}$ are its neighbours. 7020 $\equiv 6(\text{mod } 7)$ is the initial subdivision value \mathbb{K}_{y-0} . Assuming that $\mathcal{S}_{\mathcal{N}_1}' = \frac{\mathbb{K}_{y-0}}{\mathbb{V}_n} = \frac{7020}{7}$; $\pi_{11} = \mathcal{S}_{\mathcal{N}_1}' / \mathcal{O}_1' = 0.10028$; (where \mathcal{O}_1' denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{S}_{\mathcal{N}_1}'$). This has the minimum values of either ζ_{d_1} or $\zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \zeta_{d_{14}}, \zeta_{d_{15}}, \zeta_{d_6}, \zeta_{d_7}$ respectively, and is divided into the total of seven values, say $\zeta_{d_1}, \zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \zeta_{d_{14}}, \zeta_{d_{15}}, \zeta_{d_6}, \zeta_{d_7}$. By definition 2.4, the edges degree of certainty values are $\min\{\zeta_{d_1}, \zeta_{d_{11}}\}; \min\{\zeta_{d_1}, \zeta_{d_{12}}\}; \dots; \min\{\zeta_{d_1}, \zeta_{d_{15}}\}$.

The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.81$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 2: Form $\mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*$, the subsequent sub network, whose ζ_{d_2} is the centre, and $\zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \dots, \zeta_{d_{26}}$ are its neighbours. \mathbb{K}_{y-1} is given by 5520 $\equiv 4(\text{mod } 7)$ is the second subdivision value $\mathcal{S}_{\mathcal{N}_2}' = \frac{\mathbb{K}_{y-1}}{\mathbb{V}_n} = \frac{5520}{7}$; $\pi_{12} = \mathcal{S}_{\mathcal{N}_2}' / \mathcal{O}_2' = 0.788$; (where \mathcal{O}_2' denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{S}_{\mathcal{N}_2}'$). This has the minimum values of either ζ_{d_2} or $\zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \dots, \zeta_{d_{26}}$ respectively, and is divided into a total of six values, say $\zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \dots, \zeta_{d_{26}}$. By definition, the edges degree of certainty values are $\min\{\zeta_{d_2}, \zeta_{d_{21}}\}; \min\{\zeta_{d_2}, \zeta_{d_{22}}\}; \dots; \min\{\zeta_{d_2}, \zeta_{d_{26}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.044$. Similarly the false certainty values are now splitted into parts as per the networks edge counts.

Construct the third sub network, $\mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*$, with ζ_{d_3} as its centre and the following neighbours:

$\zeta_{d_{31}}, \zeta_{d_{32}}, \zeta_{d_{33}}, \dots, \zeta_{d_{35}}$. \mathbb{K}_{y-2} is given by 2860 $\equiv 4(\text{mod } 7)$ is the third subdivision value $\mathcal{S}_{\mathcal{N}_3}' = \frac{\mathbb{K}_{y-2}}{\mathbb{V}_n} = \frac{2860}{7}$; $\pi_{13} = \mathcal{S}_{\mathcal{N}_3}' / \mathcal{O}_3' = 0.408$; (where \mathcal{O}_3' denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of $\mathcal{S}_{\mathcal{N}_3}'$). This has the minimum values of either ζ_{d_3} or $\zeta_{d_{31}}, \zeta_{d_{32}}, \zeta_{d_{33}}, \dots, \zeta_{d_{35}}$.

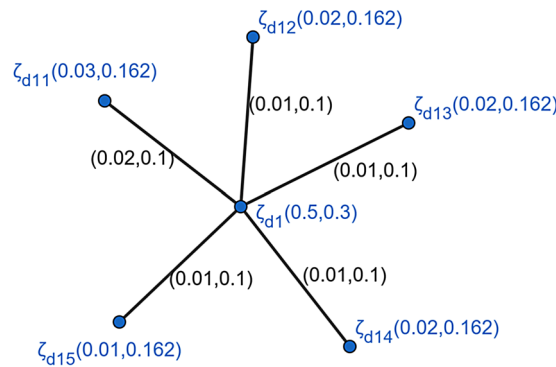


Fig. 2. Initial sub network representation with Intuitionistic fuzzy assignments ($\mathcal{M}_{\mathcal{I}N_1}^*$).

respectively, and is divided into the total of five values, say $\zeta_{d_{31}, \zeta_{d_{32}}, \zeta_{d_{33}}, \dots, \zeta_{d_{35}}}$. By definition, the edges degree of certainty values are $\min\{\zeta_{d_3}, \zeta_{d_{31}}\}; \min\{\zeta_{d_3}, \zeta_{d_{32}}\}; \dots; \min\{\zeta_{d_3}, \zeta_{d_{35}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}N_3}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.350$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 3: Construct the fourth sub network, $\mathcal{M}_{\mathcal{I}N_4}^*$, with ζ_{d_4} as its centre and the following neighbours: $\zeta_{d_{41}, \zeta_{d_{42}}, \zeta_{d_{43}}}$. \mathbb{K}_{y-3} is given by $3120 \equiv 5 \pmod{7}$ is the fourth subdivision value $\mathcal{S}'_{N_4} = \frac{\mathbb{K}_{y-3}}{\mathbb{V}_n} = \frac{3120}{7}$; $\mathcal{M}_{14} = \mathcal{S}'_{N_4} / \mathcal{O}'_4 = 0.445$; (where \mathcal{O}'_4 denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of \mathcal{S}'_4). This has the minimum values of either ζ_{d_4} or $\zeta_{d_{41}, \zeta_{d_{42}}, \zeta_{d_{43}}}$ respectively, and is divided into the total of three values, say $\zeta_{d_{41}, \zeta_{d_{42}}, \zeta_{d_{43}}}$. By definition, the edges degree of certainty values are $\min\{\zeta_{d_4}, \zeta_{d_{41}}\}; \min\{\zeta_{d_4}, \zeta_{d_{42}}\}; \min\{\zeta_{d_4}, \zeta_{d_{43}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}N_4}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.445$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 4: Construct the fifth sub network, $\mathcal{M}_{\mathcal{I}N_5}^*$, with ζ_{d_5} as its centre and the following neighbours: $\zeta_{d_{51}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}}$. \mathbb{K}_{y-4} is given by $3128 \equiv 6 \pmod{7}$ is the fifth subdivision value $\mathcal{S}'_{N_5} = \frac{\mathbb{K}_{y-4}}{\mathbb{V}_n} = \frac{3128}{7}$; $\mathcal{M}_{15} = \mathcal{S}'_{N_5} / \mathcal{O}'_5 = 1000/446.85 = 0.446$; (where \mathcal{O}'_5 denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of \mathcal{S}'_5). This has the minimum values of either ζ_{d_5} or $\zeta_{d_{51}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}}$ respectively, and is divided into the total of four values, say $\zeta_{d_{51}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}}$. By definition, the edges degree of certainty values are $\min\{\zeta_{d_5}, \zeta_{d_{51}}\}; \min\{\zeta_{d_5}, \zeta_{d_{52}}\}; \min\{\zeta_{d_5}, \zeta_{d_{53}}\}; \min\{\zeta_{d_5}, \zeta_{d_{54}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}N_5}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.3069$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 5: Construct the sixth sub network, $\mathcal{M}_{\mathcal{I}N_6}^*$, with ζ_{d_6} as its centre and the following neighbours: $\zeta_{d_{61}, \zeta_{d_{62}}, \zeta_{d_{63}}}$. \mathbb{K}_{y-5} is given by $1755 \equiv 5 \pmod{7}$ is the sixth subdivision value $\mathcal{S}'_{N_6} = \frac{\mathbb{K}_{y-5}}{\mathbb{V}_n} = \frac{1755}{7}$; $\mathcal{M}_{16} = \mathcal{S}'_{N_6} / \mathcal{O}'_6 = 0.250$; (where \mathcal{O}'_6 denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of \mathcal{S}'_6). This has the minimum values of either ζ_{d_6} or $\zeta_{d_{61}, \zeta_{d_{62}}, \zeta_{d_{63}}}$ respectively, and is divided into the total of three values, say $\zeta_{d_{61}, \zeta_{d_{62}}, \zeta_{d_{63}}}$. By Definition 2.4, the edges degree of certainty values are $\min\{\zeta_{d_6}, \zeta_{d_{61}}\}; \min\{\zeta_{d_6}, \zeta_{d_{62}}\}; \min\{\zeta_{d_6}, \zeta_{d_{63}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}N_6}^*$ is taken as follows. $\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.5625$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 6: Construct the seventh sub network, $\mathcal{M}_{\mathcal{I}N_7}^*$, with ζ_{d_7} as its centre and the following neighbours: $\zeta_{d_{71}, \zeta_{d_{72}}}$. \mathbb{K}_{y-6} is given by $1780 \equiv 2 \pmod{7}$ is the seventh subdivision value $\mathcal{S}'_{N_7} = \frac{\mathbb{K}_{y-6}}{\mathbb{V}_n} = \frac{1780}{7}$; $\mathcal{M}_{17} = \mathcal{S}'_{N_7} / \mathcal{O}'_7 = 0.254$; (where \mathcal{O}'_7 denotes the value 1 followed by a sequence of zeros equal to the count of zero digits in the integral part of \mathcal{S}'_7). This has the minimum values of either ζ_{d_7} or $\zeta_{d_{71}, \zeta_{d_{72}}}$ respectively, and is divided into the total of two values, say $\zeta_{d_{71}, \zeta_{d_{72}}}$. By definition, the edges degree of certainty values are $\min\{\zeta_{d_7}, \zeta_{d_{71}}\}; \min\{\zeta_{d_7}, \zeta_{d_{72}}\}$. The falsity membership value ($\mathcal{V}_{\sigma\mathcal{F}}(j)$) of the sub network $\mathcal{M}_{\mathcal{I}N_7}^*$ is taken as follows.

$\mathcal{V}_{\sigma\mathcal{F}}(j) = 0.5565$. Similarly, the false certainty values are now splitted into parts as per the networks edge counts.

Step 7: The sub networks $\mathcal{M}_{\mathcal{I}\mathcal{N}_1}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*, \dots, \mathcal{M}_{\mathcal{I}\mathcal{N}_7}^*$ are minimally connected and formed into a network $(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*)$ in the successive steps with certain conditions.

$\mathcal{E}_{\mu_{del}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*) = \{\zeta_{d1}\zeta_{d21}, 1 \leq j \leq \tau_2; \zeta_{d1}\zeta_{d31}, 1 \leq j \leq \tau_3; \zeta_{d1}\zeta_{d41}, 1 \leq j \leq \tau_4; \zeta_{d1}\zeta_{d51}, 1 \leq j \leq \tau_5; \zeta_{d1}\zeta_{d61}, 1 \leq j \leq \tau_6; \zeta_{d1}\zeta_{d71}, 1 \leq j \leq \tau_7\}$.

Step 10: Create the intuitionistic network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$ (Fig. 3). It includes the edges, with the exception of $\mathcal{E}_{\mu_{del}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*)$.

$\mathcal{E}_{\mu_{min}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*) = \{\zeta_{d1}\zeta_{d11}, = 1 \text{ to } 5; \zeta_{d2}\zeta_{d21}, = 1 \text{ to } 6; \zeta_{d3}\zeta_{d31}, = 1 \text{ to } 5; \zeta_{d4}\zeta_{d41}, = 1-3; \zeta_{d5}\zeta_{d51}, = 1 \text{ to } 4; \zeta_{d6}\zeta_{d61}, = 1 \text{ to } 3; \zeta_{d7}\zeta_{d71}, = 1 \text{ to } 2\} \cup \{\zeta_{d11}\zeta_{d21}, = 1 \text{ to } 5, \mathfrak{k} = 1 \text{ to } 6; \zeta_{d21}\zeta_{d31}, = 1 \text{ to } 6, \mathfrak{k} = 1 \text{ to } 5; \zeta_{d31}\zeta_{d41}, = 1 \text{ to } 5; \zeta_{d41}\zeta_{d51}, = 1 \text{ to } 3, \mathfrak{k} = 1 \text{ to } 3; \zeta_{d51}\zeta_{d61}, = 1 \text{ to } 4, \mathfrak{k} = 1 \text{ to } 3; \zeta_{d61}\zeta_{d71}, = 1 \text{ to } 3, \mathfrak{k} = 1 \text{ to } 2\}$ for only one and \mathfrak{k} .

The number of edges present in the network is 34.

The constructed network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$ requires the minimum number of edges. Likewise, the networks $\mathcal{M}_{\mathcal{I}\mathcal{N}_2}^*, \mathcal{M}_{\mathcal{I}\mathcal{N}_3}^*, \dots, \mathcal{M}_{\mathcal{I}\mathcal{N}_7}^*$ are linked with at least one edge to form a minimal spanning tree.

For the minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$, the vertex and edge certainty values are computed and shown in Tables 1 and 2, respectively.

Building upon the foundation established in the first level of encryption, the second level employs vertex order coloring to enhance the security and complexity of the encrypted network. This step involves calculating the degree of each vertex, which serves as the basis for assigning unique colors to vertices in a way that conceals adjacency relationships and reinforces the encryption scheme.

In the context of intuitionistic fuzzy graphs, each vertex and edge is characterized by both truth and falsity certainty values, reflecting degrees of certainty and uncertainty. To streamline these dual values into a single fuzzy representation, a score function is employed. The function, defined in definition 2.5 and 2.6 are applied to calculate the cardinality for vertices and edges. Performing this step prior to encryption through vertex order coloring is crucial, as it transforms the graph into a more accurate and consistent representation, thereby improving the precision and reliability of the encryption process. Now the new network is shown in Fig. 4.

The minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$, consists of 35 vertices and 34 edges. For the intuitionistic network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$ with vertices, The m -coloring of the vertices is found below. Let $\{\mathfrak{a}_1, \mathfrak{a}_2, \dots, \mathfrak{a}_m\}$ be the vertices of $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$: $(\mathbb{V}, \mathcal{V}_{\sigma}, \mathcal{E}_{\mu})$, and let $\{1, 2, \dots, m\}$ be the colors of the \mathcal{Z}^* vertices. The following condition is true since every edge in the network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$ is effective.

$\mathcal{E}_{\mu}(\zeta_i \zeta_j) = \frac{1}{2} [\mathcal{V}_{\sigma}(\zeta_i) \wedge \mathcal{V}_{\sigma}(\zeta_j)] \leq \mathcal{E}_{\mu}(\zeta_i \zeta_j)$. The degree of each vertex of the minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$ is calculated using definition 2.8 and tabulated in Table 3.

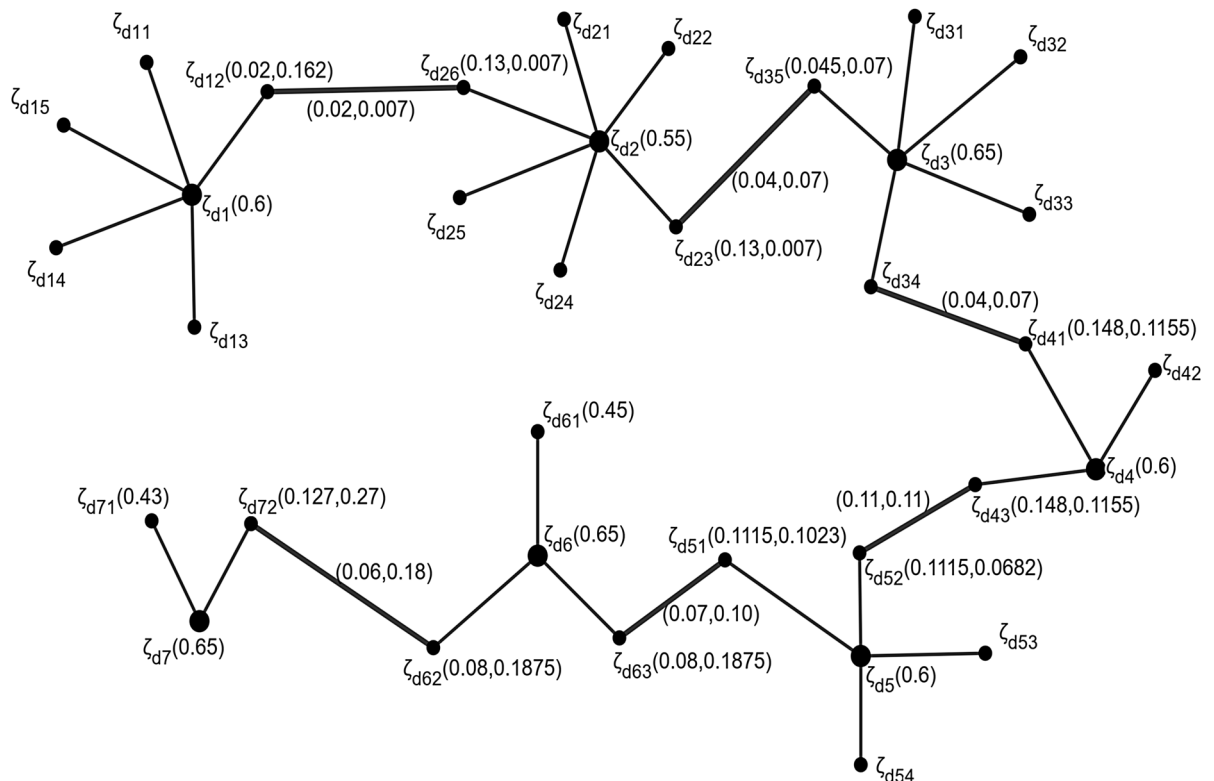


Fig. 3. Integration of sub networks into a spanning tree structure.

Vertices	Certainty values	Vertices	Certainty values	Vertices	Certainty values
ζ_{d_1}	(0.5,0.3)	$\zeta_{d_{21}}$	(0.13,0.007)	$\zeta_{d_{42}}$	(0.148,0.077)
ζ_{d_2}	(0.8,0.7)	$\zeta_{d_{22}}$	(0.13,0.007)	$\zeta_{d_{43}}$	(0.148,0.1155)
ζ_{d_3}	(0.5,0.2)	$\zeta_{d_{23}}$	(0.13,0.007)	$\zeta_{d_{51}}$	(0.1115,0.1023)
ζ_{d_4}	(0.7,0.5)	$\zeta_{d_{24}}$	(0.13,0.007)	$\zeta_{d_{52}}$	(0.1115,0.0682)
ζ_{d_5}	(0.5,0.3)	$\zeta_{d_{25}}$	(0.13,0.007)	$\zeta_{d_{53}}$	(0.1115,0.0682)
ζ_{d_6}	(0.7,0.4)	$\zeta_{d_{26}}$	(0.13,0.007)	$\zeta_{d_{54}}$	(0.1115,0.0682)
ζ_{d_7}	(0.8,0.5)	$\zeta_{d_{31}}$	(0.136,0.05)	$\zeta_{d_{61}}$	(0.08,0.1875)
$\zeta_{d_{11}}$	(0.03,0.162)	$\zeta_{d_{32}}$	(0.136,0.09)	$\zeta_{d_{62}}$	(0.08,0.1875)
$\zeta_{d_{12}}$	(0.02,0.162)	$\zeta_{d_{33}}$	(0.045,0.07)	$\zeta_{d_{63}}$	(0.08,0.1875)
$\zeta_{d_{13}}$	(0.02,0.162)	$\zeta_{d_{34}}$	(0.045,0.07)	$\zeta_{d_{71}}$	(0.127,0.27)
$\zeta_{d_{14}}$	(0.02,0.162)	$\zeta_{d_{35}}$	(0.03,0.05)	$\zeta_{d_{72}}$	(0.127,0.27)
$\zeta_{d_{15}}$	(0.01,0.162)	$\zeta_{d_{41}}$	(0.148,0.1155)		

Table 1. Vertex certainty values for minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$

Edges	Certainty values	Edges	Certainty values	Edges	Certainty values
$\mathcal{E}_{\mu}(\zeta_{d_1} \zeta_{d_{11}})$	(0.02,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_3} \zeta_{d_{32}})$	(0.12,0.07)	$\mathcal{E}_{\mu}(\zeta_{d_6} \zeta_{d_{62}})$	(0.06,0.1)
$\mathcal{E}_{\mu}(\zeta_{d_1} \zeta_{d_{12}})$	(0.01,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_3} \zeta_{d_{33}})$	(0.03,0.05)	$\mathcal{E}_{\mu}(\zeta_{d_6} \zeta_{d_{63}})$	(0.06,0.1)
$\mathcal{E}_{\mu}(\zeta_{d_1} \zeta_{d_{13}})$	(0.01,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_3} \zeta_{d_{34}})$	(0.03,0.05)	$\mathcal{E}_{\mu}(\zeta_{d_7} \zeta_{d_{71}})$	(0.1,0.2)
$\mathcal{E}_{\mu}(\zeta_{d_1} \zeta_{d_{14}})$	(0.01,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_3} \zeta_{d_{35}})$	(0.03,0.05)	$\mathcal{E}_{\mu}(\zeta_{d_7} \zeta_{d_{72}})$	(0.1,0.2)
$\mathcal{E}_{\mu}(\zeta_{d_1} \zeta_{d_{15}})$	(0.01,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_4} \zeta_{d_{41}})$	(0.1,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_{12}} \zeta_{d_{26}})$	(0.02,0.007)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{21}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_4} \zeta_{d_{42}})$	(0.1,0.02)	$\mathcal{E}_{\mu}(\zeta_{d_{23}} \zeta_{d_{31}})$	(0.04,0.07)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{22}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_4} \zeta_{d_{43}})$	(0.1,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_{34}} \zeta_{d_{41}})$	(0.04,0.07)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{23}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_5} \zeta_{d_{51}})$	(0.1,0.1)	$\mathcal{E}_{\mu}(\zeta_{d_{43}} \zeta_{d_{52}})$	(0.11,0.11)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{24}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_5} \zeta_{d_{52}})$	(0.11,0.03)	$\mathcal{E}_{\mu}(\zeta_{d_{51}} \zeta_{d_{63}})$	(0.07,0.10)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{25}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_5} \zeta_{d_{53}})$	(0.1,0.04)	$\mathcal{E}_{\mu}(\zeta_{d_{62}} \zeta_{d_{72}})$	(0.06,0.18)
$\mathcal{E}_{\mu}(\zeta_{d_2} \zeta_{d_{26}})$	(0.13,0.007)	$\mathcal{E}_{\mu}(\zeta_{d_5} \zeta_{d_{54}})$	(0.1,0.02)		
$\mathcal{E}_{\mu}(\zeta_{d_3} \zeta_{d_{31}})$	(0.1,0.01)	$\mathcal{E}_{\mu}(\zeta_{d_6} \zeta_{d_{61}})$	(0.04,0.1)		

Table 2. Edge certainty values for minimally connected network $\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*$

Based on the above table, the network has been colored using vertex order coloring technique as shown in Fig. 5

From the above network it is observed that the chromatic number to color the network is 3. The weight of α , β and γ strong vertices are 13.79, 6.067 and 16.7185 respectively. A critical element in this encryption process is network complexity, which greatly improves security by boosting complexity and attack resilience. The addition of edges under particular constraints makes a network appear more complex and unpredictable. Create the intuitionistic network, it can include any number of the edges, with the exception of $\mathcal{E}_{\mu_{del}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*)$.

$\mathcal{E}_{\mu_{max}}(\mathcal{O}_{\mathcal{M}_{\mathcal{I}\mathcal{N}}}^*) = \{\zeta_{d_{11}} \zeta_{d_{2e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{11}} \zeta_{d_{3e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{11}} \zeta_{d_{24}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{11}} \zeta_{d_{5e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{11}} \zeta_{d_{6e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{11}} \zeta_{d_{7e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{21}} \zeta_{d_{1e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{21}} \zeta_{d_{3e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{21}} \zeta_{d_{4e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{21}} \zeta_{d_{5e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{21}} \zeta_{d_{6e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{21}} \zeta_{d_{7e}}, = 1 \text{ to } 6, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{31}} \zeta_{d_{1e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{31}} \zeta_{d_{2e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{31}} \zeta_{d_{4e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{31}} \zeta_{d_{5e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{31}} \zeta_{d_{6e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{31}} \zeta_{d_{7e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{41}} \zeta_{d_{1e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{41}} \zeta_{d_{2e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{41}} \zeta_{d_{3e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{41}} \zeta_{d_{5e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{41}} \zeta_{d_{6e}}, = 1 \text{ to } 5, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{41}} \zeta_{d_{7e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{51}} \zeta_{d_{1e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{51}} \zeta_{d_{2e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{51}} \zeta_{d_{3e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{51}} \zeta_{d_{4e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{51}} \zeta_{d_{6e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{51}} \zeta_{d_{7e}}, = 1 \text{ to } 4, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{61}} \zeta_{d_{1e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{61}} \zeta_{d_{2e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{61}} \zeta_{d_{3e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{61}} \zeta_{d_{4e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{61}} \zeta_{d_{5e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 4; \zeta_{d_{61}} \zeta_{d_{7e}}, = 1 \text{ to } 3, \mathfrak{E} = 1 \text{ to } 2; \zeta_{d_{71}} \zeta_{d_{1e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 5; \zeta_{d_{71}} \zeta_{d_{2e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 6; \zeta_{d_{71}} \zeta_{d_{3e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{71}} \zeta_{d_{4e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 3; \zeta_{d_{71}} \zeta_{d_{5e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 4; \zeta_{d_{71}} \zeta_{d_{6e}}, = 1 \text{ to } 2, \mathfrak{E} = 1 \text{ to } 3;$

Vertex	Adjacent vertices	Degree	$\tilde{\alpha}_{str}$	$\tilde{\beta}_o$	$\tilde{\gamma}_o$
ζ_{d_1}	$\zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \zeta_{d_{14}}, \zeta_{d_{15}}$	2.3	ζ_{d_1}	–	–
ζ_{d_2}	$\zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \zeta_{d_{24}}, \zeta_{d_{25}}, \zeta_{d_{26}}$	4.05	ζ_{d_2}	–	–
ζ_{d_3}	$\zeta_{d_{31}}, \zeta_{d_{32}}, \zeta_{d_{33}}, \zeta_{d_{34}}, \zeta_{d_{35}}$	2.54	ζ_{d_3}	–	–
ζ_{d_4}	$\zeta_{d_{41}}, \zeta_{d_{42}}, \zeta_{d_{43}}$	1.54	ζ_{d_4}	–	–
ζ_{d_5}	$\zeta_{d_{51}}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}$	1.03	ζ_{d_5}	–	–
ζ_{d_6}	$\zeta_{d_{61}}, \zeta_{d_{62}}, \zeta_{d_{63}}$	1.43	ζ_{d_6}	–	–
ζ_{d_7}	$\zeta_{d_{71}}, \zeta_{d_{72}}$	0.9	ζ_{d_7}	–	–
$\zeta_{d_{11}}$	ζ_{d_1}	0.46	–	–	$\zeta_{d_{11}}$
$\zeta_{d_{12}}$	$\zeta_{d_1}, \zeta_{d_{26}}$	0.86	–	–	$\zeta_{d_{12}}$
$\zeta_{d_{13}}$	ζ_{d_1}	0.46	–	–	$\zeta_{d_{13}}$
$\zeta_{d_{14}}$	ζ_{d_1}	0.46	–	–	$\zeta_{d_{14}}$
$\zeta_{d_{15}}$	ζ_{d_1}	0.46	–	–	$\zeta_{d_{15}}$
$\zeta_{d_{21}}$	ζ_{d_2}	0.554	–	–	$\zeta_{d_{21}}$
$\zeta_{d_{22}}$	ζ_{d_2}	0.5495	–	–	$\zeta_{d_{22}}$
$\zeta_{d_{23}}$	$\zeta_{d_2}, \zeta_{d_{31}}$	1.398	–	$\zeta_{d_{23}}$	–
$\zeta_{d_{24}}$	ζ_{d_2}	0.554	–	–	$\zeta_{d_{24}}$
$\zeta_{d_{25}}$	ζ_{d_2}	0.554	–	–	$\zeta_{d_{25}}$
$\zeta_{d_{26}}$	$\zeta_{d_2}, \zeta_{d_{12}}$	1.349	–	$\zeta_{d_{26}}$	–
$\zeta_{d_{31}}$	$\zeta_{d_3}, \zeta_{d_{23}}$	1.045	–	–	$\zeta_{d_{31}}$
$\zeta_{d_{32}}$	ζ_{d_3}	0.525	–	–	$\zeta_{d_{32}}$
$\zeta_{d_{33}}$	ζ_{d_3}	0.49	–	–	$\zeta_{d_{33}}$
$\zeta_{d_{34}}$	$\zeta_{d_3}, \zeta_{d_{41}}$	0.89	–	–	$\zeta_{d_{34}}$
$\zeta_{d_{35}}$	ζ_{d_3}	0.49	–	–	$\zeta_{d_{35}}$
$\zeta_{d_{41}}$	$\zeta_{d_4}, \zeta_{d_{34}}$	0.9	–	$\zeta_{d_{41}}$	–
$\zeta_{d_{42}}$	ζ_{d_4}	0.54	–	–	$\zeta_{d_{42}}$
$\zeta_{d_{43}}$	$\zeta_{d_4}, \zeta_{d_{52}}$	0.8	–	–	$\zeta_{d_{43}}$
$\zeta_{d_{51}}$	$\zeta_{d_5}, \zeta_{d_{63}}$	0.8	–	$\zeta_{d_{51}}$	–
$\zeta_{d_{52}}$	$\zeta_{d_5}, \zeta_{d_{43}}$	0.84	–	$\zeta_{d_{52}}$	–
$\zeta_{d_{53}}$	ζ_{d_5}	0.53	–	–	$\zeta_{d_{53}}$
$\zeta_{d_{54}}$	ζ_{d_5}	0.54	–	–	$\zeta_{d_{54}}$
$\zeta_{d_{61}}$	ζ_{d_6}	0.47	–	–	$\zeta_{d_{61}}$
$\zeta_{d_{62}}$	$\zeta_{d_6}, \zeta_{d_{72}}$	0.78	–	$\zeta_{d_{62}}$	–
$\zeta_{d_{63}}$	$\zeta_{d_6}, \zeta_{d_{51}}$	0.78	–	–	$\zeta_{d_{63}}$
$\zeta_{d_{71}}$	ζ_{d_7}	0.45	–	–	$\zeta_{d_{71}}$
$\zeta_{d_{72}}$	$\zeta_{d_7}, \zeta_{d_{62}}$	0.75	–	–	$\zeta_{d_{72}}$

Table 3. Degree of vertices and List of $\alpha_{str}^*(\mathcal{V}_\sigma)$, $\beta_{str}^*(\mathcal{V}_\sigma)$, $\gamma_{str}^*(\mathcal{V}_\sigma)$ vertices.

The maximum number of possible edges in the network is given by $\mathcal{E}_{\mu_{max}}(\mathcal{O}_{\mathcal{M}_{IN}}^*)$. This complexity conceals patterns or relationships that intruders might exploit, ensuring the network resists analytical techniques like spanning tree analysis because the additional edges hide the original structure and decryption is impossible without the ordered triple (efficient dominating set, weight of alpha-strong vertices, and chromatic number), it also increases the reliance on the encryption key. Figure 6 is the complexified structure, by including some of the edges from $\mathcal{E}_{\mu_{max}}(\mathcal{O}_{\mathcal{M}_{IN}}^*)$, it not only makes the encryption seem natural but also replicates the anomalies of real-world networks.

For the above Fig. 7, it is observed that the minimum no. of colors used to color the vertices of the network is 4. The degrees of the vertices for Fig. 7 are calculated and tabulated in Table 4, and the weights of α_{str}^* , β_{str}^* and γ_{str}^* vertices are observed as 18.7, 18.435 and 8.975 respectively.

The secret key to decrypt the message is the ordered triple $(\mathcal{O}_{\mathcal{M}_{IN}}^\circ, \mathcal{W}_{\alpha_{str}^*}, \tilde{\chi}_o)$.

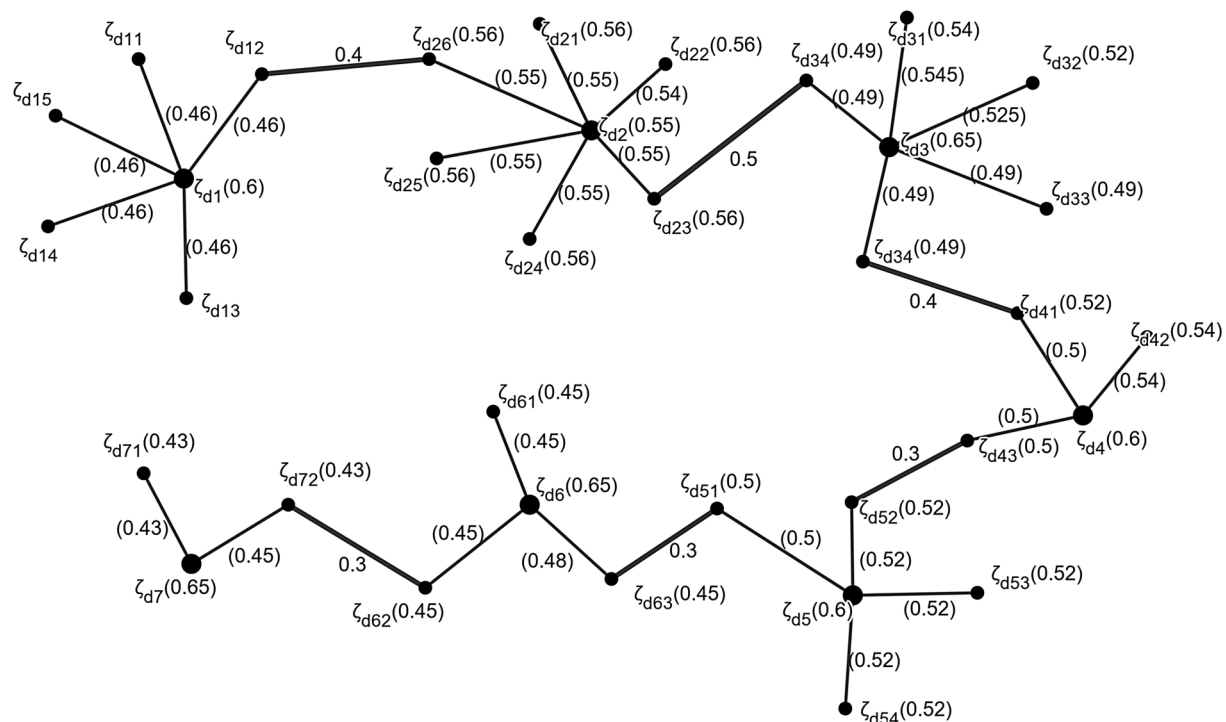


Fig. 4. Cardinality based membership analysis in a spanning tree.

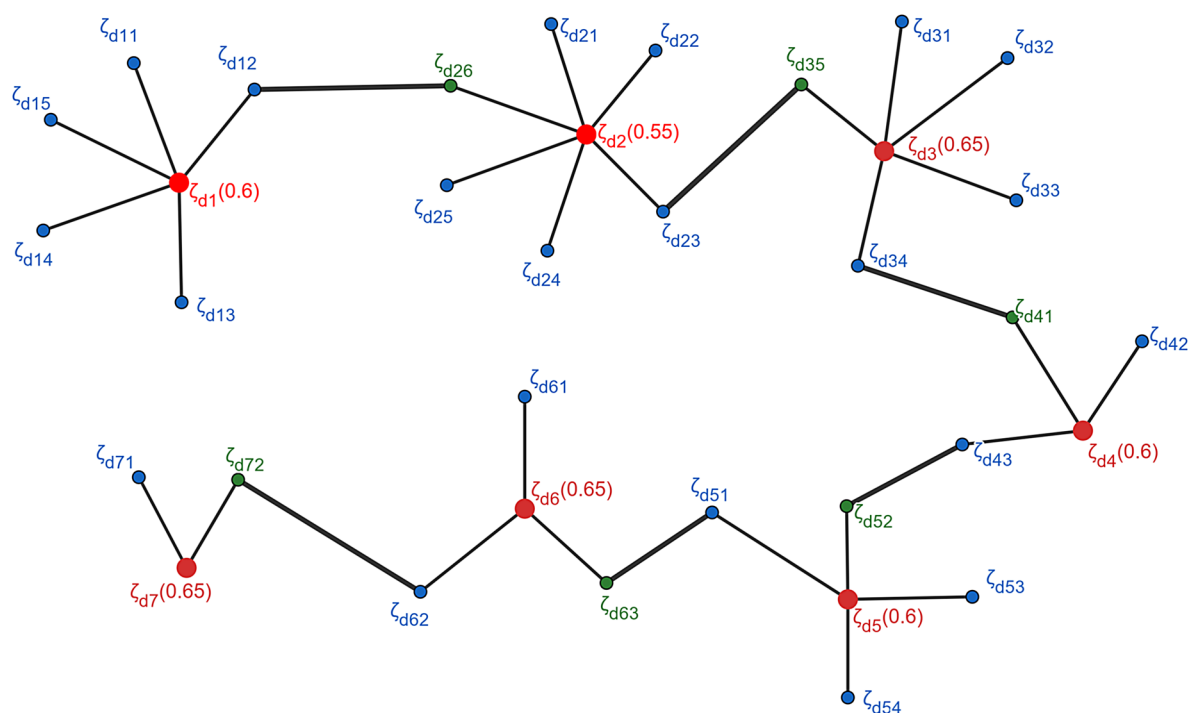


Fig. 5. Spanning tree representation with vertex order coloring.

Framework for decryption

Input: Intuitionistic Color—coded encrypted network ($\mathcal{O}_{\mathcal{M}_{IN}^*}$).

Output: $\mathbb{S}_{\mathcal{V}}^{\diamond} = 25,193$.

Efficient dominating set ($\mathcal{O}_{\mathcal{V}}^{\diamond}$), Weight of the α —strong vertex ($\mathcal{W}_{\alpha_{str}}$) and chromatic number ($\tilde{\chi}_o$) are the secret key.

Start

Step 1: Observe the Intuitionistic color—coded encrypted network and figure out the efficient dominating nodes using the efficient dominating set as its key.

Step 2: Calculate the degree of each vertex $\Delta_{deg}(v_i) = \sum \Delta_{deg}(\mathcal{A}(v_i))$. Similarly, the degree of neighboring adjacent vertices of $\zeta_{d1}, \zeta_{d2}, \zeta_{d3}, \zeta_{d4}, \zeta_{d5}, \zeta_{d6}, \zeta_{d7}$ are calculated.

Step 3: Based on the degree of the vertices in Table 4, they are categorized as α strong, β strong, and γ strong vertices. For every vertex v_i , if $\Delta_{deg}(v_i) \geq \Delta_{deg}(\mathcal{A}(v_i))$ then those v_i 's are termed as α strong vertices.

Step 4: Identify the network effective dominating nodes $\zeta_{d1}, \zeta_{d2}, \zeta_{d3}, \dots, \zeta_{d7}$ such that $\mathcal{N}_d[\zeta_{d1}] \cap \mathcal{N}_d[\zeta_{d2}] \cap \dots \cap \mathcal{N}_d[\zeta_{d7}] = \emptyset$ where $\mathcal{N}_d[\zeta_{d_i}]$ represents the neighbors of the vertex ζ_{d_i} .

Step 5: $\mathbb{K}_{y-r} = \mathbb{V}_n(\sum_{j=1}^7 \zeta_{j_s})$.

Step 6: Consider the vertex properties and label them into three types namely α -strong, β -strong, and γ -strong, following the Definitions 2.10, 2.11, 2.12.

Step 7: Observe that the networks are connected with certain constraints, and figure out the minimally connected tree by deleting edges which are simply connected to look like a complex network.

Step 8: From the minimally connected tree, decrypt the subnetworks separately.

Step 9: The decrypted seven subnetworks are now observed with fuzzy membership values.

Step 10: Break the complex network into various subnetworks $\mathbb{K}_{y-0}, \mathbb{K}_{y-1}, \mathbb{K}_{y-2}, \mathbb{K}_{y-3}, \mathbb{K}_{y-4}, \mathbb{K}_{y-5}, \mathbb{K}_{y-6}, \mathbb{K}_{y-7}$.

Step 11: From each subnetwork the integer value has to be retrieved as follows.

$$\begin{aligned} \mathbb{K}_{(y-0)} &= \mathcal{S}'_{N_1} \left(\sum_{j=1}^5 \zeta_{j_5} \right) \\ &= 0.03 + 0.02 + 0.0256 + 0.02 + 0.01 \\ \text{i.} \quad &= 0.03 + 0.02 + 0.0256 + 0.02 + 0.01 \\ &= 100000 (0.10028) \\ &= 1003.05 \\ &= \mathbb{V}_n(1003.05) = 7 (1002.8) \\ &= 7020 \end{aligned}$$

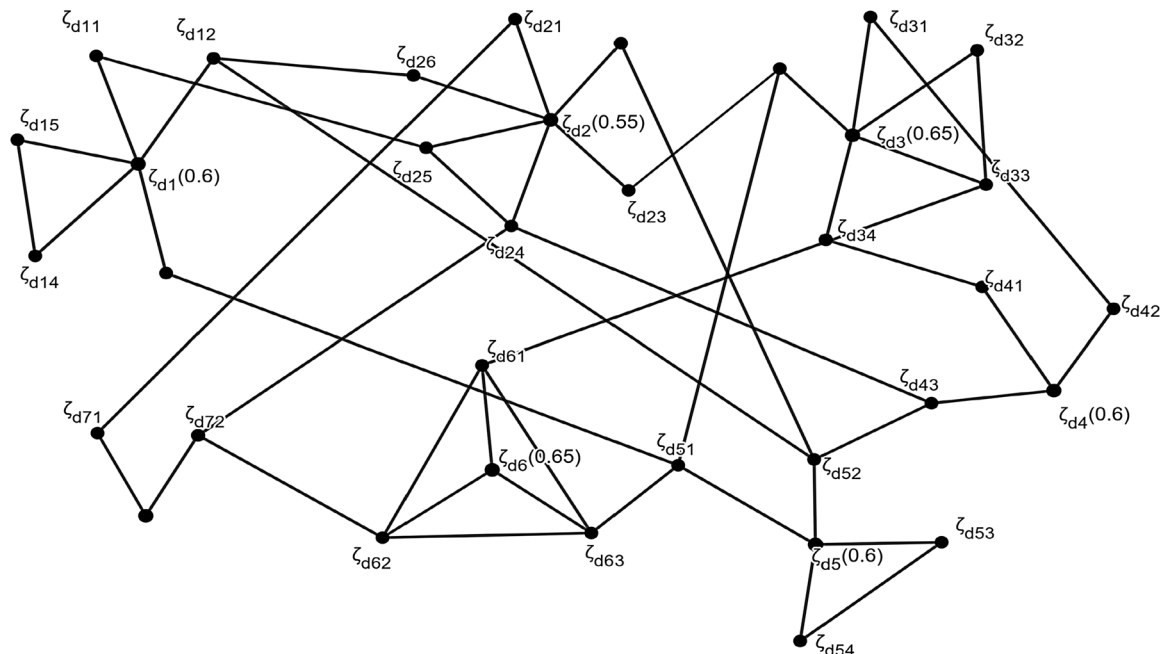


Fig. 6. Advanced network complexity with moderate edges.

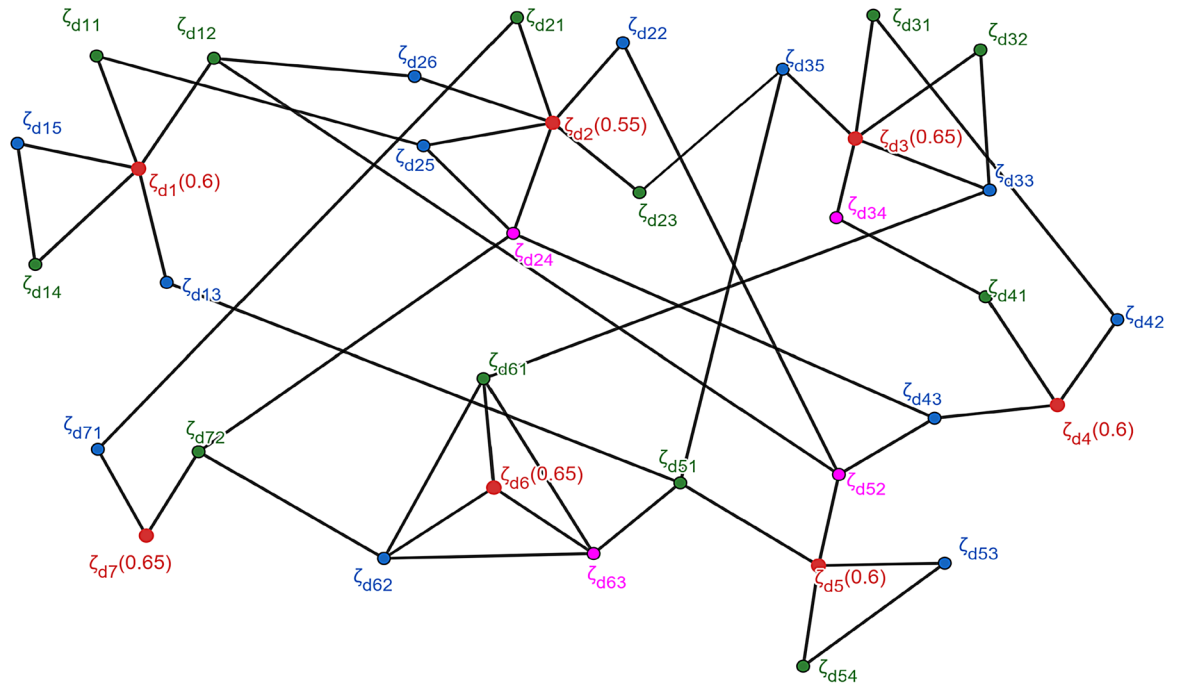


Fig. 7. Vertex classification and coloring in the complex network.

$$\begin{aligned} \text{ii. } \mathbb{K}_{y-1} &= \mathcal{S}'_{\mathcal{N}_2} \left(\sum_{j=1}^6 \zeta_{j_6} \right) \\ &= \mathbb{V}_n(788.82) = 7(788.82) \\ &= 5520 \end{aligned}$$

$$\begin{aligned} \text{iii. } \mathbb{K}_{y-2} &= \mathcal{S}'_{\mathcal{N}_3} \left(\sum_{j=1}^5 \zeta_{j_5} \right) \\ &= \mathbb{V}_n(408.75) = 7(408.75) \\ &= 2860 \end{aligned}$$

$$\begin{aligned} \text{iv. } \mathbb{K}_{y-3} &= \mathcal{S}'_{\mathcal{N}_4} \left(\sum_{j=1}^3 \zeta_{j_3} \right) \\ &= \mathbb{V}_n(445.95) = 7(445.95) \\ &= 3120 \end{aligned}$$

$$\begin{aligned} \text{v. } \mathbb{K}_{y-4} &= \mathcal{S}'_{\mathcal{N}_5} \left(4 \sum_{j=1}^3 \zeta_{j_3} \right) \\ &= \mathbb{V}_n(447.1) = 7(447.1) \\ &= 3128 \end{aligned}$$

$$\begin{aligned} \text{vi. } \mathbb{K}_{y-5} &= \mathcal{S}'_{\mathcal{N}_6} \left(\sum_{j=1}^3 \zeta_{j_3} \right) \\ &= \mathbb{V}_n(250.95) = 7(250.95) \\ &= 1755 \end{aligned}$$

$$\begin{aligned} \text{vii. } \mathbb{K}_{y-6} &= \mathcal{S}'_{\mathcal{N}_7} \left(\sum_{j=1}^2 \zeta_{j_2} \right) \\ &= \mathbb{V}_n(254.45) = 7(254.45) \\ &= 1780 \end{aligned}$$

Vertex	Adjacent vertices	Degree	$\widetilde{\alpha}_v$	$\widetilde{\beta}_v$	$\widetilde{\gamma}_v$
ζ_{d_1}	$\zeta_{d_{11}}, \zeta_{d_{12}}, \zeta_{d_{13}}, \zeta_{d_{14}}, \zeta_{d_{15}}$	2.3	ζ_{d_1}	–	–
ζ_{d_2}	$\zeta_{d_{21}}, \zeta_{d_{22}}, \zeta_{d_{23}}, \zeta_{d_{24}}, \zeta_{d_{25}}, \zeta_{d_{26}}$	4.05	ζ_{d_2}	–	–
ζ_{d_3}	$\zeta_{d_{31}}, \zeta_{d_{32}}, \zeta_{d_{33}}, \zeta_{d_{34}}, \zeta_{d_{35}}$	2.54	ζ_{d_3}	–	–
ζ_{d_4}	$\zeta_{d_{41}}, \zeta_{d_{42}}, \zeta_{d_{43}}$	1.54	ζ_{d_4}	–	–
ζ_{d_5}	$\zeta_{d_{51}}, \zeta_{d_{52}}, \zeta_{d_{53}}, \zeta_{d_{54}}$	1.03	ζ_{d_5}	–	–
ζ_{d_6}	$\zeta_{d_{61}}, \zeta_{d_{62}}, \zeta_{d_{63}}$	1.43	ζ_{d_6}	–	–
ζ_{d_7}	$\zeta_{d_{71}}, \zeta_{d_{72}}$	0.9	ζ_{d_7}	–	–
$\zeta_{d_{11}}$	ζ_{d_1}	0.96	–	–	$\zeta_{d_{11}}$
$\zeta_{d_{12}}$	$\zeta_{d_1}, \zeta_{d_{26}}$	1.36	–	$\zeta_{d_{12}}$	–
$\zeta_{d_{13}}$	ζ_{d_1}	0.86	–	–	$\zeta_{d_{13}}$
$\zeta_{d_{14}}$	ζ_{d_1}	0.76	–	$\zeta_{d_{14}}$	–
$\zeta_{d_{15}}$	ζ_{d_1}	0.76	–	$\zeta_{d_{15}}$	–
$\zeta_{d_{21}}$	ζ_{d_2}	0.95	–	$\zeta_{d_{21}}$	–
$\zeta_{d_{22}}$	ζ_{d_2}	1.04	–	–	$\zeta_{d_{22}}$
$\zeta_{d_{23}}$	$\zeta_{d_2}, \zeta_{d_{31}}$	1.39	–	$\zeta_{d_{23}}$	–
$\zeta_{d_{24}}$	ζ_{d_2}	1.75	–	$\zeta_{d_{24}}$	–
$\zeta_{d_{25}}$	ζ_{d_2}	1.55	–	$\zeta_{d_{25}}$	–
$\zeta_{d_{26}}$	$\zeta_{d_2}, \zeta_{d_{12}}$	1.34	–	–	$\zeta_{d_{26}}$
$\zeta_{d_{31}}$	$\zeta_{d_3}, \zeta_{d_{23}}$	1.045	–	$\zeta_{d_{31}}$	–
$\zeta_{d_{32}}$	ζ_{d_3}	0.925	–	–	$\zeta_{d_{32}}$
$\zeta_{d_{33}}$	ζ_{d_3}	1.29	–	$\zeta_{d_{33}}$	–
$\zeta_{d_{34}}$	$\zeta_{d_3}, \zeta_{d_{41}}$	0.89	–	–	$\zeta_{d_{34}}$
$\zeta_{d_{35}}$	ζ_{d_3}	1.39	–	$\zeta_{d_{35}}$	–
$\zeta_{d_{41}}$	$\zeta_{d_4}, \zeta_{d_{34}}$	0.9	–	$\zeta_{d_{41}}$	–
$\zeta_{d_{42}}$	ζ_{d_4}	1.04	–	$\zeta_{d_{42}}$	–
$\zeta_{d_{43}}$	$\zeta_{d_4}, \zeta_{d_{52}}$	1.2	–	–	$\zeta_{d_{43}}$
$\zeta_{d_{51}}$	$\zeta_{d_5}, \zeta_{d_{63}}$	1.6	$\zeta_{d_{51}}$	–	–
$\zeta_{d_{52}}$	$\zeta_{d_5}, \zeta_{d_{43}}$	1.74	$\zeta_{d_{52}}$	–	–
$\zeta_{d_{53}}$	ζ_{d_5}	0.93	–	–	$\zeta_{d_{53}}$
$\zeta_{d_{54}}$	ζ_{d_5}	0.94	–	$\zeta_{d_{54}}$	–
$\zeta_{d_{61}}$	ζ_{d_6}	1.57	$\zeta_{d_{61}}$	–	–
$\zeta_{d_{62}}$	$\zeta_{d_6}, \zeta_{d_{72}}$	1.08	–	$\zeta_{d_{62}}$	–
$\zeta_{d_{63}}$	$\zeta_{d_6}, \zeta_{d_{51}}$	1.18	–	$\zeta_{d_{63}}$	–
$\zeta_{d_{71}}$	ζ_{d_7}	0.85	–	–	$\zeta_{d_{71}}$
$\zeta_{d_{72}}$	$\zeta_{d_7}, \zeta_{d_{62}}$	1.05	–	$\zeta_{d_{72}}$	–

Table 4. Degree of vertices and List of $\alpha_{\text{str}}^*(\mathcal{V}_\sigma), \beta_{\text{str}}^*(\mathcal{V}_\sigma), \gamma_{\text{str}}^*(\mathcal{V}_\sigma)$ vertices of the moderate edges.

$$\begin{aligned}
 \text{viii. } \mathbb{K}_{y-r} &= \mathbb{V}_n \left(\sum_{j=1}^7 \zeta_{j_v} \right) \\
 &= 7 (1003.05 + 788.82 + 408.75 + 445.95 + 447.1 + 250.95 + 254.45) \\
 &= 25193
 \end{aligned}$$

End

The original message, retrieved through a multi-step decryption process, demonstrates the robustness and reliability of advanced cryptographic techniques. This process, built on a foundation of efficient domination and vertex order coloring, ensures enhanced security, making unauthorized access or decryption highly improbable.

The suggested encryption technique is based on graph-structured transformations, which may need more advanced key management techniques to provide secure and effective key distribution. Storing encrypted data in graph format and keeping extra graph-related parameters may add storage overhead, making it less effective for resource-limited environments. Future research might involve developing safe and effective key management methods that may utilize block chain technology or quantum key distribution for improving the security and usability of encryption keys. Future studies could concentrate into optimizing the storage needs, e.g., compressing the graph model or creating lightweight encoding schemes without undermining security.

Algorithm and flowchart

Pseudocode of the proposed cryptographic framework

Encryption

```
def encrypt_integer(N):
    # Step 1: Split the integer using modular arithmetic (7 subnetworks)
    components = split_integer(N, modulus=7)
    subnetworks = []

    # Step 2: Construct subnetworks based on modular values
    for ci in components:
        center_node = create_center_node(ci)
        neighbors = create_neighbors(center_node, count=5 or 6)
        subnetwork = construct_subnetwork(center_node, neighbors)
        subnetworks.append(subnetwork)

    # Step 3: Connect subnetworks following the given rules
    encrypted_tree = connect_subnetworks(subnetworks)

    # Step 4: Apply additional constraints to enhance security
    encrypted_complex_network = form_complex_network(encrypted_tree)

    # Step 5: Compute graph properties
    vertex_cardinality = compute_vertex_cardinality(encrypted_complex_network)
    edge_cardinality = compute_edge_cardinality(encrypted_complex_network)
    vertex_degrees = compute_vertex_degrees(encrypted_complex_network)

    # Step 6: Categorize vertices (Alpha, Beta, Gamma strong)
    categorized_vertices = categorize_vertices(vertex_degrees)

    # Step 7: Apply intuitionistic vertex order coloring
    colored_network = apply_coloring(categorized_vertices)

    # Step 8: Generate encryption keys
    encryption_keys = generate_encryption_keys(efficient dominating set, weight of alpha
    strong vertices, chromatic number)

    return colored_network, encryption_keys
```

Decryption

```
def decrypt_encrypted_network(colored_network, keys):
    # Step 1: Extract encryption keys
    efficient_dominating_set = keys["Dominating_Set"]
    alpha_weight = keys["Alpha_Weight"]
    chromatic_number = keys["Chromatic_Number"]

    # Step 2: Reverse coloring process
    ordered_vertices = reverse_coloring(colored_network, chromatic_number)

    # Step 3: Retrieve fuzzy values from categorized vertices
    recovered_fuzzy_values = retrieve_fuzzy_values(ordered_vertices)

    # Step 4: Extract numerical components from fuzzy values
    numerical_components = convert_fuzzy_to_numbers(recovered_fuzzy_values)

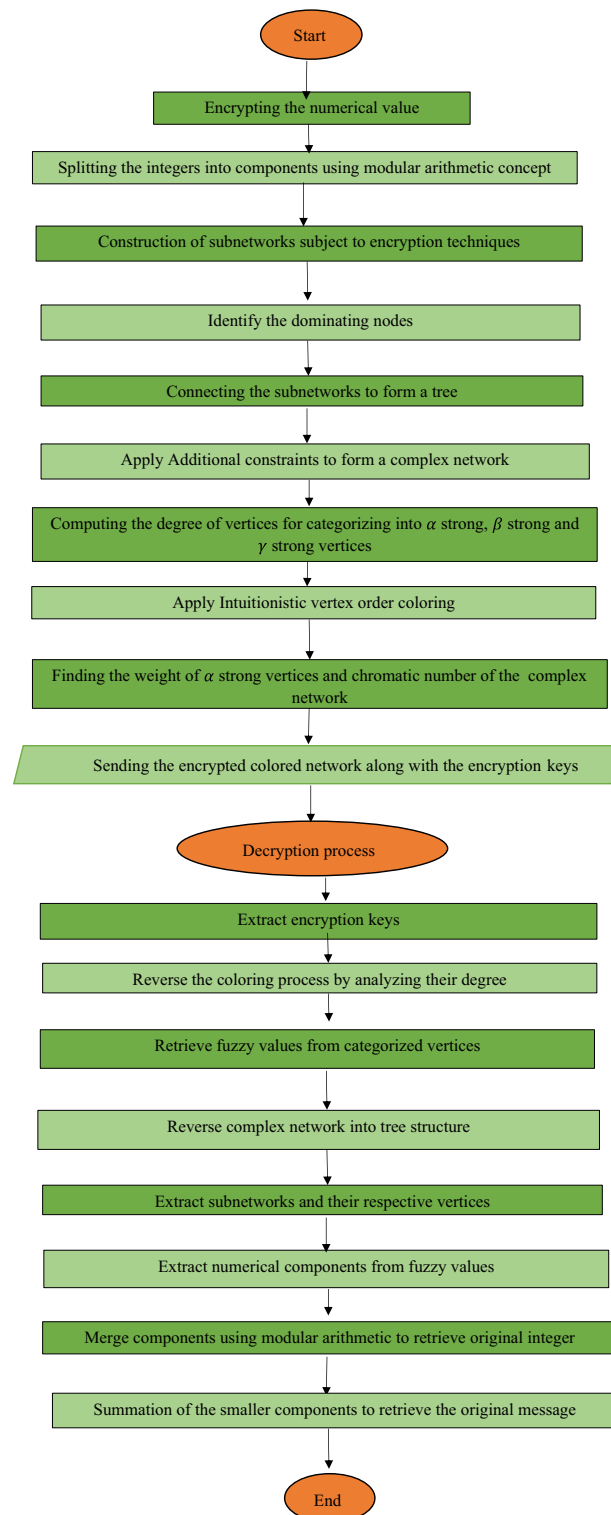
    # Step 5: Reverse complex network into tree structure
    reconstructed_tree = reverse_complex_network(colored_network)

    # Step 6: Extract subnetworks and their respective vertices
    extracted_subnetworks = extract_subnetworks(reconstructed_tree)

    # Step 7: Merge components using modular arithmetic to retrieve original integer
    original_integer = merge_components(numerical_components, modulus=7)

    return original_integer
```

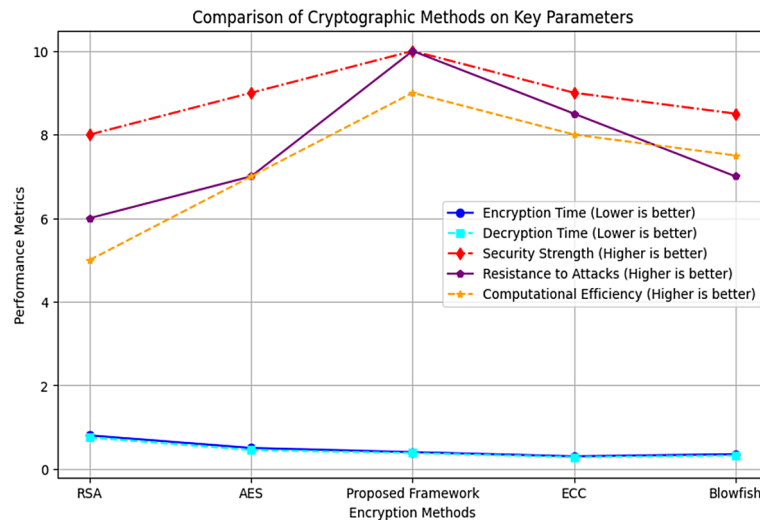
Flowchart of the proposed framework



Comparative study

Conventional cryptographic techniques like RSA, AES, ECC, and Blowfish find it difficult to strike the ideal balance between attack resistance, computational complexity, and security strength in light of the growing sophistication of cyberattacks. This motivated the need for a comparative study to analyze how our multi-layered encryption framework improves upon existing methods. The suggested framework exhibits enhanced efficiency in encryption and decryption than conventional techniques. Splitting the integers using modular arithmetic, assigning fuzzy values, and intuitionistic vertex order coloring facilitates computation optimization, resulting

in less processing time. The comparison indicates that the encryption time of the suggested framework is less than RSA and AES, thus it is faster for secure communication. Decryption time is also minimized because of the organized strategy of reverse graph transformation and effective key retrieval mechanisms. Safety is one of the key considerations in assessing a cipher algorithm. The multi layered encryption mechanism of our framework provides better cryptographic strength compared to RSA and Blowfish by using intuitionistic fuzzy graphs for intensive key generation. The framework as suggested shows maximum resilience to cryptanalysis methods with the formation of the intricate network structure based on connectivity rules between graphs, not allowing reconstruction of encrypted data. The application of intuitionistic vertex order coloring additionally secures it by making extraction of keys from adversaries challenging. In comparison with standard cryptographic solutions, our process is computationally more efficient in the form of graph-theoretic transformation in preserving the encryption strength at lowered processing costs.



Conclusion

A robust method for encrypting confidential information via a network structure has been suggested as a multi-layer encryption technique. It begins with an elementary number and progresses stepwise with the number of steps: subdivision of networks, intuitionistic fuzzy assignments, construction of a spanning tree, the addition of complex edges, and finally, graph coloring in fuzzy colorings. All these stages of encryption make the message complicated in structure and strong enough to prevent malicious decoding. The application of alpha-strong, beta-strong, and gamma-strong vertex classification and their respective weights and a chromatic-based ordered triple as the key decrypts the information to be highly reliant on the proper interpretation of multiple layers. This reliance greatly enhances security by making the data impossible to decode without the key and virtually precluding unauthorized access. This technique uniquely integrates intuitionistic fuzzy logic, fuzzy graph coloring, and encryption methodologies into a single model, demonstrating efficiency and effectiveness. Future research could focus on optimizing key management and storage efficiency. The reliance on graph-structured transformations necessitates advanced key distribution methods, potentially using block chain technology or quantum key distribution for enhanced security. Additionally, reducing storage overhead through graph compression or lightweight encoding schemes could improve efficiency for resource-constrained environments. These advancements will further strengthen the proposed encryption model, making it a powerful tool for modern cryptographic systems.

Data availability

The datasets generated and/or analyzed during the current study are not publicly available due to confidentiality but are available from the corresponding author upon reasonable request.

Received: 6 February 2025; Accepted: 9 May 2025

Published online: 01 July 2025

References

1. Wilson, R. Graph theory: The evolution of an engineering tool. *J. Graph Theory* **23**(4), 123–145 (1979).
2. Zhang, F. & Chartrand, G. Graph theory: Theory and applications. *J. Math. Struct.* **45**(3), 200–215 (2006).
3. Deo, N. *Graph theory with applications to engineering and computer science* (Dover Publications, 2016).
4. Atanassov, K. T. *On intuitionistic fuzzy sets theory* (Springer, 2012).
5. Parvathi, R. & Karunambigai, V. Structural characteristics of intuitionistic fuzzy graphs: A mathematical approach. *Int. J. Fuzzy Syst.* **14**(2), 78–94 (2006).
6. Sahoo, S. K. & Pal, S. Insightistic Fuzzy competition graphs and their applications in social networks. *J. Math. Soc. Sci.* **40**(6), 150–165 (2016).

7. Talebi, A. A., Kacprzyk, J., Rashmanlou, H. & Sadati, S. H. A new concept of an intuitionistic fuzzy graph with applications. *J. Multip.-Valued Logic Soft Comput.* **35** (2020).
8. Kosari, S., Shao, Z., Rao, Y., Liu, X., Cai, R. & Rashmanlou, H. Some types of domination in vague graphs with application in medicine. *J. Multiple-Valued Logic Soft Comput.* **41** (2023).
9. Shao, Z., Kosari, S., Rashmanlou, H. & Shoaib, M. New concepts in intuitionistic fuzzy graph with application in water supplier systems. *Mathematics* **8**(8), 1241 (2020).
10. Chen, Z. et al. A video processing algorithm using temporal intuitionistic fuzzy sets. *J. Intell. Fuzzy Syst.* **43**(6), 8057–8072 (2022).
11. Zadeh, L. A. Fuzzy sets. *Inf. Control* **8**(3), 338–353 (1965).
12. Gani, A. N. & Shajitha Begum, S. Degree, order, and size in intuitionistic fuzzy graphs. *Int. J. Algorithms Comput. Math.* **3**(3), 11–16 (2010).
13. Sadati, S. H., Rashmanlou, H. & Talebi, A. A. (2021). Domination in intuitionistic fuzzy incidence graph. In *52nd Annual Iranian Mathematics Conference (AIMC)* (pp. 88–94). IEEE (2021).
14. Rajeshkumar, R. & Anto, A. M. Some domination parameters in intuitionistic fuzzy graphs. In *American Institute of Physics Conference Series*, Vol. 2516, No. 1, p. 200016 (2022, November).
15. Yaqoob, I. et al. Optimizing cellular networks with intuitionistic fuzzy graphs: A simulation based approach. *J. Netw. Optim.* **28**(4), 310–325 (2019).
16. Mahapatra, R. et al. Colouring of COVID-19 affected region based on fuzzy directed graphs. *Comput. Mater. Contin.* **68**(1), 1219–1233 (2021).
17. Rao, Y. et al. New concepts of intuitionistic fuzzy trees with applications. *Int. J. Comput. Intell. Syst.* **14**, 1–12 (2021).
18. Kumaran, N., Meenakshi, A., Mahdal, M., Prakash, J. U. & Guras, R. Application of fuzzy network using efficient domination. *Mathematics* **11**(10), 2258 (2023).
19. Meenakshi, A., Kannan, A., Cep, R. & Elangovan, M. Efficient graph network using total magic labeling and its applications. *Mathematics* **11**(19), 4132 (2023).
20. Meenakshi, A., Senbagamalar, J. & Kannan, A. Application of intuitionistic fuzzy network using efficient domination. *Fuzzy Logic Appl. Comput. Sci. Math.* **11**, 213–232 (2023).
21. Meenakshi, A. & Babujee, J. Encryption through labeling using efficient domination. *Asian J. Res. Soc. Sci. Human.* **6**(9), 1967–1974 (2016).
22. Masud, K. I., Hasan, M. R., Hoque, M. M., Nath, U. D. & Rahman, M. O., A new approach of cryptography for data encryption and decryption. In *5th International Conference on Computing and Informatics (ICCI)*, pp. 234–239 (2022).
23. Kaushik, B., Malik, V. & Saroha, V. Review paper on data encryption and decryption. *Int. J. Res. Appl. Sci. Eng. Technol. (IJRASET)*, ISSN, 2321–9653 (2023).
24. Meenakshi, A., Dhanushiya, S., Qin, H. & Elangovan, M. Optimal network analysis through vertex order coloring of intuitionistic fuzzy graph operations. *J. Math.* **2024**, 8898813 (2024).
25. Zuo, C., Zhang, X., Yan, L. & Zhang, Z. GUGEN: Global user graph enhanced network for next POI recommendation. *IEEE Trans. Mobile Comput.* **23**(12), 14975–14986 (2024).
26. Cheng, G. et al. HyperPart: A Hypergraph-based abstraction for deduplicated storage systems. *IEEE Trans. Cloud Comput.* **13**, 1 (2024).
27. Wang, Z., Zhang, Z., Qi, W., Yang, F. & Xu, J. FreqGAN: Infrared and visible image fusion via unified frequency adversarial learning. *IEEE Trans. Circuits Syst. Video Technol.* (2024).
28. Zhou, Z., Li, Z., Zhou, W., Chi, N., Zhang, J. & Dai, Q. Resource saving and high robustness image sensing based on binary optical computing. *Laser Photon. Rev.* **24**00936 (2024).
29. Chu, L. et al. A deniable encryption method for modulation-based DNA storage. *Interdiscip. Sci. Comput. Life Sci.* **16**(4), 872–881 (2024).
30. Hu, J., Jiang, H., Xiao, Z., Chen, S., Dustdar, S. & Liu, J. HeadTrack: Real time human computer interaction via wireless earphones. *IEEE J. Sel. Areas Commun.* **42**(4), 990–1002 (2023).
31. Sun, G., Sheng, L., Luo, L. & Yu, H. Game theoretic approach for multipriority data transmission in 5G vehicular networks. *IEEE Trans. Intell. Transp. Syst.* **23**(12), 24672–24685 (2022).
32. Guan, H. et al. Dominations in intuitionistic fuzzy directed graphs with applications towards influential graphs. *Mathematics* **12**(6), 872 (2024).
33. Meenakshi, A. & Dhanushiya, S. Fuzzy network using vertex order coloring and efficient domination. In *2023 First International Conference on Advances in Electrical, Electronics and Computational Intelligence (ICAEECI)*, Tiruchengode, India, pp. 1–11 (2023).
34. Meenakshi, A. & Dhanushiya, S. Optimizing network structures through neutrosophic graph product operations and its coloring: A comprehensive approach for enhanced connectivity and robustness. *Neutrosophic Sets Syst.* **76**(1), 22 (2024).
35. Gani, A. N. & Kani, B. F. Fuzzy vertex order coloring. *Int. J. Pure Appl. Math.* **107**(3), 601–614 (2016).

Author contributions

A. Meenakshi: Conceptualization, Methodology, Writing—Original Draft Preparation. S. Dhanushiya: Data Curation, Formal Analysis, Writing—Review & Editing. Leo Mrcic: Software Development, Validation, Visualization, Funding Acquisition. Antonios Kalampakas: Investigation, Resources, Supervision. Sovan Samanta: Project Administration, Writing—Review & Editing.

Declarations

Competing interests

The authors declare no competing interests.

Additional information

Correspondence and requests for materials should be addressed to S.S.

Reprints and permissions information is available at www.nature.com/reprints.

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Open Access This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License, which permits any non-commercial use, sharing, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if you modified the licensed material. You do not have permission under this licence to share adapted material derived from this article or parts of it. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by-nc-nd/4.0/>.

© The Author(s) 2025