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Platonic configurations of points and lines*

Jurij Kovič†
IMFM, Ljubljana and UP FAMNIT, Koper, Slovenia

Aleksander Simonič
School of Science, The University of New South Wales (Canberra), ACT, Australia

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Abstract

We present some methods for constructing connected spatial geometric configurations (p_q, n_k) of points and lines, preserved by the same isometries of Euclidean space E^3 as the predetermined Platonic solid. In this paper, we are mainly interested in configurations (n_3), (n_4), and (n_5), but also in unbalanced configurations (p_3, n_4), (p_3, n_5), and (p_4, n_5).

Keywords: Configuration of points and lines, symmetry group, Platonic solid, centrally symmetric solid, projection from a point.

Math. Subj. Class.: 51A20, 51M20

1 Basic notions, formulation of the problem

Definition 1.1. A *geometric configuration* consists of points and (straight) lines in the Euclidean space E^3. Points incident with q lines are called *q-valent*, and likewise *k-valent* lines are lines incident with k points.

Remark 1.2. Here we use this ambient space, which serves a better purpose of our paper (see Definition 1.4), instead of the more usual Euclidean plane that is embedded into the real projective plane.

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E-mail addresses: jurij.kovic@siol.net (Jurij Kovič), aleks.simonic@gmail.com (Aleksander Simonič)

For denoting configurations we use the same notation as Grünbaum in [7]. Let (p_q, n_k) be a configuration of p q-valent points and n k-valent lines; it is also called a (q, k)-configuration. If $p = n$ and $q = k$, it is called a *balanced* configuration and is denoted simply as (n_k); it is also called a k-configuration. In this paper, we use the following natural generalization. A configuration may contain $p_1, p_2, p_3, \ldots$ points of valence $q_1, q_2, q_3, \ldots$, respectively, and $n_1, n_2, n_3, \ldots$ lines of valence $k_1, k_2, k_3, \ldots$, respectively. The notation (p_q, n_k) may also be generalized in a natural way. For example, the type of configuration shown in Figure 1 is denoted by $(6_4 12_3, 15_4)$. Note that in this case the well-known elementary combinatorial rule $pq = nk$ also extends to $p_1q_1 + p_2q_2 + p_3q_3 + \cdots = n_1k_1 + n_2k_2 + n_3k_3 + \cdots$, see [8, page 257].

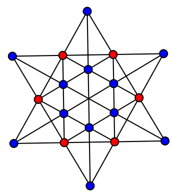

Figure 1: A configuration $(6_4 12_3, 15_4)$ having 6 points of valence 4, 12 points of valence 3 and 15 lines of valence 4.

Remark 1.3. Such generalized configurations appeared for the first time in the configuration literature in the paper by Zindler [9]. The very interesting topic of Zindler degrees of regularity of such structures, which admit an infinite hierarchy of regularity, is mentioned in [8]. As Grünbaum remarks in [7], “it seems that Zindler’s general challenge has never been met”.

We are especially interested in symmetrical configurations. *Symmetry* (or, more precisely *E^n-symmetry*) of any combinatorial object X (e.g., solid, configuration, graph, etc.), geometrically realized as $Y = Y(X)$ in the Euclidean space E^n, is an isometry (rotation or reflection or combinations of them) of E^n preserving Y. The *(full) symmetry group* $\mathrm{Sym}(Y)$ of $Y = Y(X) \subset E^n$ consists of all the isometries of the Euclidean space E^n preserving Y; it is a subgroup of the group of automorphisms $\mathrm{Aut}(X)$ of the corresponding X. The (rotational) symmetry group $\mathrm{Sym}_R(Y)$ of $Y = Y(X) \subset E^n$ consists of all the rotations of E^n preserving Y.

Various tools and techniques have been used to construct symmetrical planar geometric configurations (n_k) of points and lines, see [2, 3] for example. It is well known that a *planar* geometric configuration $Y = Y(X) \subset E^2$ may only have “cyclical” or “dihedral” symmetry. However, the same underlying configuration X, which has a planar realisation $Y = Y(X)$, may have (and reveal) more (hidden) symmetries, if it is realised as a spatial configuration $Z = Z(X)$ in some higher-dimensional space. Recently some authors started investigating spatial configurations and symmetrical configurations more systematically; e.g., in [4, 5] Gévay presents a number of examples whose underlying structure is some convex polyhedron and even, in some cases, higher-dimensional convex polytope with a non-trivial symmetry group.

Definition 1.4. A "full" *Platonic configuration* is a geometric configuration $Z \subset E^3$ with all symmetries of a Platonic solid $P \in \{T, C, O, D, I\}$ (preserved by all isometries of E^3 preserving P, thus $\mathrm{Sym}(Z) = \mathrm{Sym}(P)$). A "rotational" Platonic configuration Z is preserved only by *rotational symmetries* of P, thus $\mathrm{Sym}(Z) = \mathrm{Sym}_R(P)$. For any $P \in \{T, C, O, D, I\}$ let $P\,(p_q, n_k)$ and $P_R\,(p_q, n_k)$ denote the classes of "full" and "rotational" (p_q, n_k) Platonic configurations, respectively.

For example, the configurations obtained from the skeletons (vertices and edges) of the five Platonic solids T, C, O, D, I belong to the classes $T\,(4_3, 6_2)$, $C\,(8_3, 12_2)$, $O\,(6_4, 12_2)$, $D\,(20_3, 30_2)$, $I\,(12_5, 30_2)$, respectively.

Now we can formulate the problem on which we focus in this paper:

Problem 1.5. For each of the five Platonic solids P construct examples of connected Platonic configurations $Z \in P\,(p_q, n_k)$ and $Z \in P_R\,(p_q, n_k)$ of points and lines such that $3 \leq q \leq k \leq 5$.

2 How to construct Platonic configurations

To construct examples of connected Platonic configurations we combine some known methods with some new ones. Our approach is based on the very general idea (allowing many variations) of connecting isomorphic copies of some initial (planar) configuration A, placed "symmetrically" around P.

The idea of obtaining larger configurations by linking together small isomorphic building blocks has been explored by several authors, e.g., in [1, 3, 4, 6, 7], see also the description of Grünbaum's incidence calculus in [8, pages 243–263].

We will use the "vector" notation $P = (v, e, f, d, m)$ for the polyhedron P with v vertices of valence d, with e edges and f m-gonal faces. Thus we have the following parameters for the five Platonic solids.

	v	e	f	d	m
tetrahedron	4	6	4	3	3
cube	8	12	6	3	4
octahedron	6	12	8	4	3
dodecahedron	20	30	12	3	5
icosahedron	12	30	20	5	3

Table 1: Parameters of Platonic solids.

To construct examples of Platonic configurations $Z \in (p_q, n_k)$ with the symmetry group of the Platonic solid $P = (v, e, f, d, m)$ the following construction may be applied, at least for the cases $k \in \{3, 4, 5\}$ and $3 \leq q \leq k$.

Construction 2.1. Step 1: *Start with a planar configuration A with cyclical C_m or dihedral D_m symmetry; all of its lines must have valence k and each of its vertices must have a valence at most k.*

Step 2: *Place a copy of A on each of the f faces of the Platonic solid P; the center of each copy of A should coincide with the central point of the corresponding face to obtain a configuration B with the symmetries of P.*

Step 3: *Identifying some points of different copies of A or connecting them with new lines try to get a connected configuration B_c.*

Step 4: *If necessary, add some new points and lines to increase the valences of some points to get a desired configuration $Z \in (p_q, n_k)$.*

Thus we obtain 1-*layer Platonic configurations*. To obtain s-*layer configurations* we use several concentric and homothetic copies of P. Radial projection from the center of P through the concentric and homothetic copies of P increases (adds 1 to) the valences of those points that are connected either by "radial rays" (Figure 2 left) or by "antipodal lines" (Figure 2 right).

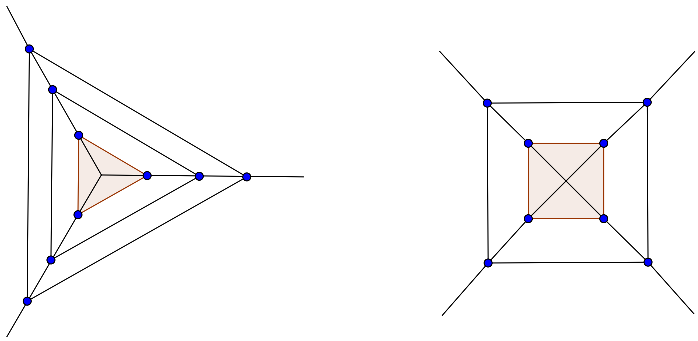

Figure 2: Lines connecting points in concentric and homothetic copies of P.

The numbers of points and lines in a 1-layer Platonic configuration can easily be obtained as follows:

Proposition 2.2. *Let Z be a 1-layer Platonic configuration whose points and lines all lie on the boundary of the chosen Platonic solid $P = (v, e, f, d, m)$. Let x, y, z denote the number of points of Z in each vertex (represented by yellow points in our figures), in the interior of each edge (red points) and in the interior of each face (blue points) of P, respectively. Then $x \in \{0, 1\}$ and the number of points of Z is*

$$p(Z) = xv + ye + zf.$$

Let u, w denote the number of lines of Z incident with each edge and with (the interior of) each face of P, respectively. Then $u \in \{0, 1\}$ and the number of lines in Z is

$$l(Z) = ue + wf.$$

In Table 2 we provide formulas for $p(Z)$ and $l(Z)$ for all Platonic solids by using Table 1 and Proposition 2.2.

3 Examples of Platonic (n_k) configurations

In this section we apply Construction 2.1 in order to obtain examples of most of the Platonic (p_q, n_k) configurations for $k \in \{3, 4, 5\}$ and $3 \leq q \leq k$.

P	$p(Z)$	$l(Z)$
tetrahedron	$4x+6y+4z$	$6u+4w$
cube	$8x+12y+6z$	$12u+6w$
octahedron	$6x+12y+8z$	$12u+8w$
dodecahedron	$20x+30y+12z$	$30u+12w$
icosahedron	$12x+30y+20z$	$30u+20w$

Table 2: Numbers of points and lines in a 1-layer Platonic configuration Z.

For any Platonic solid $P \in \{T, C, O, D, I\}$ let P_k denote the class of all Platonic configurations (n_k) whose group of symmetry is the same as the full symmetry group of P. Likewise, let $P_{k,R}$ denote the class of all Platonic configurations (n_k) with the rotational symmetry group of P.

Let Cyc_m denote the class of all (planar) configurations, preserved by the cyclical group of rotations for the multiples of $\frac{2\pi}{m}$, and let Dih_m denote the class of all (planar) configurations, preserved by the dihedral group D_m.

Example 3.1. Place copies of the planar configuration $A = (3_2 3_1, 3_3) \in \mathrm{Cyc}_3$ with 3-fold rotational symmetry (Figure 3) on the faces of T such that the 1-valent vertices of each copy of A get into the vertices of the tetrahedron. The obtained configuration $B_c = (4_3 12_2, 12_3)$ is connected and contained in the class $T_{3,R}$. To obtain a balanced configuration Z, we have to increase the valence of 12 2-valent points in B_c to 3. Take 3 concentric and homothetic copies of B_c to get $B_1 = (12_3 36_2, 36_3)$. Add 12 lines from the central point of T through the 12 triples of 2-valent points. Now we have $36+12 = 48$ 3-valent lines and $12+36 = 48$ 3-valent points. Hence, we obtained (a 3-layer) $Z \in T_{3,R}$ for $n = 48$.

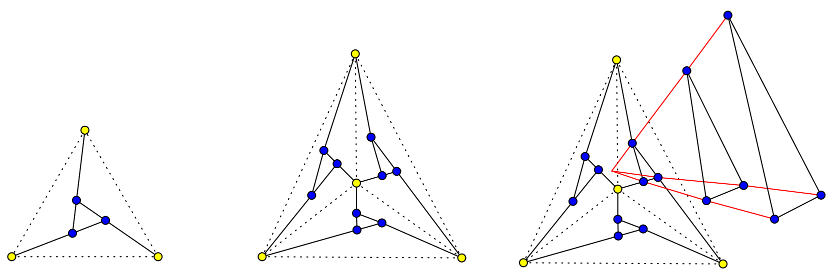

Figure 3: Construction of a 3-layer $Z \in T_R$ of type (48_3) (right) from $A = (3_2 3_1, 3_2)$ (left) and $B_c = (4_3 12_2, 12_3)$ (middle).

Problem 3.2. Construct a Platonic configuration $Z \in T_3$ of points and lines with the full symmetry group of the tetrahedron T.

Solution. Take the barycentric subdivision of each face of T by the three perpendicular bisectors of its edges, see Figure 4 (left). These $3f = 12$ perpendicular bisectors are the lines of the configuration with $v+e+f = 4+6+4 = 14$ points. The four v-points and the 4 f-points have valence 3, the 6 e-points have valence 2. To increase their valence, take 3 concentric and homothetic copies of T and connect the triples of 2-valent points by 6 radial lines from the center of T. Thus, we get $14 \times 3 = 42$ 3-valent points and $12 \times 3 + 6 = 42$ 3-valent lines, hence a configuration of type (42_3). □

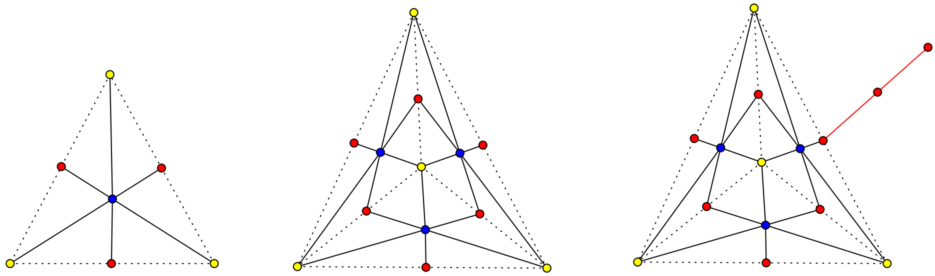

Figure 4: Construction of a 3-layer $Z \in T_3$ of type (42_3) starting from $A = (1_3 6_1, 3_3)$ and using 3 copies of $B_c \in T_3$ of type $(8_3 6_2, 12_3)$.

Many configurations may be obtained from the Pappus configuration (9_3), as the following examples show.

Problem 3.3. Construct a Platonic configuration $Z \in (n_3)$ of points and lines with the full symmetry group of the tetrahedron T, or octahedron O, or icosahedron I. Do this by placing the copies of the same planar configuration A on each of the faces of these solids.

Solution. Draw on each face of $P \in \{T, O, I\}$ a copy of the Pappus configuration $A = (9_3)$ in the 3-fold rotational symmetry form. The lines of these copies of A intersect each edge of P in 3 points. Include the lines through the edges of P among the lines of configuration Z. Exclude the vertices of P and the 3 corresponding points of each copy from the points of Z, see Figure 5. □

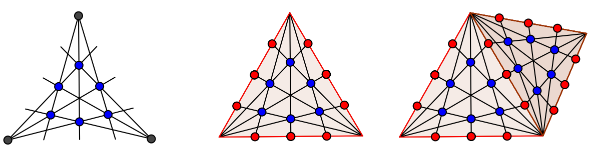

Figure 5: Starting with the Pappus configuration $A = (9_3)$ placed on each face of P, we obtain a Platonic configuration.

On each face there are $z = 6$ interior points of valence 3, on each edge there are $y = 3$ points of valence 3. So, in accordance with Proposition 2.2, we get $6f+3e = 4e+3e = 7e$ points of valence 3, and $9f+e = 6e+e = 7e$ lines of valence 3, and the obtained Platonic configuration is balanced $(7e_3) = \left(\left(\frac{21f}{2}\right)_3\right)$. The same construction works for any solid P such that all of its faces are equilateral triangles, hence $3f = 2e$.

Very often a solution of one problem leads to a solution of another problem, yet some modifications in the construction are needed. The Pappus configuration lies in the background of the following construction, too.

Example 3.4. Divide each pentagonal face of a dodecahedron into 5 congruent isosceles triangles and draw into each triangle the same configuration as in Figure 5. The resulted configuration has $(6+3) \times (5 \times f) = 9 \times 60 = 540$ 3-valent points in the interiors of faces of D and $3 \times e = 90$ 3-valent points on the edges of D, and $(9+5) \times f + e = 540 + 60 + 30 = 630$ 3-valent lines. Thus, we get a (630_3) configuration $Z \in D_3$ with the full symmetry group of the dodecahedron, see Figure 6.

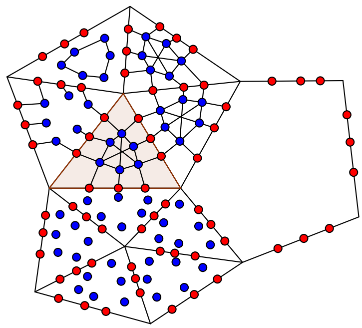

Figure 6: "Connecting the dots" in the triangulated pentagonal faces of D. As a result we obtain a configuration $Z \in D_3$ of type (630_3).

Problem 3.5. Construct a Platonic configuration $Z \in D_{3,R}$ with the rotational symmetry group $\mathrm{Sym}_R(D)$ of the dodecahedron.

Solution. Draw on each face of D the configuration $(15_3 15_1, 20_3)$ as in Figure 7; include the edges of D among the lines of configuration, exclude the vertices of D from the points of the configuration. Thus, we get a configuration with $12 \times 15 + 30 \times 3 = 270$ 3-valent points and $12 \times 45 + 30 = 270$ 3-valent lines, hence a (270_3) configuration. This is in accordance with Proposition 2.2 since $x = 0$, $y = 3$ and $z = 15$, and dodecahedral formula for points is $20x + 30y + 12z$. Note that on each edge the position of three points remains the same if the edge is reflected about its midpoint, see Figure 7. □

Problem 3.6. Construct examples of Platonic configurations $Z \in C\,(p_3, n_4)$ and $Z \in C\,(n_4)$ with the full symmetry of the cube.

Solution. Subdivide each face of two concentric and homothetic copies of a cube into a square grid having 4×4 points and $4 + 4$ lines. We have $2 \times (8 + 12 \times 2) = 64$ 3-valent points (in the vertices and on the edges) and $2 \times 6 \times 4 = 48$ 2-valent points (in the interiors of faces). We have $2 \times 3 \times 12 = 72$ 4-valent lines parallel with one of the faces. Add to them $24/2 = 12$ lines connecting 2 pairs of 2-valent antipodal points, see Figure 8. We obtain a configuration with $64 + 48 = 112$ 3-valent points and $72 + 12 = 84$ 4-valent points, thus a $(112_3, 84_4)$ configuration Z. □

Placing on each face of a cube a copy of the configuration $A = (32_4 16_2, 40_4)$ in such a way that 2-valent points are on the edges whose lines are not included among the lines of Z, see Figure 9, we get a (240_4) configuration with full symmetry of a cube.

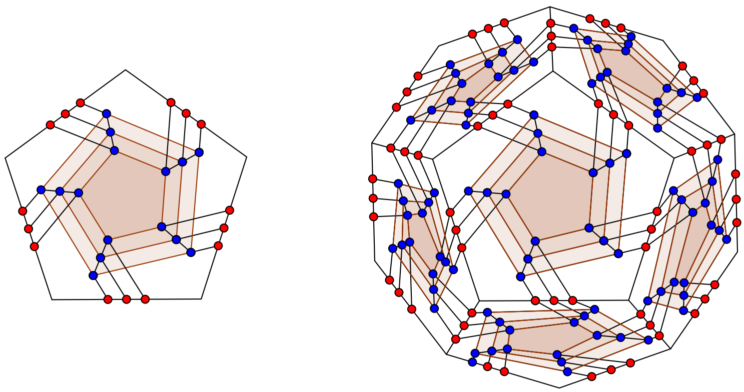

Figure 7: Placing the copies of $(15_3 15_1, 20_3)$ to each face raises the valences of 1-valent points to 2 while adding the lines of the edges of D raises them to 3.

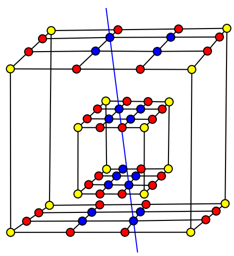

Figure 8: We obtain a $(112_3, 84_4)$ configuration by connecting antipodal pairs of 2-valent points on the faces of two concentric and homothetic cubes.

Problem 3.7. Construct a Platonic configuration $Z \in C_{3,R}$ of points and lines with the symmetry group of the rotations of the cube C.

Solution. Place on each face of the cube a copy of $A = (4_3 1_2 4_1, 6_3)$ and identify 1-valent points with the 8 vertices of the cube to obtain a configuration $B = ((8 + 24)_3 6_2, 36_3)$. After adding the central point of the cube and 3 lines connecting the antipodal 2-valent points of B, we obtain $Z = (39_3)$, see Figure 10. □

Problem 3.8. For $P \in \{T, O, I\}$ construct $Z \in P_{3,R}$. Construct also $Z \in P_{3,R}$ for $P \in \{C, D\}$.

Solution. Place the copies of a configuration $A = (9_3 9_1, 12_3) \in \mathrm{Cyc}_3$, see Figure 11, on each of the triangular faces of $P(v, e, f, d, m) \in \{T, O, I\}$, so that the 1-valent points lie on the edges of P, which are included among the lines of configuration. The points on each edge must be equidistant and the middle one must be the center of the edge. We get 3-valent Platonic configurations with $p = 9f + 3e$ points and $l = 12f + e$ lines. Since $3f = 2e$, we have $p = 6e + 3e = 9e$ and $l = 8e + e = 9e$. Thus, we get configurations $(54_3) \in T_{3,R}$, $(108_3) \in O_{3,R}$, and $(270_3) \in I_{3,R}$. □

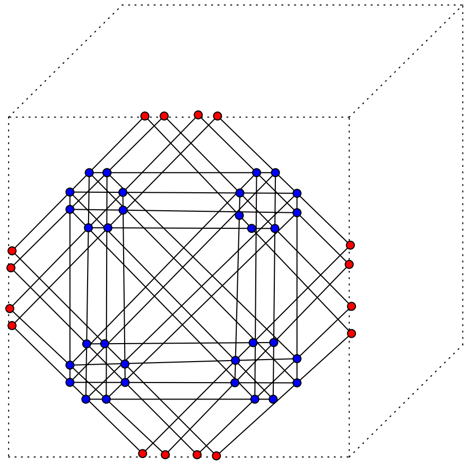

Figure 9: The 2-valent points of the 6 copies of $A = (32_4 16_2, 40_4)$ become 4-valent after they are placed on the edges of a cube.

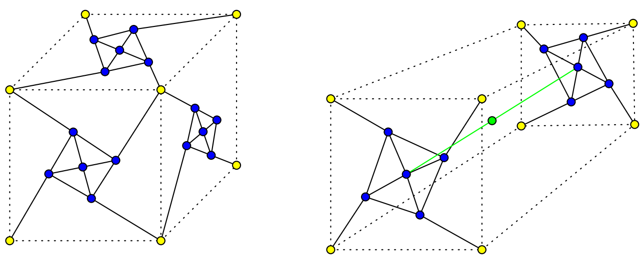

Figure 10: Platonic configuration (39_3) with rotational symmetry of the cube.

Triangulating the faces of the cube and dodecahedron (with congruent isosceles triangles each having one vertex in the center of the face) and placing the same A as in Figure 8 into each of these triangles, we obtain 3-valent configurations with rotational symmetry of the cube and the dodecahedron. In the case of the cube (in which $4f = 2e$) the number of points is $p = 48 \times f + 3e = 24e + 3e = 27 \times 12 = 324$ and the number of lines is $52 \times f + e = 26e + e = 324$. Hence, we have $(324_3) \in C_{3,R}$. In the case of the dodecahedron (in which $5f = 2e$) we have $p = 60f + 3e = 24e + 3e = 27e = 810$ points and $n = 65f + e = 26e + e = 27e = 810$ lines. Hence, we have $(810_3) \in C_{3,R}$.

Remark 3.9. Construction of $A = (9_3 9_1, 12_3) \in \mathrm{Cyc}_3$ using triangular lattice in Figure 11 not only exactly defines the locations of its points and lines in a triangular face but also serves as a necessary proof of its existence (even a small change in locations of some of its elements may cause that the triples of blue points are not collinear any more).

Problem 3.10. Construct a Platonic configuration $Z \in O_4$ with the full symmetry group of the octahedron.

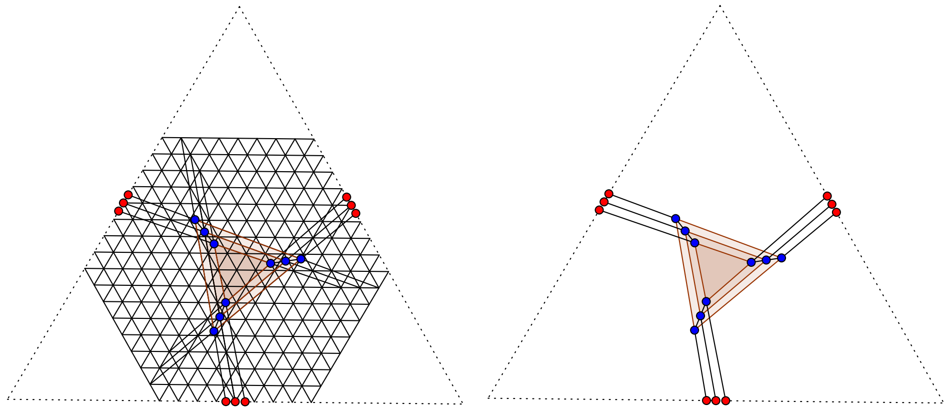

Figure 11: Configuration $A = (9_3 9_1, 12_3) \in \mathrm{Cyc}_3$.

Solution. Place the copy of $A = (3_1 6_2 7_3, 9_4)$ on each of the faces of the octahedron in such a way that their 1-valent points coincide with the vertices of the faces (Figure 12) to obtain $B = (30_4 56_3, 72_4)$ with $2e + v = 24 + 6 = 30$ 4-valent points and $7 \times f = 56$ 3-valent points and $9f = 72$ 4-valent lines. Now take two concentric and homothetic copies of B and connect antipodal points of valence 3 with 4-valent lines. Then all the points and lines have valence 4. The number of lines is $72 \times 2 + \frac{56}{2} = 144 + 28 = 172$. The number of points is $2 \times 86 = 172$. So we have a (172_4) configuration. □

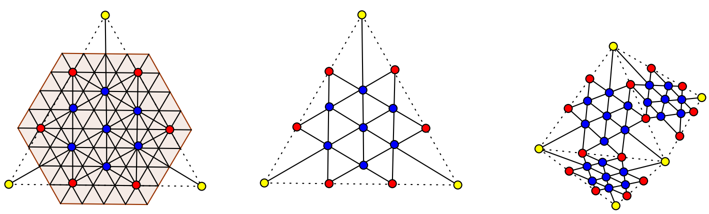

Figure 12: Construction of $B = (30_4 56_3, 72_4)$ with octahedral symmetry.

Solution. Place the copies of $A = (15_3 3_1, 12_4)$ on each of the 8 faces of the octahedron and identify the 1-valent points of A with the vertices (Figure 13). Now, these 1-valent points produce 6 4-valent points, and we have a configuration $(6_4 120_3, 96_4)$. To obtain a balanced (n_4) configuration, take two concentric copies of the octahedron, and connect 60 antipodal pairs of 3-valent points with 60 4-valent lines. Thus, we get a (252_4) configuration. □

In the next example the Pappus configuration is useful again.

Example 3.11. Placing copies of $A = (10_3 15_1, 9_5)$ on the faces of $P \in \{T, O, I\}$, see Figure 14, and including the edges, we obtain $B = ((5e + 10f)_3, (e + 9f)_5)$.

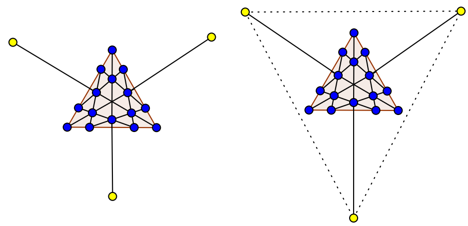

Figure 13: Placing 1-valent points into 4-valent vertices of the octahedron.

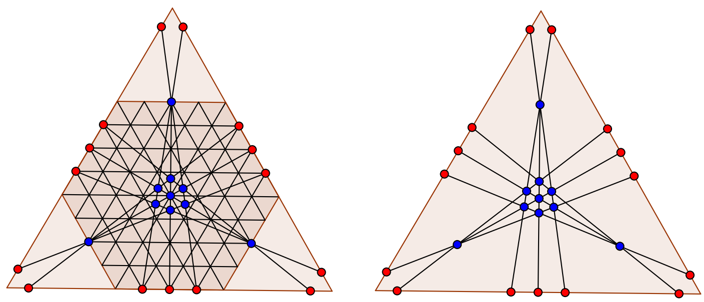

Figure 14: Construction of $A = (10_3 15_1, 9_5)$.

Example 3.12. Place copies of $A = (45_3, 27_5) \in \mathrm{Cyc}_3$ on the faces of T, see Figure 15, and exclude the lines on the edges to get $B = (30_4 120_3, (96)_5)$ with 4-valent points on edges, 3-valent points in the interiors of faces and 5-valent lines. Connect 3-valent points in 5 concentric and homothetic copies of T with 120 5-valent radial lines to get $C = ((150 \times 5)_4, (96 \times 5 + 120)_5) = (750_4, 600_5)$ having rotational symmetry of tetrahedron.

Example 3.13. Place copies of $A = (12_3 3_2 3_1, 15_3) \in \mathrm{Cyc}_3$, see Figure 16, on the faces of $P \in \{T, O, I\}$ to obtain $Z \in P_R\,((2e + 12f)_3, (15f)_3)$ whose 3-valent edge points have valence 2 in one triangle and valence 1 in the adjacent triangle. Thus, we get configurations Z in classes $T_R(60_3)$, $O_R(120_3)$ and $I_R(300_3)$.

Example 3.14. Placing 3 points X_i, Y_i, Z_i on each of the 4 rotational axes a_i through the vertices of T in such a way that for any $i, j \in \{1, 2, 3, 4\}$ the 3 lines $X_i Z_j$, $Y_i Y_j$ and $Z_i X_j$ meet in the point $T_{i,j}$, we obtain a configuration (18_3), see Figure 17. It has the full symmetry of T if the points X_i, Y_i, Z_i on each axis a_i are at distances x, y and $z = z(x, y)$ from the center of T.

We could also say that the 6 copies of $A = (3_1 1_6, 3_3)$, see Figure 17 (left), with 1-valent vertices placed an axes on T produce (18_3). The (yellow) points X_i, Y_i, Z_i are in the vertices of 3 concentric and homothetic tetrahedra, while the (red) points $T_{i,j}$ are the midpoints of the edges of the middle tetrahedron.

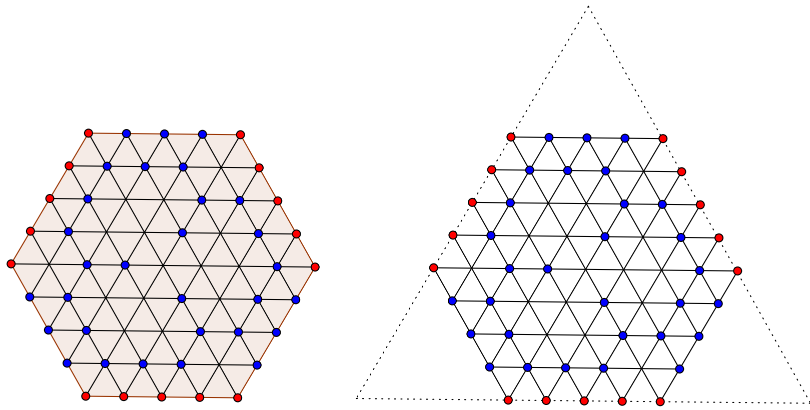

Figure 15: Configuration $A = (45_3, 27_5)$ with 3-fold rotational symmetry.

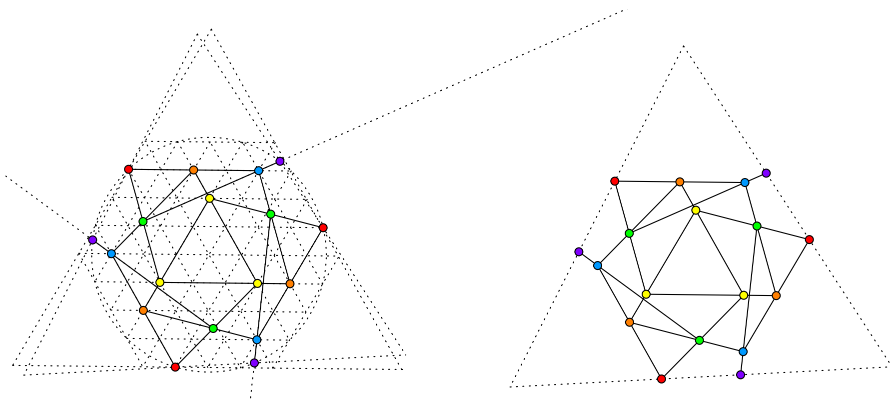

Figure 16: A copy of $A = (12_3 3_2 3_1, 15_3) \in \mathrm{Cyc}_3$ on a face of P.

4 Concluding remarks

Most of the constructions presented in this paper may be used to produce infinite families of examples and may be easily generalised into theorems. But to make the constructions easier to understand we rather presented concrete examples of Platonic configurations together with their visual representation.

ORCID iDs

Jurij Kovič https://orcid.org/0000-0003-0567-2626

Aleksander Simonič https://orcid.org/0000-0003-1298-9031

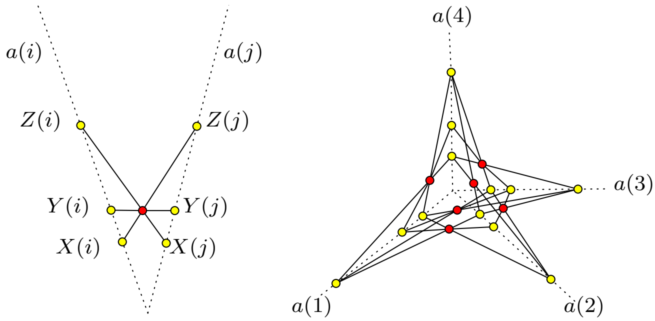

Figure 17: $A = (3_1 1_6, 3_3)$ and $C = (18_3)$.

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