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Pythagorean linguistic information-based green supplier selection using quantum-based group decision-making methodology and the MULTIMOORA approach

Prasenjit Mandal[1] · Leo Mrsic[1,2] · Antonios Kalampakas[3] · Tofigh Allahviranloo[4,5] · Sovan Samanta[1,4,6]

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Abstract

The selection of environmentally sustainable suppliers has been a significant challenge in management decision-making (DM). Multicriteria group decision-making (MCGDM) is a ranking methodology used to select suppliers, but it is complex and influenced by the different opinions of decision-makers. Once again, extensive research on MCGDM has exposed inadequacies in the trustworthiness of experts' judgements, which profoundly impact the ultimate ranking results. The Pythagorean linguistic number (PLN) concept has been used to address MCGDM by considering experts' confidence levels and real-world scenarios. This study introduces an extensive technique using a quantum scenario-based Bayesian network (QSBN) and Deng entropy-based belief entropy to account for the interference of beliefs. The goal is to replicate the subjectivity of experts' opinions during different stages of DM, including the accumulation of experts' weights and alternative probabilities. The correlation coefficient of PLNs is introduced for determining criterion weights and employing new techniques based on entropy methods for experts' weights. The MULTIMOORA approach consolidates the probability of alternatives in QSBN among all experts, and the interference value is computed using belief entropy, an index for quantifying the probability of uncertainty. The study provides a numerical example to illustrate the proposed methodology, specifically focusing on selecting environmentally sustainable suppliers, and demonstrates its applicability and effectiveness.

Keywords Pythagorean linguistic set · MULTIMOORA · Quantum probability theory · MCGDM

1 Introduction

MCGDM is a prevalent methodology for assessing and prioritising solutions amidst competing qualities across many applications. MCGDM methodologies have been suggested (She et al. 2021; Li and Cao 2019; Opricovic and Tzeng 2004; Ren et al. 2019; Zhang

Extended author information available on the last page of the article and Deng 2019; Han et al. 2021). As real situations get increasingly complicated, several experts are consulted to enhance clarity in DM into MCGDM (He et al. 2018). MCGDM consists of many experts who evaluate various alternatives and subsequently select the most suitable alternative. A generic MCGDM issue may be structured into three phases: preparation phase, aggregation phase, and selection phase (He et al. 2018; Liu et al. 2016). Numerous aspects contribute to the complexity of MCGDM challenges, with the articulation of experts' cognitive knowledge during the DM process which is paramount (Li et al. 2021). Current MCGDM methodologies are inadequate for addressing such issues (Wang 2021). In practical scenarios, MCGDM is a complex cognitive process that encompasses the varied cognition of experts and several uncertainties and ambiguities (Li et al. 2021; Yeni and Ozcelik 2019; Li et al. 2021). An increasing number of MCGDM methodologies, that account for experts' ambiguous cognitive information, has been introduced.

The LT (Xu 2005; Mandal et al. 2022) serves as a potent instrument for adeptly conveying experts' epistemic doubt and it has been adapted into several forms, including the HFLTS (Wu et al. 2022), PLTS (Wang et al. 2022), and flexible verbal expression (Wu et al. 2019). Nevertheless, these representations are inadequate for conveying the uncertainty associated with relativistic information, such as the confidence levels of experts. For instance, suppose a scenario in which we solicit an expert to evaluate two providers, x_1 and x_2. Upon assessing suppliers x_1 and x_2 based on product quality criteria, he/she considers supplier x_2 to be "good", expressing 93% confidence and 11% uncertainty over this conclusion. The term "good" pertains to the expert's relative information, including confidence levels such as 93% certainty and 11% uncertainty. To obtain this sort of information, the PLS is created and it examined their operational rules in (Liu et al. 2020). (Mandal et al. 2021, 2023a, b) further showed that the PLNs of PLS may be interpreted as the psychological representation of Z-numbers, while the experts employed Yager's Pythagorean fuzzy numbers to articulate their varying degrees of confidence. In PLNs, the initial component represents the original expert opinion, whereas the subsequent component denotes the expert's confidence level in the first element. Consequently, as per PLN, the data regarding supplier x_2 is represented as $\langle good, (0.93, 0.11)\rangle$.

Resolving linguistic dispersion in MCGDM, selection of appropriate MCGDM methodologies is required. No single MCGDM strategy excels in all aspects, as each offers unique advantages. MCGDM approaches are typically classified into three categories (Hafezalkotob et al. 2019):

(1) Methods of Value Measurement: WASPAS (Weighted Aggregated Sum Product Assessment) (Mardani et al. 2017) and SAW (Simple Additive Weighting) (Seyedmohammadi et al. 2018).
(2) Models of Reference Level Models: EDAS (Evaluation based on Distance from Average Solution) (Ghorabaee et al. 2017), VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) (Wu et al. 2019), and TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) (Rani et al. 2020).
(3) Techniques of alternative ranking: GLDS (Gained and Lost Dominance Score) (Wu and Liao 2019), ELECTRE (ELimination Et Choix Traduisant la REalité) (Liao et al. 2019), PROMETHEE (Preference Ranking Organisation METHod for Enrichment of Evaluations) (Makan and Fadili 2020), ORESTE (Organisation, Rangement Et Synthèse de données relationnelles) (Wang et al. 2020), and MULTIMOORA (Multi-Objective

Optimisation based on Ratio Analysis plus the full MULTIplicative form) (Maghsoodi et al. 2020).

In practical scenarios, decision-makers may express their preferences based on personal perspectives, which can vary across issues. Therefore, it is crucial to consider the risk attitudes of decision-makers in MCGDM. The MULTIMOORA method stands for less time, high stability, simple calculation, and more concise by comparing TODIM, ORESTE, TOPSIS, and VIKOR according to rigorous studies by Hafezalkotob et al. (2019). Therefore, the MULTIMOORA method is a comprehensive MCGDM tool. It is designed to assess a range of alternatives against a backdrop of varied and often conflicting criteria. The approach is particularly noted for:

(1) **RS approach:** It is based on the ratio of the beneficial criteria and the non-beneficial criteria. Each criterion is given equal weight, and the alternative with the highest ratio is considered the best.
(2) **RP approach:** This involves comparing alternatives to a reference point, which usually represents the most desirable performance levels across all criteria. The alternative closest to this reference point is preferred.
(3) **FMF approach:** This component multiplies the performance ratings of each alternative across all criteria. The alternative with the highest product is deemed the most suitable. The MULTIMOORA approach is appreciated for its simplicity, comprehensiveness, and ability to provide a more balanced decision-making process by combining these three methods. It is used in engineering, economics, and healthcare, to facilitate complex DM scenarios.

However, experts often possess varied histories, originated from distinct domains, resulting in diverse, complex, and ambiguous cognitions. Consequently, the opinions of various experts may interfere with one another or be influenced by the surrounding environment (He et al. 2018; Li et al. 2021). Human cognition and DM are inherently probabilistic and unpredictable. In certain instances, human behaviour may not align effectively with CPT and anticipated utility theory (Khrennikova et al. 2014; Moreira and Wichert 2018). When alternative routes are unobserved, individuals' behavioural outcomes may contravene the sure thing principle and the rule of total probability (He et al. 2018). In the aggregation of alternative views, the beliefs of several experts may interact, like the particle dynamics in physical systems (He et al. 2018). QPT is a recently introduced framework for developing probabilistic and dynamic systems (She et al. 2021; He et al. 2018; Busemeyer and Wang 2018). The QT elucidates two facets of decision-making issues: the context of individuals' judgements or decisions parallels interference in QT (She et al. 2021; Anshu et al. 2020; Basieva et al. 2019; Broekaert et al. 2020); quantum entanglement and cognitive events lack compositional characteristics (She et al. 2021; Busemeyer and Bruza 2012). Numerous decisions, based on QT contradict human intuition and the results derived from CPT, as QPT possesses the capability to uncover concealed interference and offers significant advantages in elucidating the violations of the sure thing principle that remain inexplicable within the framework of CPT (She et al. 2021; He et al. 2018; Moreira and Wichert 2018; Al-Nowaihi and Dhami 2017; Chen et al. 2020; He and Jiang 2018). Khrennikov and Haven (2009) and Khrennikov (2019) introduced a quantum-concerning method by including an interference value to elucidate the rationale for violating the sure thing principle. Moreira and Wichert (2016) proposed the QSBN, a component of QPT, to elucidate the infringement of the sure thing principle in multi-attribute classification DM issues. In contrast to conventional theories and prior conjectures on QP, it can deduce the outcome of an event in the gaming domain and dynamically forecast the interference parameter based on varying contexts (Liu et al. 2016). Atmanspacher et al. (2020) presented scientific justifications for linking DM with QT, prompting an increasing number of researchers to adopt QT as a framework for elucidating complex findings and persistent issues in the domain of DM.

The evaluation procedure of MCGDM issues may align with the “superposition state" in QT (Moreira and Wichert 2018). The DM process parallels the quantum collapse process. QPT is more appropriate for MCGDM issues (He et al. 2018). The interference value is crucial in addressing MCGDM difficulties with QPT (She et al. 2021). The interference value delineates the angle between the two quantum spaces and the impact of the interaction between the two experts on the outcome. The interference value fluctuates unpredictably and it is challenging to quantify during the DM process (She et al. 2021). He et al. (2018) introduced a quantum framework to model subjectivity arising from the interference of expert’s beliefs during the DM process. They computed the differential value between the phase angles of two experts using the similarity heuristic method, where the phases signify the subjective belief attitudes regarding experts’ independence. This differential value is a crucial parameter for assessing the interference effect between the two experts. Cai et al. (2022) developed a decision framework that incorporates superposition and interference effects in GDM inside social networks. Gao and Deng (2023) introduced an inferable dynamic Markov DM model to objectively assess and ascertain the importance of interference effects in the DM process. Huang et al. (2019) employed belief entropy to assess total uncertainty, using this uncertainty in place of cosine values to determine final alternative preferences. Given that the hypothetical information in the aforementioned research is challenging to apply to real-world issues, and the absence of sufficient rigour in merely substituting all cosines with total uncertainty, She et al. (2021) developed an MCGDM model that incorporates the interference effects among attributes using a QSBN and belief entropy. They proposed a method to compute the entropy measure between each pair of attributes and utilised the corresponding cosine entropy measure to determine the final preference. Wu et al. (2022) established a quantum framework to represent interference effects in the linguistic distribution MCGDM process, utilising a linguistic distribution evaluation approach. Wu et al. (2021) created a behavioural MCGDM model that integrates the extended generalised TODIM technique with QDT. Jiang and Liu (2022) introduced a QSBN to investigate the interference effects of experts’ psychology in GDM, with experts’ weights being adjusted according to their contributions affected by inter-expert interference. Liu and Zhu (2022) utilised the quantum framework to revise the expert state transition in multistage GDM scenarios. As per Han and Liu (2023) an enhancement of the MCGDM approach utilising a QSBN accounts for belief interference. Mandal et al. (2024) utilise TODIM and PROMETHEE II methods in QDT to investigate quantum scenario-based MCGDM.

With increasing environmental awareness and businesses’ pursuit of sustainable growth, GSCM has become a key factor in corporate competitiveness (Qu et al. 2020; Konys 2019). Therefore, the development of GSCM has profoundly shaped business practices and marketing, making the selection of eco-friendly suppliers as essential for effective GSCM. This area has drawn significant interest from both scholars and practitioners. Organizations must consider economic, environmental, and social factors in their evaluations, making GSS as quintessential MCGDM challenge (Qian and Hou 2016). Researchers have recently developed various MCGDM methodologies and decision models to address GSS. For example, Deng et al. (2014) proposed the D-AHP technique, enhancing the traditional analytic hierarchy process (AHP) by incorporating D numbers to manage ambiguous information. With the help of the TOPSIS approach, Boran et al. (2009) proposed intuitionistic fuzzy information-based MCGDM. Integrating ELECTRE and TOPSIS approaches, Qu et al. (2020) analyzed a case study of a Chinese e-commerce company to identify suitable green suppliers. The above studies are effectively handling GSCM. In Ruan and Yan (2024), Ruan et al. proposed a model for GSS under Fermatean probabilistic hesitant fuzzy information. However, we observe that it has some drawbacks in studies of GSCM, which are listed in the following.

(1) The selection methods that are currently in use frequently fail to take into account the interdependencies that exist between certain characteristics and do not sufficiently handle the ambiguity and variability that are inherent in assessment data.
(2) Avoid expert confidence and doubt for providing their judgment.
(3) For aggregating individual assessments, it is assumed that decision-makers are independent and uninfluenced by one another.

1.1 Motivation

Contemporary research has rarely explored MCGDM models that incorporate group linguistic assessments, the psychological behaviors of experts, and the impacts of interference among their perspectives. Here we combine the MULTIMOORA approach with QDT within the framework of Pythagorean linguistic assessment-based MCGDM. The primary objectives of this study are as follows:

(1) Experts express their primary opinions based on secondary information, such as confidence and doubt levels. This allows them to communicate their decisions thoroughly and accurately, aligning with natural human thought processes. However, many studies overlook this aspect.
(2) In the MCGDM process, Wu et al. (2018) pointed out that for the determination of criterion weights, correlation coefficients among criteria must be considered. However, many studies overlook this aspect.
(3) In the MCGDM process, experts often exhibit bounded rationality, characterized by loss aversion and reference dependence. These behaviors are effectively captured by the MULTIMOORA technique. Therefore, extending the MULTIMOORA technique to the PLN context is essential, making the decision process as realistic as possible.
(4) Integrating individual evaluation, results can lead to interference among opinions. This interference can be accurately characterized and managed within the QPT framework. Therefore, exploring the impact of interference among different experts' perspectives in the MCGDM process using QPT is both insightful and practical.
(5) As mentioned in Harmati et al. (2022), we can quantify the quality of the expert's assessment using the appropriate entropy. However, many studies overlook this aspect, applying the existing expert weights determination approach to calculate the first

layer probability in the aggregation process of individual opinions under the QSBN framework.

In this paper, a quantum-based Pythagorean linguistic MCGDM model is proposed. This model takes into account the constrained rationality of experts as well as the impact of interference that occurs among different viewpoints. The following is a list of the key contributions that this paper highlights:

(1) We provide the correlation coefficients among criteria-based criterion weight determinations.
(2) We suggest a new way, based on study in (Harmati et al. 2022) to figure out the first layer probability in the QSBN framework by using entropy. This is part of the process of collecting individual opinions.
(3) An improved PLI-based MULTIMOORA approach accounts for experts' limited rationality, less time, high stability, simple calculations, and more conciseness and also it resolves some paradoxes of the traditional MULTIMOORA approach. We use this approach to determine the second layer probability in the process of aggregating individual opinions within the QSBN framework.
(4) A QDT-based method for combining expert opinion results, which helps us for better understanding of human psychology through the effects of interference.
(5) A case study demonstrating the effectiveness of our approach in the GSCM. Comparative and sensitivity analyses show the flexibility and robustness of our method.

The structure of the paper is as follows: In Sect. 2, recall some basic definitions and the background of LTSs, the PLS, QPT, QSBN, and Deng entropy, respectively. The proposed MCGDM model is discussed in Sect. 3. This section presents the correlation coefficient of PLNs, which subsequently guides the calculation of criteria weights. This section suggests a new way to figure out the first-layer probability using the entropy method and the second-layer probability using the MULTIMOORA method. A case study for the GSCM with comparative analysis is provided in Sect. 4. Section 5 concludes our study.

The meaning of the acronyms and abbreviations used in the article is explained in Table 1.

2 Preliminaries

Here we recall some related definitions and results that are being used in this paper.

2.1 LTS

Here we assume an LTS is: $\mathscr{L} = \{\ell_\lambda \mid \lambda = -\iota, \cdots, -1, 0, 1, \cdots, \iota\}$ with cardinality $2\lambda + 1$ and holds the following (Xu 2005):

(1) $\ell_{\lambda_1} \geq \ell_{\lambda_2}$ if and only if $\lambda_1 \geq \lambda_2$.
(2) $neg(\ell_{\lambda_1}) = \ell_{-\lambda_1}$.
(3) $\max(\ell_{\lambda_1}, \ell_{\lambda_2}) = \ell_{\lambda_1}$ if $\ell_{\lambda_1} \geq \ell_{\lambda_2}$.
(4) $\min(\ell_{\lambda_1}, \ell_{\lambda_2}) = \ell_{\lambda_1}$ if $\ell_{\lambda_1} \leq \ell_{\lambda_2}$.

Table 1 List of abbreviations

Abbreviation	Full Form
DM	Decision-making
GDM	Group decision making
MCGDM	Multicriteria group decision-making
LT	Linguistic term
LI	Linguistic information
LTS	Linguistic term set
HFLTS	Hesitant fuzzy linguistic term set
PLTS	Probabilistic linguistic term set
PLS	Pythagorean linguistic set
PLWA	Pythagorean linguistic weighted averaging
PLOWA	Pythagorean linguistic ordered weighted averaging
PLWG	Pythagorean linguistic weighted geometric
PLOWG	Pythagorean linguistic ordered weighted geometric
PLN	Pythagorean linguistic number
PLI	Pythagorean linguistic information
RS	Ratio system
RP	Reference point
FMF	Full multiplicative form
PT	Probability theory
CP	Classical probability
CPT	Classical probability theory
HS	Hilbert space
QP	Quantum probability
QPT	Quantum probability theory
QT	Quantum theory
BN	Bayesian network
QSBN	Quantum-scenario-based Bayesian network
QDT	Quantum decision theory
GSCM	Green supply chain management
DAG	Directed acyclic graph
MORV	Maximal objective reference vector

By Ref. (Xu 2005), the continuous LTS can be represented as: $\overline{\mathscr{L}} = \{\ell_{-t} \leq \ell_{\lambda} \leq \ell_{t} \mid \lambda \in [-t, t]\}$, $t > \iota$.

The subscript index of any $\ell_{\lambda} \in \overline{\mathscr{L}}$, represented by the following functions:

$$\mathscr{I} : \overline{\mathscr{L}} \to [-t, t],\ \mathscr{I}(\ell_{\lambda}) = \lambda;\ \mathscr{I}^{-1} : [-t, t] \to \overline{\mathscr{L}},\ \mathscr{I}^{-1}(\lambda) = \ell_{\lambda}.$$

Let $\mathscr{F}$ be the linguistic conversation function for any LT $L \in \overline{\mathscr{L}}$, which is defined in the following way:

$$\mathscr{F} : L \to [0, 1],\ \mathscr{F}(L) = \left\{ \frac{\lambda}{2t} + \frac{1}{2} \mid \lambda \in [-t, t] \right\} = \kappa.$$

2.2 PLS

Definition 2.1 (Liu et al. 2020; Mandal et al. 2020) A PLS denoted by $\mathscr{P}$ in an alternative set X for continuous LTS $\overline{\mathscr{L}} = \{\ell_{-t} \leq \ell_\lambda \leq \ell_t \mid \lambda \in [-t, t]\}$ is represented in the following way:

$$\mathscr{P} = \left\{ \left\langle \ell_{\theta(x)}, \left(\mu_{\mathscr{P}(x)}, \nu_{\mathscr{P}(x)}\right) \right\rangle \mid x \in X \right\}, \tag{1}$$

where $\ell_{\theta(x)} \in \overline{\mathscr{L}}$, $\mu_{\mathscr{P}} : X \to [0,1]$, and $\nu_{\mathscr{P}} : X \to [0,1]$ are the membership and non-membership degree of x to $\ell_{\theta(x)}$ such that $\mu^2_{\mathscr{P}}(x) + \nu^2_{\mathscr{P}}(x) \leq 1$.
In Definition 2.1, the element in $\mathscr{P}$ is called PLN, and is denoted by $\wp = \left\langle \ell_{\theta(\wp)}, (\mu(\wp), \nu(\wp)) \right\rangle$ with $\ell_{\theta(\wp)} \in \overline{\mathscr{L}}$, $\mu(\wp), \nu(\wp) \in [0,1]$, $\mu^2(\wp) + \nu^2(\wp) \leq 1$.

Remark 2.1 In Definition 2.1, if $\mu_{\mathscr{P}(x)} = 1$, then the PLS presented in equation (1) cab be degenerate is $\mathscr{P} = \left\{ \left\langle \ell_{\theta(x)}, (1, 0) \right\rangle \mid x \in X \right\}$. Consequently, the membership degree of all LTs in Definition 2.1 is equal to 1. Nonetheless, in certain circumstances, it may be feasible, and the expert may be uncertain of his or her language preferences with a membership degree of 1, as the expert does not possess complete understanding of the issue. This indicates that, in certain circumstances, the expert may lack complete confidence in their judgement due to the intricacy of the issue or their own limitations of knowledge. In this context, a PLN is very appropriate for an expert to articulate their alternatives. When an expert evaluates two alternatives based on a PLN, the LTs reflect the preferred degree or intensity of one alternative relative to the other. The expert expresses their confidence level through a membership grade function, termed self-confidence level, and their level of doubt through a nonmembership grade function, referred to as self-doubting level. Thus, a PLN embodies not only the qualitative preferences of specialists but also their levels of self-confidence and self-doubt, reflecting their knowledge level.

Definition 2.2 (Mandal et al. 2020) Let $\wp = \left\langle \ell_{\theta(\wp)}, (\mu(\wp), \nu(\wp)) \right\rangle$ be an PLN, then the expected value of $\wp$ is denoted by Ξ and defined in the following way:

$$\Xi(\wp) = \ell_{\frac{\theta(\wp) \times \left(\mu^2(\wp) + 1 - \nu^2(\wp)\right)}{2}}. \tag{2}$$

Definition 2.3 (Wang et al. 2019) Let $\wp_1 = \left\langle \ell_{\theta(\wp_1)}, (\mu(\wp_1), \nu(\wp_1)) \right\rangle$ and $\wp_2 = \left\langle \ell_{\theta(\wp_2)}, (\mu(\wp_2), \nu(\wp_2)) \right\rangle$ be two PLNs. Then the distance between them is defined in the following way:

$$\begin{aligned} d(\wp_1, \wp_2) = \frac{1}{2} \Big| &\left(1 + \mu^2(\wp_1) - \nu^2(\wp_1)\right) * \mathscr{F}\left(\ell_{\theta(\wp_1)}\right) \\ &- \left(1 + \mu^2(\wp_2) - \nu^2(\wp_2)\right) * \mathscr{F}\left(\ell_{\theta(\wp_2)}\right) \Big|. \end{aligned} \tag{3}$$

2.3 HS and QPT

The HS is a vector space of complex numbers covered by a collection of orthogonal basis vectors $\mathscr{H} = \{|\mathscr{A}\rangle, |\mathscr{B}\rangle\}$ (refer to Fig. 1a). Events in a HS are represented by complex numbers. Each event comprises two dimensions: the actual component and the imagined component (He et al. 2018).

QPT is a novel theory for the construction of probabilistic and dynamic systems, as thoroughly elucidated in Busemeyer and Bruza (Busemeyer and Bruza 2012). In QPT, events are represented as subspaces of the fundamental set $\mathscr{H}$ rather than as sets. Events may be mutually exclusive, or they may represent an intersection or union of fundamental states. An event $|\mathscr{S}\rangle$ is defined as a superposition of all fundamental states. In the presence of two fundamental states (refer to Fig. 1b): (i) complete belief in $\mathscr{A}$ (represented as $|\mathscr{A}\rangle$) and (ii) complete belief in $\mathscr{B}$ (represented as $|\mathscr{B}\rangle$), $|\mathscr{S}\rangle$ can be expressed as

$$|\mathscr{S}\rangle = \varphi_{\mathscr{A}} e^{i\zeta_{\mathscr{A}}} |\mathscr{A}\rangle + \varphi_{\mathscr{B}} e^{i\zeta_{\mathscr{B}}} |\mathscr{B}\rangle, \ |\mathscr{A}\rangle = (1,0), |\mathscr{B}\rangle = (0,1), \tag{4}$$

where the probability amplitudes associated with the states $|\mathscr{A}\rangle$ and $|\mathscr{B}\rangle$ are represented by $\varphi_{\mathscr{A}} e^{i\zeta_{\mathscr{A}}}$ and $\varphi_{\mathscr{B}} e^{i\zeta_{\mathscr{B}}}$, respectively. The probability amplitude quantifies the amplitude of a wave through a complex number, whereas the word $e^{i\zeta}$ represents the phase of the amplitude.

Born's rule establishes a connection between an amplitude and classical probability (CP). Let there be n events $\mathscr{V}_i (i = 1, 2, \cdots, n)$. The probability of each occurrence is denoted as $Pr(\mathscr{V}_i)$, where $i = 1, 2, \cdots, n$, and it is represented as

$$Pr(\mathscr{V}_i) = \left| \varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}} \right|^2, \ i = 1, 2, \cdots, n, \tag{5}$$

where $\varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}}$ represents the amplitude of probability associated with the state $|\mathscr{V}_i\rangle$.

It is necessary for the total of the squared magnitudes of each amplitude of all occurrences to equal 1, that is, $Pr(\mathscr{V}_i) = 1$, that is,

$$\begin{aligned} &\sum_{i=1}^{n} \left| \varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}} \right|^2 = 1 \\ &\sum_{i=1}^{n} \varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}} \left(\varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}} \right)^* = \varphi_{\mathscr{V}_i} e^{i\zeta_{\mathscr{V}_i}} \cdot \varphi_{\mathscr{V}_i} e^{-i\zeta_{\mathscr{V}_i}} = \sum_{i=1}^{n} \varphi_{\mathscr{V}_i}^2 = 1. \end{aligned} \tag{6}$$

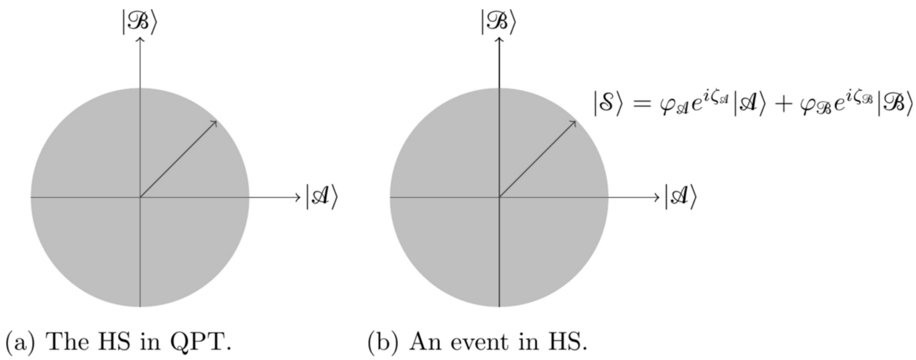

Fig. 1 Two-dimensional HS and quantum probability and an associated event

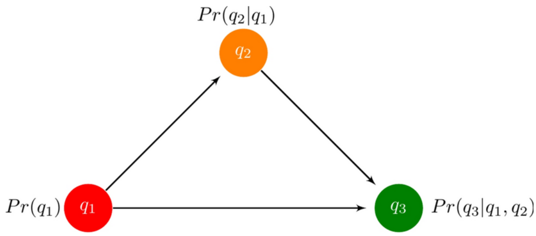

Fig. 2 The classical BN

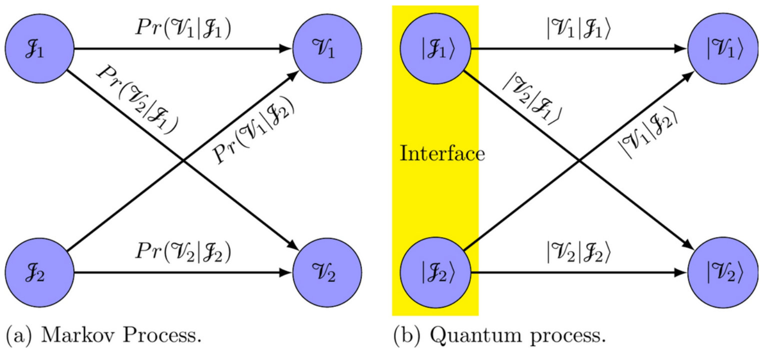

Fig. 3 DM in a Markov and quantum process

Consequently, $\varphi_{\mathscr{V}_i}$ in an amplitude probability may be articulated as a CP by extracting its square root, i.e., $\varphi_{\mathscr{V}_i} = \sqrt{Pr(\mathscr{V}_i)}$.

2.4 QSBN

A BN is often referred to as a belief network. A classical BN is a DAG consisting of nodes that represent variables and directed edges that connect these nodes. The directed edges between the nodes signify the reciprocal interaction among the nodes. The conditional probability quantifies the strength of the association (refer to Fig. 2).

A DM process may be represented as a Markov process (He et al. 2018; Busemeyer et al. 2009) according to traditional Bayesian paradigm (See Fig. 3a). The aggregate of the individual probabilities of each path is the probability of a final state. This perspective of all experts is regarded independently.

Let $\Omega = (I, E)$ be a DAG, where I signifies the set of all nodes and E denotes the set of directed edges. Additionally, let $\mathscr{V}_i$ $(i = 1, 2, \cdots, n)$ represent a random variable association with node i in the DAG. Consider a committee including two experts ($\mathscr{J}_1$ and $\mathscr{J}_2$) who provide assessments on the possibility of $\mathscr{V}_i$. Therefore, the joint probability of $\mathscr{V}_i$ can be calculated in the following way:

$$\begin{aligned} Pr(\mathscr{V}_i) &= Pr(\mathscr{V}_i \mathscr{J}_1) + Pr(\mathscr{V}_i \mathscr{J}_2) = Pr(\mathscr{V}_i|\mathscr{J}_1)Pr(\mathscr{J}_1) + Pr(\mathscr{V}_i|\mathscr{J}_2)Pr(\mathscr{J}_2) \\ &= \sum_{k=1}^{2} Pr(\mathscr{V}_i \mathscr{J}_k)Pr(\mathscr{J}_k), \end{aligned} \tag{7}$$

where $Pr(\mathscr{J}_k)$ indicates the k expert probability, and $Pr\left(\mathscr{V}_i|\mathscr{J}_k\right)$ represents the conditional probability that articulates the joint probability of $\mathscr{V}_i$ $(i = 1, 2, \cdots, n)$ given k expert.

The DM process inside a quantum framework is illustrated in Fig. 3b. All occurrences constitute subspaces of a HS. The state is a superposition of many fundamental states, which may be represented as wave functions (She et al. 2021; He et al. 2018). Therefore, in quantum framework, the joint probability of $\mathscr{V}_i$ can be calculated in the following way:

$$\begin{aligned} Pr(\mathcal{V}_i) &= \left|\varphi_{\mathcal{J}_1} e^{i\zeta_{\mathcal{J}_1}} \varphi_{\mathcal{V}_i|\mathcal{J}_1} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_1}} + \varphi_{\mathcal{J}_2} e^{i\zeta_{\mathcal{J}_2}} \varphi_{\mathcal{V}_i|\mathcal{J}_2} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_2}}\right|^2 \\ &= \left|\varphi_{\mathcal{J}_1} e^{i\zeta_{\mathcal{J}_1}} \varphi_{\mathcal{V}_i|\mathcal{J}_1} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_1}} + \varphi_{\mathcal{J}_2} e^{i\zeta_{\mathcal{J}_2}} \varphi_{\mathcal{V}_i|\mathcal{J}_2} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_2}}\right| \\ &\quad \left|\varphi_{\mathcal{J}_1} e^{i\zeta_{\mathcal{J}_1}} \varphi_{\mathcal{V}_i|\mathcal{J}_1} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_1}} + \varphi_{\mathcal{J}_2} e^{i\zeta_{\mathcal{J}_2}} \varphi_{\mathcal{V}_i|\mathcal{J}_2} e^{i\zeta_{\mathcal{V}_i|\mathcal{J}_2}}\right|^* \\ &= \sum_{k=1}^{2} \left|\varphi_{\mathcal{J}_k} \varphi_{\mathcal{V}_i|\mathcal{J}_k}\right|^2 + 2\left|\varphi_{\mathcal{J}_1} \varphi_{\mathcal{V}_i|\mathcal{J}_1}\right| \left|\varphi_{\mathcal{J}_2} \varphi_{\mathcal{V}_i|\mathcal{J}_2}\right| \\ \cos(\zeta_1 - \zeta_2) &= \sum_{k=1}^{2} \left|\varphi_{\mathcal{J}_k} \varphi_{\mathcal{V}_i|\mathcal{J}_k}\right|^2 + interface(i), \end{aligned} \tag{8}$$

where $\zeta_1 = \zeta_{\mathscr{J}_1} + \zeta_{\mathscr{V}_i|\mathscr{J}_1}$, $\zeta_2 = \zeta_{\mathscr{J}_2} + \zeta_{\mathscr{V}_i|\mathscr{J}_2}$, $\cos(\zeta_1 - \zeta_2) = \frac{e^{i(\zeta_1-\zeta_2)}+e^{i(\zeta_2-\zeta_1)}}{2}$, $\varphi_{\mathscr{J}_k} = \sqrt{Pr(\mathscr{J}_k)}$, and $\varphi_{\mathscr{V}_i|\mathscr{J}_k} = \sqrt{Pr(\mathscr{V}_i|\mathscr{J}_k)}$.

The quantum framework takes the interference term $2\left|\varphi_{\mathscr{J}_1}\varphi_{\mathscr{V}_i|\mathscr{J}_1}\right|\left|\varphi_{\mathscr{J}_2}\varphi_{\mathscr{V}_i|\mathscr{J}_2}\right|\cos(\zeta_1 - \zeta_2)$ into account; otherwise, the quantum method will revert to its classical counterpart, as

$$\begin{aligned} Pr(\mathscr{V}_i) &= \left|\varphi_{\mathscr{J}_1} e^{i\zeta_{\mathscr{J}_1}} \varphi_{\mathscr{V}_i|\mathscr{J}_1} e^{i\zeta_{\mathscr{V}_i|\mathscr{J}_1}} + \varphi_{\mathscr{J}_2} e^{i\zeta_{\mathscr{J}_2}} \varphi_{\mathscr{V}_i|\mathscr{J}_2} e^{i\zeta_{\mathscr{V}_i|\mathscr{J}_2}}\right|^2 \\ &= \sum_{k=1}^{2} \left|\varphi_{\mathscr{J}_k} \varphi_{\mathscr{V}_i|\mathscr{J}_k}\right|^2 = \sum_{k=1}^{2} Pr(\mathscr{V}_i|\mathscr{J}_k) Pr(\mathscr{J}_k). \end{aligned} \tag{9}$$

2.5 Deng entropy

Let $\hbar$ be a basic probability assignment that is specified on the frame of discerning Θ. Then by Ref. (Deng 2016), the Deng entropy is:

$$\mathscr{E}(\hbar) = -\sum_{Y \subseteq \Theta} \hbar(Y) \log_2 \left[\frac{\hbar(Y)}{2^{|Y|} - 1}\right], \tag{10}$$

where the cardinality of Y is $|Y|$.

3 The proposed MCGDM under PLI with MULTIMOORA and quantum scenario-based frameworks

This section describes the new approach of quantum cognition MCGDM based on the MULTIMOORA method with PLI. The method evaluates multiple competing alternatives in the presence of diverse and conflicting criteria according to RS, RP, and FMF approaches. Initially, each expert delivers their assessment via PLNs utilising established LTSs. The personal assessment outcome is thereafter computed with an augmented generalised PLN MULTIMOORA methodology. Finally, a quantum-based aggregation method is provided to examine the effects of interference among the evaluation results of experts.

3.1 Statement of the problem

In a MCGDM situation, a team of q experts $\{\mathscr{J}_1, \mathscr{J}_2, \cdots, \mathscr{J}_q\}$ provides evaluations for n alternatives $\{\mathscr{V}_1, \mathscr{V}_2, \cdots, \mathscr{V}_n\}$ concerning m criteria $\{\mathscr{C}_1, \mathscr{C}_2, \cdots, \mathscr{C}_m\}$. Since the experts comes from different fields, thus, different experts provides different opinions. Therefore, the different experts have different weights, and the importance of criterion in eye of different experts are different. In view of this, let $\{\mathscr{W}_1^k, \mathscr{W}_2^k, \cdots, \mathscr{W}_m^k\}$ be a set of criterion weights computed from $\mathscr{J}_k (k = 1, 2, \cdots, q)$ expert opinion judgement matrix with $\mathscr{W}_j^k \in [0, 1]$, and $\sum_{j=1}^{k} \mathscr{W}_j^k = 1$. In this connection, we encourage the experts for providing their opinions using PLNs, in the form of following decision matrix:

$$\mathcal{P}^k = (p_{ij}^k)_{nm} = \begin{matrix} & \mathscr{C}_1 & \mathscr{C}_2 & \cdots & \mathscr{C}_m \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \cdots \\ \mathscr{V}_n \end{matrix} \begin{bmatrix} p_{11}^k & p_{12}^k & \cdots & p_{1m}^k \\ p_{21}^k & p_{22}^k & \cdots & p_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^k & p_{n2}^k & \cdots & p_{nm}^k \end{bmatrix}, \tag{11}$$

where $p_{ij}^k = \left\langle \ell_{\theta_{ij}^k}, (\mu_{ij}^k, \nu_{ij}^k) \right\rangle$ is a PLN, called PLI of k-th expert $\mathscr{J}_k$ for the alternative $\mathscr{V}_i$ concerning the criteria $\mathscr{C}_j$.

Our objective is to consolidate the perspectives with the form of Eq. (11) of q number of experts and establish a hierarchy of preferences among i number of alternatives.

3.2 Determining the criterion weights

The coefficient of correlations is used to generate the objective criteria weights, which eliminates the possibility of bias caused by strongly linked qualities as well as the effect of the subjective preferences of experts. Most objective criteria weighting approaches are based on the "contrast intensity" of each criterion. Some examples of these methods are the standard deviation and the multiple objective programming model. These methods offer a larger weight to the attribute with a broad range of performance alternatives. Due to the fact that there are obvious disparities between the alternatives, a question has been asked regarding whether or not the characteristic inherently has a large impact on the results. However, it is not reasonable to assign a high weight to the criteria because it would be an overstatement of its importance.

The independence of criteria is a crucial need for MCGDM. Nonetheless, several important traits are present in actual MCGDM issues. Typically, we consolidate the dependent qualities or eliminate them directly. The direct removal of dependent qualities is arbitrary and significantly results in the loss of crucial information. Consequently, a minimal weight is an appropriate criterion for the traits with elevated coefficients. By employing this method, we will circumvent the irrational outcomes produced by the strongly linked qualities and save significantly more important information. Consequently, we utilise the coefficient of correlations among characteristics to obtain the objective criteria weights. In this connection, in the following, we introduce the coefficient of correlations between the two PLSs.

Definition 3.1 Let $\mathscr{P}_1 = \{\mathscr{p}_{1j} \mid j = 1, 2, \cdots, m\}$ and $\mathscr{P}_2 = \{\mathscr{p}_{2j} \mid j = 1, 2, \cdots, m\}$ be two PLSscontains$m$PLNs$\mathscr{p}_{1j} = \langle \mathscr{h}_{\theta(\mathscr{p}_{1j})}, (\mu(\mathscr{p}_{1j}), \nu(\mathscr{p}_{1j})) \rangle$and$\mathscr{p}_{2j} = \langle \mathscr{h}_{\theta(\mathscr{p}_{2j})}, (\mu(\mathscr{p}_{2j}), \nu(\mathscr{p}_{2j})) \rangle$. Then the coefficient of correlations $\mathscr{R}(\mathscr{P}_1, \mathscr{P}_1)$ is defined according to the Pearson coefficient of correlations in the following way:

$$\mathscr{R}(\mathscr{P}_1, \mathscr{P}_2) = \frac{\sum_{j=1}^{m} \left(\left(\frac{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_{1j})}{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_1^-)} - \frac{1}{m} \sum_{j=1}^{m} \frac{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_{1j})}{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_1^-)} \right) * \left(\frac{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_{2j})}{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_2^-)} - \frac{1}{m} \sum_{j=1}^{m} \frac{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_{2j})}{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_2^-)} \right) \right)}{\sqrt{\sum_{j=1}^{m} \left(\frac{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_{1j})}{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_1^-)} - \frac{1}{m} \sum_{j=1}^{m} \frac{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_{1j})}{\mathscr{d}(\mathscr{p}_1^+, \mathscr{p}_1^-)} \right)^2} * \sqrt{\sum_{j=1}^{m} \left(\frac{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_{2j})}{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_2^-)} - \frac{1}{m} \sum_{j=1}^{m} \frac{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_{2j})}{\mathscr{d}(\mathscr{p}_2^+, \mathscr{p}_2^-)} \right)^2}}, \tag{12}$$

where $\mathscr{p}_1^+ = \langle \max_j \mathscr{h}_{\theta(\mathscr{p}_{1j})}, (\max_j \mu(\mathscr{p}_{1j}), \min_j \nu(\mathscr{p}_{1j})) \rangle$, $\mathscr{p}_1^- = \langle \min_j \mathscr{h}_{\theta(\mathscr{p}_{1j})}, (\min_j \mu(\mathscr{p}_{1j}), \max_j \nu(\mathscr{p}_{1j})) \rangle$, and similarly computing $\mathscr{p}_2^+$ and $\mathscr{p}_2^-$.

Theorem 3.1 The coefficient of correlations between two PLSs defined in Eq. (12), holds the following properties:

(1) $\mathscr{R}(\mathscr{P}_1, \mathscr{P}_2) = \mathscr{R}(\mathscr{P}_2, \mathscr{P}_1)$.
(2) $\mathscr{R}(\mathscr{P}_1, \mathscr{P}_1) = 1$.
(3) $-1 \leq \mathscr{R}(\mathscr{P}_1, \mathscr{P}_2) \leq 1$.

Proof (1) and (2) are obvious. By the Cauchy-Schwarz inequality item (3) is straightforward. □

Now, based on Definition 3.1 in the following we provide the computation process of weights of criteria from decision matrix 11. The weights of criterion $\{\mathscr{W}_1^k, \mathscr{W}_2^k, \cdots, \mathscr{W}_m^k\}$ for expert $\mathscr{J}_k (k = 1, 2, \cdots, q)$ are computed in the following way:

$$\mathscr{W}_j^k = \frac{\sum_{s=1}^{m} (1 - \mathscr{R}_{js})}{\sum_{j=1}^{m} \left(\sum_{s=1}^{m} (1 - \mathscr{R}_{js}) \right)}, j = 1, 2, \cdots, m, \tag{13}$$

where

$$\mathscr{R}_{js} = \frac{\sum_{i=1}^{n}\left(\left(\frac{\mathscr{d}(\wp_j^{+,k},\wp_{ij}^k)}{\mathscr{d}(\wp_j^{+,k},\wp_j^{-,k})} - \frac{1}{n}\sum_{i=1}^{n}\frac{\mathscr{d}(\wp_j^{+,k},\wp_{ij}^k)}{\mathscr{d}(\wp_j^{+,k},\wp_j^{-,k})}\right) * \left(\frac{\mathscr{d}(\wp_s^{+,k},\wp_{is}^k)}{\mathscr{d}(\wp_s^{+,k},\wp_j^{-,k})} - \frac{1}{n}\sum_{i=1}^{n}\frac{\mathscr{d}(\wp_s^{+,k},\wp_{is}^k)}{\mathscr{d}(\wp_s^{+,k},\wp_s^{-,k})}\right)\right)}{\sqrt{\left(\frac{\mathscr{d}(\wp_j^{+,k},\wp_{ij}^k)}{\mathscr{d}(\wp_j^{+,k},\wp_j^{-,k})} - \frac{1}{n}\sum_{i=1}^{n}\frac{\mathscr{d}(\wp_j^{+,k},\wp_{ij}^k)}{\mathscr{d}(\wp_j^{+,k},\wp_j^{-,k})}\right)^2} * \sqrt{\left(\frac{\mathscr{d}(\wp_s^{+,k},\wp_{is}^k)}{\mathscr{d}(\wp_s^{+,k},\wp_j^{-,k})} - \frac{1}{n}\sum_{i=1}^{n}\frac{\mathscr{d}(\wp_s^{+,k},\wp_{is}^k)}{\mathscr{d}(\wp_s^{+,k},\wp_s^{-,k})}\right)^2}}. \quad (14)$$

Let $\mathcal{B}$ and $\mathcal{C}$ be means the sets of benefit and cost criterion. Then, in Eq. (14), $\wp_j^{+,k} = \begin{cases} \max_i\{\wp_{ij}^k\}, & \text{for } \mathscr{C}_j \in \mathcal{B} \\ \min_i\{\wp_{ij}^k\}, & \text{for } \mathscr{C}_j \in \mathcal{C} \end{cases}$, $\wp_j^{-,k} = \begin{cases} \min_i\{\wp_{ij}^k\}, & \text{for } \mathscr{C}_j \in \mathcal{B} \\ \max_i\{\wp_{ij}^k\}, & \text{for } \mathscr{C}_j \in \mathcal{C} \end{cases}$; and similarly we can computed that of $\wp_s^{+,k}$ and $\wp_s^{-,k}$.

3.3 The PLN MULTIMOORA method

Among a variety of MCGDM methods, MULTIMOORA is a significant MCGDM technique that combines three subordinate preference values of the alternatives obtained by the fully compensatory, non-compensatory and incompletely compensatory models entitled RS, RP, and FMF approaches. This approaches are extended in the setting of PLNs, which are described here. For this, first we normalize the decision matrix 11. Let $\mathcal{A}^k = (\mathscr{a}_{ij}^k)_{nm}$ be the normalized decision matrix, where

$$\mathscr{a}_{ij}^k = \begin{cases} \frac{\mathscr{F}(\Xi(\wp_{ij}^k))}{\sum_{i=1}^{n}\mathscr{F}(\Xi(\wp_{ij}^k))}, & \text{for } \mathscr{C}_j \in \mathcal{B} \\ 1 - \frac{\mathscr{F}(\Xi(\wp_{ij}^k))}{\sum_{i=1}^{n}\mathscr{F}(\Xi(\wp_{ij}^k))}, & \text{for } \mathscr{C}_j \in \mathcal{C} \end{cases}. \quad (15)$$

Now here we extend the MULTIMOORA method to the PLN MULTIMOORA method. This is included the following steps:

Step 1: The PLN RS approach

In this step, the overall utility is derived from the normalized decision matrix $\mathcal{A}^k = (\mathscr{a}_{ij}^k)_{nm}$ of each expert $\mathscr{J}_k$ for ith alternative $\mathscr{V}_i$, which is expressed as follows:

$$\Psi_1^k(\mathscr{V}_i) = \left|\sum_{j\in\mathcal{B}}\mathscr{W}_j^k\mathscr{a}_{ij}^k - \sum_{j\in\mathcal{C}}\mathscr{W}_j^k\mathscr{a}_{ij}^k\right|. \quad (16)$$

According to the greater value of $\Psi_1^k(\mathscr{V}_i)$ assigned a higher rank of alternative, which is denoted by $\mathscr{a}_1^k(\mathscr{V}_i)$ for expert $\mathscr{J}_k$.

Step 2: The PLN RP approach

Let $\phi^k = (\phi_1^k, \phi_2^k, \cdots, \phi_j^k, \cdots, \phi_m^k)$ be the MORV found from the normalized decision matrix $\mathcal{A}^k = (\mathscr{a}_{ij}^k)_{nm}$ of each expert $\mathscr{J}_k$ according to two following principles:

(I) For the benefit criteria type, we have $\phi_j^k = \max_{i=1}^n \mathscr{a}_{ij}^k$, $\mathscr{C}_j \in \mathcal{B}$.

(II) For the cost criteria type, we have $\phi_j^k = \min_{i=1}^n \mathscr{a}_{ij}^k$, $\mathscr{C}_j \in \mathcal{C}$.

Now from the MORV, the collective decision of the ith alternative $\mathscr{V}_i$ of each expert $\mathscr{J}_k$ calculate in the following way:

$$\Psi_2^k\left(\mathscr{V}_i\right)=\max_j\left|\mathscr{W}_j^k\mathscr{a}_{ij}^k-\mathscr{W}_j^k\phi_j^k\right|. \tag{17}$$

According to the lower value of $\Psi_1^k\left(\mathscr{V}_i\right)$ assigned a higher rank of alternative, which is denoted by $\mathscr{o}_2^k(\mathscr{V}_i)$ for expert $\mathscr{J}_k$.

Step 3: The PLN FMF approach

The overall utility is derived from the normalized decision matrix $\mathcal{A}^k=(\mathscr{a}_{ij}^k)_{nm}$ of each expert $\mathscr{J}_k$ for the ith alternative $\mathscr{V}_i$, which is expressed as follows:

$$\Psi_3^k\left(\mathscr{V}_i\right)=\frac{\Upsilon_i^{k,+}}{\Upsilon_i^{k,-}}, \tag{18}$$

where $\Upsilon_i^{k,+}=\prod_{j\in\mathcal{B}}\mathscr{W}_j^k\mathscr{a}_{ij}^k$ and $\Upsilon_i^{k,+}=\prod_{j\in\mathcal{C}}\mathscr{W}_j^k\mathscr{a}_{ij}^k$.

According to the greater value of $\Psi_3^k\left(\mathscr{V}_i\right)$ assigned a higher rank of alternative, which is denoted by $\mathscr{o}_3^k(\mathscr{V}_i)$ for expert $\mathscr{J}_k$.

Step 4: Borda rule-based final ranking approach

(I) By Ref. (Wu et al. 2018), we assume the PLN RS, PLN RP and PLN FMF approaches of each expert $\mathscr{J}_k$ are three attributes denoted by $PLN-RS(\xi_1^k)$, $PLN-RP(\xi_2^k)$, and $PLN-FMF(\xi_1^k)$. Therefore, each alternative for each expert $\mathscr{J}_k$ has three types of ranking values denoted by $\Psi_\tau^k(\mathscr{V}_i)(\tau=1,2,3)$ and three types of ranking orders denoted by $\mathscr{o}_\tau^k(\mathscr{V}_i)(\tau=1,2,3)$ with respect to the criterion ξ_1^k, ξ_2^k, and ξ_3^k. Thus, the final ranking values of each alternative as multi-criteria DM contains utility matrix $\mathscr{U}^k=\left(\Psi_\tau^k(\mathscr{V}_i)\right)_{n\times 3}$ (Presented in Table 2) and ranking matrix $\mathscr{O}^k=\left(\mathscr{o}_\tau^k(\mathscr{V}_i)\right)_{n\times 3}$ (presented in Table 3).

(II) The normalized utility matrix of $\mathscr{U}^k=\left(\Psi_\tau^k(\mathscr{V}_i)\right)_{n\times 3}$ is denoted by $\mathscr{N}\left(\mathscr{U}^k\right)=\left(\mathscr{N}\left(\Psi_\tau^k(\mathscr{V}_i)\right)\right)_{n\times 3}$, where

$$\mathscr{N}\left(\Psi_\tau^k(\mathscr{V}_i)\right)=\frac{\Psi_\tau^k(\mathscr{V}_i)}{\sum_{i=1}^n\Psi_\tau^k(\mathscr{V}_i)}. \tag{19}$$

(III) The index of alternative $\mathscr{V}_i$ for expert $\mathscr{J}_k$ is denoted by Γ_i^k and obtain in the following way:

$$\begin{aligned}\Gamma_i^k=&\left|\mathscr{N}\left(\Psi_1^k(\mathscr{V}_i)\right)\times\frac{n-\mathscr{o}_1^k(\mathscr{V}_i)}{n(n+1)/2}-\mathscr{N}\left(\Psi_2^k(\mathscr{V}_i)\right)\times\frac{\mathscr{o}_2^k(\mathscr{V}_i)}{n(n+1)/2}\right.\\&\left.+\mathscr{N}\left(\Psi_3^k(\mathscr{V}_i)\right)\times\frac{n+1-\mathscr{o}_3^k(\mathscr{V}_i)}{n(n+1)/2}\right|,(i=1,2,\cdots,n).\end{aligned} \tag{20}$$

Table 2 The utility matrix $\mathscr{U}^k=\left(\Psi_\tau^k(\mathscr{V}_i)\right)_{n\times 3}$

	$PLN-RS(\xi_1^k)$	$PLN-RP(\xi_2^k)$	$PLN-FMF(\xi_1^k)$
$\mathscr{V}_1$	$\Psi_1^k(\mathscr{V}_1)$	$\Psi_2^k(\mathscr{V}_1)$	$\Psi_1^k(\mathscr{V}_1)$
$\mathscr{V}_i$	$\Psi_1^k(\mathscr{V}_i)$	$\Psi_2^k(\mathscr{V}_i)$	$\Psi_1^k(\mathscr{V}_i)$
$\mathscr{V}_n$	$\Psi_1^k(\mathscr{V}_n)$	$\Psi_2^k(\mathscr{V}_n)$	$\Psi_1^k(\mathscr{V}_n)$

Table 3 The ranking matrix $\mathscr{O}^k = \left(c_\tau^k(\mathscr{V}_i)\right)_{n\times 3}$

	$PLN-RS(\xi_1^k)$	$PLN-RP(\xi_2^k)$	$PLN-FMF(\xi_1^k)$
$\mathscr{V}_1$	$c_1^k(\mathscr{V}_1)$	$c_2^k(\mathscr{V}_1)$	$c_1^k(\mathscr{V}_1)$
$\mathscr{V}_i$	$c_1^k(\mathscr{V}_i)$	$c_2^k(\mathscr{V}_i)$	$c_1^k(\mathscr{V}_i)$
$\mathscr{V}_n$	$c_1^k(\mathscr{V}_n)$	$c_2^k(\mathscr{V}_n)$	$c_1^k(\mathscr{V}_n)$

3.4 The aggregation process relies on quantum scenarios for expert evaluation

Once the enlarged PLN MULTIMOORA technique produces the outcomes of the experts' evaluations, an effective information regarding fusion tool is required to merge the various findings. This section uses a quantum-based aggregation mode to look at the consequences of expert opinions interfering with one another.

During this phase, we converted the pre-collected data into probabilities and built a BN for decision-making. This is because the quantum framework is a partial extension of the BN. Figure 4 illustrates a straightforward two-layer BN representation of the DM process in an MCGDM case. Therefore, according to Fig. 4, here we determine the first layer and second layer probabilities of alternatives.

3.4.1 Determination of first layer probabilities

Let the probability of first layer of expert $\mathscr{J}_k$ is $Pr(\mathscr{J}_k)(k=1,2,\cdots,q)$. Then we have the following steps for determination of $Pr(\mathscr{J}_k)$.

Step 1: Construct normalized weighted decision matrix, using equation (15) and the set of criterion weights $\{\mathscr{W}_1^k, \mathscr{W}_2^k, \cdots, \mathscr{W}_m^k\}$ of each expert $\mathscr{J}_k$, as:

$$\mathscr{N}_k = (\ell_{ij}^k)_{nm}, \;\; \ell_{ij}^k = \mathscr{W}_j^k a_{ij}^k, \tag{21}$$

where $i=1,2,\cdots,n; j=1,2,\cdots,m.$

Step 2: In Eq. (21), construct arithmetic mean of $\mathscr{N}_k$ is as:

$$\mathscr{N}_* = (\ell_{ij})_{nm}, \;\; \ell_{ij} = \frac{1}{q}\sum_{k=1}^{q} \ell_{ij}^k, \tag{22}$$

where $i=1,2,\cdots,n; j=1,2,\cdots,m.$

Step 3: In Eq. (21), the entropy of $\mathscr{N}_k$ is as:

$$Ent(\mathscr{N}_k) = \begin{cases} -\frac{1}{\ln(nm)}\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}^k \ln c_{ij}^k, & \text{if } c_{ij}^k \neq 0, \\ 0 \text{ if } c_{ij}^k = 0, \end{cases} \tag{23}$$

where $c_{ij}^k = \frac{\ell_{ij}^k}{\sum_{i=1}^{n}\sum_{j=1}^{m}\ell_{ij}^k}$.

Step 4: In Eq. (22), the entropy of $\mathscr{N}_k$ is as:

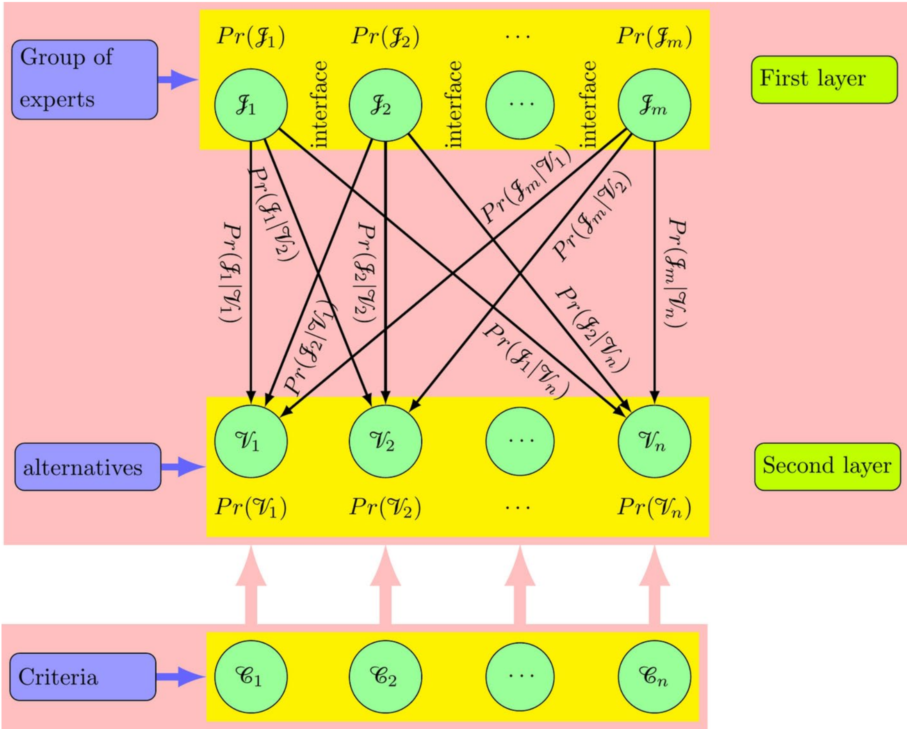

Fig. 4 The QSBN for combining alternative probabilities in a MCGDM issue

$$Ent(\mathscr{N}_*) = \begin{cases} -\frac{1}{\ln(nm)} \sum_{i=1}^{n} \sum_{j=1}^{m} \mathscr{e}_{ij}^{*} \ln \mathscr{e}_{ij}^{*}, \text{ if } \mathscr{e}_{ij}^{*} \neq 0, \\ 0 \text{ if } \mathscr{e}_{ij}^{*} = 0, \end{cases} \tag{24}$$

where $\mathscr{e}_{ij}^{*} = \frac{\mathscr{e}_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{m} \mathscr{e}_{ij}}$.

Step 5: From Eqs. (23) and (24), the proximity of relative f each expert $\mathscr{J}_k$ is as:

$$PR_k = \frac{Ent(\mathscr{N}_*)}{Ent(\mathscr{N}_*) + |Ent(\mathscr{N}_k) - Ent(\mathscr{N}_*)|}, \tag{25}$$

where $k = 1, 2, \cdots, q$.

Step 5: From Eq. (25), the Shannon entropy-based first layer probability of each expert $\mathscr{J}_k$ is as:

$$Pr(\mathscr{J}_k) = \frac{PR_k}{\sum_{k=1}^{q} PR_k}, \tag{26}$$

where $k = 1, 2, \cdots, q$.

3.4.2 Determination of second layer probabilities

Here we find out the second layer probability by PT. In this connection, according to the approach proposed in (He et al. 2018; He and Jiang 2018; Jiang and Liu 2022; Mandal et al. 2024), the index of alternative $\mathscr{V}_i$ of each expert $\mathscr{J}_k$ obtain in Eq. (20) of Sect. 3.3 are converted into conditional probabilities, as illustrated in Eq. (27), which connects experts and alternatives inside the BN.

$$Pr(\mathscr{V}_i \mid \mathscr{J}_k) = \frac{\Gamma_i^k}{\sum_{i=1}^{n} \Gamma_i^k}. \tag{27}$$

Therefore, the second layer probabilities of $\mathscr{V}_i$ is obtained as:

$$Pr(\mathscr{V}_i) = \sum_{k=1}^{q} Pr(\mathscr{J}_k) Pr(\mathscr{V}_i \mid \mathscr{J}_k). \tag{28}$$

Consequently, we may make determinations based on the $Pr(\mathscr{V}_i)$ values, as elevated probabilities, which signify a greater ranking. The conventional technique fails to include the interference effect, which may impact the final rankings. To resolve this issue, BNs are extended into a QSBN that effectively models a practical solution to genuine interference in MCGDM.

The CPT calculates the probability of alternatives using the total probability formula. Nonetheless, an interference effect among experts may influence the ranking outcomes of alternatives. Consequently, it is imperative to expand the BN to QPT for the purpose of modelling this impact named QSBN. As shown by the QPT and QSBN in Sects. 2.3 and 2.4, the probability of $\mathscr{V}_i$ in the second layer of the QPT and QSBN is derived using Eq. (29).

$$\begin{aligned} Pr(\mathscr{V}_i) &= \frac{1}{\Re} \left| \sum_{k=1}^{q} \varphi_{\mathscr{J}_k} \varphi_{\mathscr{V}_i \mid \mathscr{J}_k} e^{i\zeta_k} \right|^2 \\ &= \frac{1}{\Re} \left| \sum_{k=1}^{q} \sqrt{Pr(\mathscr{J}_k)} \sqrt{Pr(\mathscr{V}_i \mid \mathscr{J}_k)} e^{i\zeta_k} \right|^2, i = 1, 2, \cdots, n, \end{aligned} \tag{29}$$

where $\Re = \sum_{i=1}^{n} \left| \sum_{k=1}^{q} \varphi_{\mathcal{J}_k} \varphi_{\mathcal{V}_i \mid \mathcal{J}_k} e^{i\zeta_k} \right|^2 = \sum_{i=1}^{n} \left| \sum_{k=1}^{q} \sqrt{Pr(\mathcal{J}_k)} \sqrt{Pr(\mathcal{V}_i \mid \mathcal{J}_k)} e^{i\zeta_k} \right|^2.$

We get Eq. (30) from expanding Eq. (29).

$$\begin{aligned} Pr(\mathscr{V}_i) &= \frac{1}{\Re} \left[\sum_{k=1}^{q} Pr(\mathscr{J}_k) Pr(\mathscr{V}_i \mid \mathscr{J}_k) + 2 \sum_{k=1}^{q-1} \sum_{s=k+1}^{q} \sqrt{Pr(\mathscr{J}_k) Pr(\mathscr{V}_i \mid \mathscr{J}_k)} \cos(\zeta_k - \zeta_s) \right] \\ &= \frac{1}{\Re} \left[\sum_{k=1}^{q} Pr(\mathscr{J}_k) Pr(\mathscr{V}_i \mid \mathscr{J}_k) + interface(i) \right]. \end{aligned} \tag{30}$$

Remark 3.1 The probability of alternatives computed by Eq. (28) is known as CP of alternatives, and that of by Eq. (30) is known as QP.

3.4.3 Determination of the interface(i)

In Eq. (30), *interface*(i) is the interference effects between two experts, $\mathscr{J}_k$ and $\mathscr{J}_s$. $\cos(\zeta_k - \zeta_s)$ denotes the phase angle between two separate experts, $\mathscr{J}_k$ and $\mathscr{J}_s$, which signifies the connection vector between them. In existing studies (He et al. 2018; Moreira and Wichert 2016; Jiang and Liu 2022), the phase angle between two separate experts $\cos(\zeta_k - \zeta_s)$ is determined by the similarity heuristic-based cosine approach. However, Han and Liu (2023) and Mandal et al. (2024) indicated that in practical scenarios, the phase angle $\cos(\zeta_k - \zeta_s)$ is challenging to quantify. External factors may conflict or influence the subjective opinions of experts, leading to significant variability and erroneous outcomes. They suggested a Deng entropy-based belief approach for determining the phase angle $\cos(\zeta_k - \zeta_s)$. Being inspired by their studies, here we determine the phase angle $\cos(\zeta_k - \zeta_s)$ by using the Deng entropy-based belief approach. The steps are as follows.

Step 1: In QSBN displayed in Fig. 4, $\varpi = \frac{1}{nq}$ be the average probability. Then for alternative $\mathscr{V}_i$, the distance of the belief concerning ϖ is obtained in the following way:

$$\mathscr{M}_{iu} = \left| \sigma_{iu} + \frac{|\sigma_{iu} - \varpi| - |\varrho_{iu} - \varpi|}{|\sigma_{iu} - \varpi| + |\varrho_{iu} - \varpi|} \right|, i = 1, 2, \cdots, n; u = 1, 2, \cdots, \frac{q(q-1)}{2}, \tag{31}$$

where

$$\begin{cases} \sigma_{iu} = Pr(\mathscr{J}_k)Pr(\mathscr{V}_i|\mathscr{J}_k) \\ \varrho_{iu} = Pr(\mathscr{J}_s)Pr(\mathscr{V}_i|\mathscr{J}_s) \end{cases}, i = 1, 2, \cdots, n; k = 1, 2, \cdots, q-1; s = k+1, k+2, \cdots, \frac{q(q-1)}{2}.$$

In Eq. (31), u denotes the value allocated to the correlation among q experts; ϖ signifies the probability value assigned by each expert to each alternative; $\frac{q(q-1)}{2}$ is the total number of unique pairwise correspondences among q experts, excluding duplicates; $\mathscr{M}_{iu}$ indicates the degree of deviation from ϖ; $|\sigma_{iu} - \varpi|$ and $|\varrho_{iu} - \varpi|$ represent the magnitudes of the deviation degrees from ϖ concerning alternative $\mathscr{V}_i$. If $|\sigma_{iu} - \varpi| < |\varrho_{iu} - \varpi|$, then the positions of σ_{iu} and ϱ_{iu} are interchangeable.

Step 2: The Deng entropy of $\mathscr{M}_{iu}$ is calculated in the following way:

$$\Theta_{iu}(\mathscr{M}_{iu}) = -\mathscr{M}_{iu} \log_2 \frac{\mathscr{M}_{iu}}{2^m - 1}, i = 1, 2, \cdots, n; u = 1, 2, \cdots, \frac{q(q-1)}{2}. \tag{32}$$

Step 3: To ensure the value of $\cos(\zeta_k - \zeta_s)$ in $[-1, 1]$, Deng entropy is standardized in the following way:

$$\cos(\zeta_k - \zeta_s) = \beta_{iu}(\mathscr{M}_{iu}) = 2\frac{\Theta_{iu}(\mathscr{M}_{iu}) - \min\{\Theta_{iu}(\mathscr{M}_{iu})\}}{\max\{\Theta_{iu}(\mathscr{M}_{iu})\} - \min\{\Theta_{iu}(\mathscr{M}_{iu})\}} - 1, \tag{33}$$

where $i = 1, 2, \cdots, n; u = 1, 2, \cdots, \frac{q(q-1)}{2}$.

Step 4: Using Eq. (33), the *interface*(i) can be calculated in the following way:

$$interface(i) = 2\sum_{k=1}^{q-1}\sum_{s=k+1}^{q}\sqrt{Pr(\mathscr{J}_k)Pr(\mathscr{V}_i \mid \mathscr{J}_k)}\beta_{iu}(\mathscr{M}_{iu}), \tag{34}$$

where $i = 1, 2, \cdots, n; u = 1, 2, \cdots, \frac{q(q-1)}{2}$.

According to the proposed MCGDM under PLI with MULTIMOORA and quantum scenario-based frameworks in Sect. 3, in Algorithm 1 we summarized the steps.

Input: $\mathcal{P}^k = (\mathcal{p}_{ij}^k)_{nm}$, $\mathcal{p}_{ij}^k = \left\langle \ell_{\theta_{ij}^k}, (\mu_{ij}^k, \nu_{ij}^k) \right\rangle$, $(i = 1, 2, \cdots, n; j = 1, 2, \cdots, m; k = 1, 2, \cdots, q)$.
Output: Decision result.

Step 1: By equation (13), obtain the criteria weights $\mathscr{W}_1^k, \mathscr{W}_2^k, \cdots, \mathscr{W}_m^k$ of for each expert $\mathscr{J}_k$ $(k = 1, 2, \cdots, q)$.

Step 2: From equations (21) - (26), we can obtain the first layer probabilities $Pr(\mathscr{J}_k)$.

Step 3: Using equations (15) - (20), we can obtain the second layer probabilities $Pr(\mathscr{V}_i)$ by equation (28).

Step 4: From equations (31) - (34), we can obtain the $interface(i)$.

Step 5: According to equation (30), the overall probability $Pr(\mathscr{V}_i)$ for each alternative $\mathscr{V}_i$ is established by considering the inferential effects among the experts' viewpoints.

Step 6: The ultimate conclusion may be derived from the values of $Pr(\mathscr{V}_i)$.

Step 7: End.

Algorithm 1 Algorithm of the proposed MCGDM under PLI with MULTIMOORA and quantumscenario-based frameworks

4 A practical example with comparative analysis

This section presents a new approach to MCGDM that integrates QDT with PLI, based on the MULTIMOORA approach. Below, we present the detailed procedure for the combined MULTIMOORA approach.

4.1 A case study

With ongoing economic and social development, coupled with escalating global environmental challenges, many countries and regions are promoting the production of low-carbon for green lifestyles. Consumer demand for sustainable products which have been steadily increased. To enhance the appeal of green products, a growing number of companies are adopting environmentally sustainable and low-carbon strategies. GSCM has become a crucial approach for organizations to boost and achieve sustainable growth and competitiveness (Srivastava 2007)

In 1996, GSCM was included for the first time in the supply chain framework because of environmental protection as well as for achieving both economic and environmental benefits. It is also rapidly used for recycling disposal and waste management. GSCM is now extensively implemented in industries such as automotive, manufacturing, construction, and services, with a key emphasis on selecting suppliers who are environmentally responsible (Gupta et al. 2019). By aligning supplier selection with their production plans as well as development strategies, core enterprises can influence the sustainable practices of both up and downstream partners. This approach helps in meeting environmental objectives, enhancing economic performance, and improving the overall GSCM within the supply chain network. Consequently, green enterprises must adopt systematic, rational, and efficient methods for evaluating and selecting green supplier in corporate development (Wang et al. 2022). In recent years, driven by India's dual carbon goals, various companies have started to assess and select green suppliers. Now India is accelerating and promoting the green and low-carbon for the upgradation of traditional industries, and establishing a clear direction for GSCM. To achieve sustainable growth, supply chain management must thoroughly assess the green capabilities of potential suppliers, often requiring careful comparisons of key factors such as product quality, pricing, and service levels.

The sustainable manufacturing standards are essential for a new energy vehicle manufacturer to meet its developmental needs. After investigating and evaluating market sources, the manufacturer selects five contenders: $\{\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4, \mathscr{V}_5\}$. To identify the most suitable candidates, the manufacturer assembles an expert group for the evaluation of the supplier. After defining the assessment objectives and criteria for selecting green suppliers, the firm assembles a panel of five experts $(\mathscr{J}_1, \mathscr{J}_2, \mathscr{J}_3, \mathscr{J}_4, \mathscr{J}_5)$ with professional relevant backgrounds and expertise. This panel, comprises esteemed academics and industry professionals, who brings substantial knowledge in technological innovation and supply chain management. They gather relevant information and conduct a detailed analysis of the prospective organizations' for their development. Through in-depth discussions and the exchange of viewpoints, the panel reaches an agreement on four key evaluation indicators, guided by their collective judgment.

$\mathscr{C}_1$: Performance of price

Companies must evaluate suppliers based on product offerings, carbon emissions, transportation, recycling, and pricing performance. Consistent, competitive pricing is essential for maintaining profitability and optimizing supply chain efficiency. Shah et al. (2022) illustrated a mathematical model that is environmentally sustainable initiative which significantly influence corporate pricing strategies, thereby affecting sales success. Therefore, the pricing effectiveness of suitable green suppliers must align with business goals for sustainable growth.

$\mathscr{C}_2$: Quality of product

Not only are product factors essential for the market operations of both upstream and downstream companies, but also they are essential for the sustainable development of a green supply chain. Product factors are significant indications of a supplier's capacity to innovate environmental sustainability (Kar and Pani 2014). Product quality is an essential criterion that indicates not only the quality assurance framework of the supplier but also their capacity to manufacture low-carbon, ecologically sustainable products that are in line with the requirements of the green marketing industry. According to recent research, the manufacturing sector in developing countries places a special emphasis on the need to maintain product quality and adhere to delivery timetables. This realization highlights a particular approach that may be taken to enhance supply chain systems methodically.

$\mathscr{C}_3$: Social influence

For a selection of green suppliers for sustainable growth, firms must assess their social responsibility which includes environmental obligations, protection of labor rights, and adherence to social and ethical standards. These factors impact the environmental performance of the company's, green innovation capabilities, economic advantages, and social reputation. Furthermore, governance practices related to obligation towards society, based on the selection of suppliers, enhance the social and environmental performance of the core firm within the supply chain (Yadlapalli et al. 2017).

$\mathscr{C}_4$: Capability of service

In today's highly competitive economy, providers that offer superior service are more favored. Key metrics for evaluating a supplier's overall service capabilities include management engagement, service disposition, delivery punctuality, and market adaptability. The logistics quality and their development of an assessment are influenced by the service level for green suppliers noted in (Wang et al. 2017). Therefore, businesses must rigorously evaluate the capabilities of service of prospective suppliers throughout their process of selection.

4.1.1 Decision procedure

After defining the assessment indicators, we use our suggested MULTIMOORA method, which integrates QDT with PLI, to examine the GSCM MAGDM scenario. Here we offer the LTSs, $\overline{\mathscr{L}} = \{\ell_{-3}: \textit{very bad},\ \ell_{-2}: \textit{bad},\ \ell_{-1}: \textit{slightly bad},\ \ell_0: \textit{moderate},\ \ell_1: \textit{slightly good},\ \ell_2: \textit{good},\ \ell_3: \textit{very good}\}$ for the evaluation of alternatives $(\mathscr{V}_1, \mathscr{V}_2, \mathscr{V}_3, \mathscr{V}_4, \mathscr{V}_5)$ about the criteria $(\mathscr{C}_1, \mathscr{C}_2, \mathscr{C}_3, \mathscr{C}_4)$. Now we follow the algorithm 1 for the ranking of alternatives.

Input: Eqs. (35–39) include the assessment details of all experts.

$$\mathcal{P}^1 = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad\mathscr{C}_1 \quad & \quad\mathscr{C}_2\quad & \quad\mathscr{C}_3\quad & \quad\mathscr{C}_4\quad \end{array} \\ \begin{bmatrix} \langle \ell_1, (0.9, 0.1)\rangle & \langle \ell_2, (0.6, 0.6)\rangle & \langle \ell_2, (0.7, 0.2)\rangle & \langle \ell_0, (0.8, 0.3)\rangle \\ \langle \ell_3, (0.8, 0.3)\rangle & \langle \ell_{-1}, (0.7, 0.45)\rangle & \langle \ell_{-2}, (0.6, 0.7)\rangle & \langle \ell_2, (0.7, 0.4)\rangle \\ \langle \ell_2, (0.7, 0.5)\rangle & \langle \ell_0, (0.8, 0.35)\rangle & \langle \ell_1, (0.5, 0.6)\rangle & \langle \ell_1, (0.6, 0.6)\rangle \\ \langle \ell_{-1}, (0.8, 0.4)\rangle & \langle \ell_{-3}, (0.9, 0.25)\rangle & \langle \ell_1, (0.8, 0.4)\rangle & \langle \ell_{-2}, (0.5, 0.7)\rangle \\ \langle \ell_2, (0.9, 0.2)\rangle & \langle \ell_1, (0.7, 0.1)\rangle & \langle \ell_2, (0.7, 0.3)\rangle & \langle \ell_1, (0.6, 0.5)\rangle \end{bmatrix} \end{array}; \tag{35}$$

$$\mathcal{P}^1 = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad\mathscr{C}_1 \quad & \quad\mathscr{C}_2\quad & \quad\mathscr{C}_3\quad & \quad\mathscr{C}_4\quad \end{array} \\ \begin{bmatrix} \langle \ell_2, (0.8, 0.3)\rangle & \langle \ell_3, (0.6, 0.6)\rangle & \langle \ell_{-1}, (0.9, 0.1)\rangle & \langle \ell_3, (0.6, 0.5)\rangle \\ \langle \ell_{-2}, (0.7, 0.4)\rangle & \langle \ell_2, (0.7, 0.7)\rangle & \langle \ell_2, (0.8, 0.3)\rangle & \langle \ell_{-3}, (0.5, 0.7)\rangle \\ \langle \ell_3, (0.8, 0.5)\rangle & \langle \ell_1, (0.5, 0.8)\rangle & \langle \ell_2, (0.7, 0.4)\rangle & \langle \ell_2, (0.8, 0.4)\rangle \\ \langle \ell_1, (0.7, 0.3)\rangle & \langle \ell_{-2}, (0.6, 0.7)\rangle & \langle \ell_{-1}, (0.8, 0.2)\rangle & \langle \ell_1, (0.9, 0.4)\rangle \\ \langle \ell_2, (0.8, 0.2)\rangle & \langle \ell_1, (0.8, 0.4)\rangle & \langle \ell_3, (0.9, 0.3)\rangle & \langle \ell_2, (0.7, 0.6)\rangle \end{bmatrix} \end{array}; \tag{36}$$

$$\mathcal{P}^3 = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{C}_1 & \mathscr{C}_2 & \mathscr{C}_3 & \mathscr{C}_4 \end{array} \\ \begin{bmatrix} \langle \ell_{-2}, (0.6, 0.6) \rangle & \langle \ell_3, (0.8, 0.1) \rangle & \langle \ell_2, (0.9, 0.1) \rangle & \langle \ell_{-1}, (0.8, 0.2) \rangle \\ \langle \ell_2, (0.8, 0.4) \rangle & \langle \ell_2, (0.7, 0.4) \rangle & \langle \ell_3, (0.6, 0.2) \rangle & \langle \ell_2, (0.7, 0.6) \rangle \\ \langle \ell_1, (0.7, 0.6) \rangle & \langle \ell_{-2}, (0.6, 0.5) \rangle & \langle \ell_1, (0.4, 0.7) \rangle & \langle \ell_3, (0.6, 0.4) \rangle \\ \langle \ell_2, (0.5, 0.7) \rangle & \langle \ell_1, (0.9, 0.1) \rangle & \langle \ell_2, (0.8, 0.4) \rangle & \langle \ell_0, (0.4, 0.5) \rangle \\ \langle \ell_3, (0.6, 0.1) \rangle & \langle \ell_0, (0.8, 0.2) \rangle & \langle \ell_{-2}, (0.9, 0.2) \rangle & \langle \ell_1, (0.3, 0.1) \rangle \end{bmatrix} \end{array}, \tag{37}$$

$$\mathcal{P}^4 = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{C}_1 & \mathscr{C}_2 & \mathscr{C}_3 & \mathscr{C}_4 \end{array} \\ \begin{bmatrix} \langle \ell_{-2}, (0.3, 0.7) \rangle & \langle \ell_{-2}, (0.8, 0.4) \rangle & \langle \ell_0, (0.6, 0.7) \rangle & \langle \ell_2, (0.9, 0.25) \rangle \\ \langle \ell_3, (0.8, 0.4) \rangle & \langle \ell_2, (0.9, 0.3) \rangle & \langle \ell_{-2}, (0.7, 0.4) \rangle & \langle \ell_{-3}, (0.7, 0.1) \rangle \\ \langle \ell_2, (0.9, 0.1) \rangle & \langle \ell_3, (0.7, 0.2) \rangle & \langle \ell_3, (0.3, 0.6) \rangle & \langle \ell_1, (0.6, 0.7) \rangle \\ \langle \ell_{-1}, (0.7, 0.2) \rangle & \langle \ell_{-2}, (0.8, 0.2) \rangle & \langle \ell_{-1}, (0.7, 0.5) \rangle & \langle \ell_{-2}, (0.8, 0.4) \rangle \\ \langle \ell_{-2}, (0.9, 0.3) \rangle & \langle \ell_{-1}, (0.9, 0.1) \rangle & \langle \ell_3, (0.9, 0.2) \rangle & \langle \ell_3, (0.9, 0.1) \rangle \end{bmatrix} \end{array}, \tag{38}$$

$$\mathcal{P}^5 = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{C}_1 & \mathscr{C}_2 & \mathscr{C}_3 & \mathscr{C}_4 \end{array} \\ \begin{bmatrix} \langle \ell_{-3}, (0.3, 0.9) \rangle & \langle \ell_{-1}, (0.9, 0.1) \rangle & \langle \ell_{-3}, (0.5, 0.2) \rangle & \langle \ell_2, (0.8, 0.3) \rangle \\ \langle \ell_2, (0.8, 0.5) \rangle & \langle \ell_{-2}, (0.8, 0.4) \rangle & \langle \ell_2, (0.7, 0.3) \rangle & \langle \ell_3, (0.9, 0.1) \rangle \\ \langle \ell_3, (0.9, 0.12) \rangle & \langle \ell_0, (0.6, 0.5) \rangle & \langle \ell_1, (0.8, 0.5) \rangle & \langle \ell_1, (0.5, 0.6) \rangle \\ \langle \ell_{-1}, (0.7, 0.6) \rangle & \langle \ell_2, (0.9, 0.1) \rangle & \langle \ell_{-2}, (0.9, 0.4) \rangle & \langle \ell_2, (0.7, 0.4) \rangle \\ \langle \ell_2, (0.6, 0.4) \rangle & \langle \ell_1, (0.7, 0.3) \rangle & \langle \ell_3, (0.4, 0.2) \rangle & \langle \ell_2, (0.6, 0.2) \rangle \end{bmatrix} \end{array} (\tag{39}$$

Step 1: By Eq. (13), the weights of the criterion for each expert $\mathscr{J}_k$ is shown in Table 4.

Step 2: From Eqs. (21–26), the first layer probabilities are as: $Pr(\mathscr{J}_1) = 0.1918$, $Pr(\mathscr{J}_2) = 0.2040$, $Pr(\mathscr{J}_3) = 0.1975$, $Pr(\mathscr{J}_4) = 0.2033$, and $Pr(\mathscr{J}_5) = 0.2034$.

Step 3: Eq. (27) is utilised to compute the conditional probability for each alternative as assessed by each expert. The detailed results are presented in Table 5.

Using the results in Table 5, the second layer probabilities without considering interface effects among experts are computed by Eq. (28) as follows: $Pr(\mathscr{V}_1) = 0.1583$, $Pr(\mathscr{V}_2) = 0.1627$, $Pr(\mathscr{V}_3) = 0.2542$, $Pr(\mathscr{V}_4) = 0.1475$, and $Pr(\mathscr{V}_5) = 0.2772$.

Step 4: Two experts' distances of belief about each alternative are determined using Eq. (31), and the results are shown in Table 6.

The Deng entropy of $\mathscr{M}_{iu}$ is commutated by Eq. (32), and the results are shown in Table 7.

According to Eq. (33), the standardised Deng entropy is presented in Table 8.

Form Table 8 and using Eq. (34) we get $interface(1) = 0.0428$, $interface(2) = 0.0590$, $interface(3) = 0.3784$, $interface(4) = 0.1740$, and $interface(5) = -0.0478$.

Table 4 The weights of the criterion for each expert $\mathscr{J}_k$

$\mathscr{J}_k$	$\mathscr{W}_1^k$	$\mathscr{W}_2^k$	$\mathscr{W}_3^k$	$\mathscr{W}_4^k$
$k = 1$	0.1648	0.3105	0.3727	0.1521
$k = 2$	0.1858	0.3473	0.2798	0.1871
$k = 3$	0.1883	0.3832	0.2093	0.2192
$k = 4$	0.1853	0.1844	0.2506	0.3798
$k = 5$	0.3019	0.2194	0.2298	0.2489

Table 5 The computational results of $Pr(\mathscr{V}_i \mid \mathscr{J}_k)$ for each expert $\mathscr{J}_k (k = 1, 2, \cdots, 5)$

$Pr(\mathscr{V}_i \mid \mathscr{J}_k)$	$\mathscr{J}_1$	$\mathscr{J}_2$	$\mathscr{J}_3$	$\mathscr{J}_4$	$\mathscr{J}_5$
$\mathscr{V}_1$	0.2163	0.1347	0.0421	0.0989	0.2996
$\mathscr{V}_2$	0.0596	0.0858	0.5293	0.0846	0.0595
$\mathscr{V}_3$	0.0908	0.2590	0.1197	0.5891	0.1994
$\mathscr{V}_4$	0.2844	0.1312	0.0839	0.1736	0.0706
$\mathscr{V}_5$	0.3488	0.3894	0.2251	0.0539	0.3709

Table 6 Two experts' distances of belief about each alternative

$\mathscr{M}_{iu}$	$\mathscr{V}_1$	$\mathscr{V}_2$	$\mathscr{V}_3$	$\mathscr{V}_4$	$\mathscr{V}_5$
$u = 1(\mathscr{J}_1, \mathscr{J}_2)$	0.4106	0.0968	0.8581	0.6174	0.2029
$u = 2(\mathscr{J}_1, \mathscr{J}_3)$	0.6198	0.1600	0.1228	0.7055	0.5732
$u = 3(\mathscr{J}_1, \mathscr{J}_4)$	0.5157	0.0921	0.3459	0.4725	0.3286
$u = 4(\mathscr{J}_1, \mathscr{J}_5)$	0.0842	0.0204	0.5686	0.7187	0.1346
$u = 5(\mathscr{J}_2, \mathscr{J}_3)$	0.2709	0.2356	0.7539	0.1533	0.7622
$u = 6(\mathscr{J}_2, \mathscr{J}_4)$	0.1129	0.0128	0.8694	0.2512	0.0612
$u = 7(\mathscr{J}_2, \mathscr{J}_5)$	0.3736	0.0590	0.4866	0.1840	0.1516
$u = 8(\mathscr{J}_3, \mathscr{J}_4)$	0.1732	0.3532	0.4277	0.4053	0.7069
$u = 9(\mathscr{J}_3, \mathscr{J}_5)$	0.5924	0.2846	0.4969	0.0154	0.5978
$u = 10(\mathscr{J}_4, \mathscr{J}_5)$	0.4841	0.0546	0.8816	0.3803	0.2217

Table 7 The Deng entropy of $\mathscr{M}_{iu}$

$\Theta_{iu}(\mathscr{M}_{iu})$	$\mathscr{V}_1$	$\mathscr{V}_2$	$\mathscr{V}_3$	$\mathscr{V}_4$	$\mathscr{V}_5$
$u = 1(\mathscr{J}_1, \mathscr{J}_2)$	2.1316	0.7042	3.5418	2.8417	1.2597
$u = 2(\mathscr{J}_1, \mathscr{J}_3)$	2.8492	1.0479	0.8513	3.1114	2.6996
$u = 3(\mathscr{J}_1, \mathscr{J}_4)$	2.5073	0.6769	1.8811	2.3569	1.8112
$u = 4(\mathscr{J}_1, \mathscr{J}_5)$	0.6295	0.1939	2.6845	3.1503	0.9155
$u = 5(\mathscr{J}_2, \mathscr{J}_3)$	1.5687	1.4116	3.2527	1.0135	3.2766
$u = 6(\mathscr{J}_2, \mathscr{J}_4)$	0.7965	0.1306	3.5723	1.4822	0.4859
$u = 7(\mathscr{J}_2, \mathscr{J}_5)$	1.9902	0.4714	2.4067	1.1683	1.0050
$u = 8(\mathscr{J}_3, \mathscr{J}_4)$	1.1146	1.9102	2.1952	2.1117	3.1154
$u = 9(\mathscr{J}_3, \mathscr{J}_5)$	2.7620	1.6278	2.4427	0.1529	2.7794
$u = 10(\mathscr{J}_4, \mathscr{J}_5)$	2.3978	0.4427	3.6047	2.0162	1.3479

Step 5: Using the interface effect among experts, the comprehensive probabilities of alternatives are computed by Eq. (30) as follows: $Pr(\mathscr{V}_1) = 0.1252$, $Pr(\mathscr{V}_2) = 0.1380$, $Pr(\mathscr{V}_3) = 0.3938$, $Pr(\mathscr{V}_4) = 0.2001$, and $Pr(\mathscr{V}_5) = 0.1428$.

Step 6: According to the value of $Pr(\mathscr{V}_i)(i = 1, 2, 3, 4, 5)$ of each alternative, we get $\mathscr{V}_3 \succ \mathscr{V}_4 \succ \mathscr{V}_5 \succ \mathscr{V}_2 \succ \mathscr{V}_1$.

4.2 Analysis of results

This part addresses the confirmation of our numerical results by effect of interface among experts and effect of the expert confidence levels.

Table 8 The standardised Deng entropy of $\mathscr{M}_{iu}$

$\beta_{iu}(\mathscr{M}_{iu})$	$\mathscr{V}_1$	$\mathscr{V}_2$	$\mathscr{V}_3$	$\mathscr{V}_4$	$\mathscr{V}_5$
$u=1(\mathscr{J}_1,\mathscr{J}_2)$	0.3534	−0.3553	0.9544	0.7941	−0.4454
$u=2(\mathscr{J}_1,\mathscr{J}_3)$	1	0.0310	− 1	0.9740	0.5865
$u=3(\mathscr{J}_1,\mathscr{J}_4)$	0.6920	−0.3860	−0.2520	0.4706	−0.0502
$u=4(\mathscr{J}_1,\mathscr{J}_5)$	− 1	−0.9288	0.3316	1	−0.6921
$u=5(\mathscr{J}_2,\mathscr{J}_3)$	−0.1537	0.4397	0.7443	−0.4258	1
$u=6(\mathscr{J}_2,\mathscr{J}_4)$	−0.8496	− 1	0.9765	−0.1130	− 1
$u=7(\mathscr{J}_2,\mathscr{J}_5)$	0.2260	−0.6169	0.1298	−0.3225	−0.6280
$u=8(\mathscr{J}_3,\mathscr{J}_4)$	−0.5629	1	−0.0238	0.3070	0.8845
$u=9(\mathscr{J}_3,\mathscr{J}_5)$	0.9215	0.6826	0.1559	− 1	0.6437
$u=10(\mathscr{J}_4,\mathscr{J}_5)$	0.5933	−0.6492	1	0.2433	−0.3823

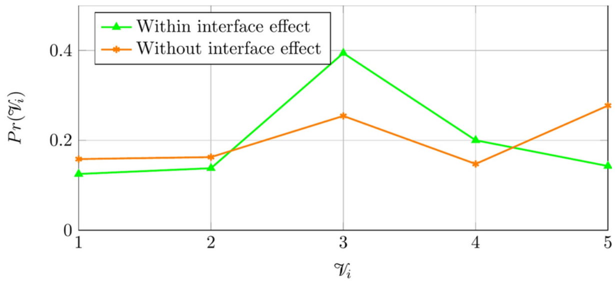

Fig. 5 The interface effect on ranking values

4.2.1 Interface effect among experts on decision result

Decisions can be made based on the $Pr(\mathscr{V}_i)(i=1,2,3,4,5)$ values as specified in Eq. (28), where greater probabilities signify a superior ranking. The conventional technique fails to include the interference effect, which may affect the final rankings, as seen in Fig. 5. The ranking outcome of alternatives, as per Eq. (28), is $\mathscr{V}_5 \succ \mathscr{V}_3 \succ \mathscr{V}_2 \succ \mathscr{V}_1 \succ \mathscr{V}_4$. The ranking outcome is $\mathscr{V}_3 \succ \mathscr{V}_4 \succ \mathscr{V}_5 \succ \mathscr{V}_2 \succ \mathscr{V}_1$ as per Eq. (30) when considering the interface impact between two experts regarding an alternative.

4.2.2 Effect of the expert confidence levels on decision result

In this section, first, we examine how much effect the expert confidence levels have on decision results. By Ref. (Mandal et al. 2020), if $\mu_{ij}^k = 1$ and $\nu_{ij}^k = 0$, the expert decision matrix shown in Eq. (11), can be degenerate into a LI-based decision matrix, where $\mathscr{A}_{ij}^k = \left\langle \mathscr{S}_{\theta_{ij}^k}, (1,0) \right\rangle$. Then we rank the alternatives by Algorithm 1 for PLI and LI, the details of the comparison results are shown in Fig. 6.

From 6, we can observe that if we avoid the expert confidence levels, the ranking order of the GSS is $\mathscr{V}_5 \succ \mathscr{V}_3 \succ \mathscr{V}_1 \succ \mathscr{V}_2 \succ \mathscr{V}_4$, which is different from the ranking order of the GSS if we consider the expert confidence levels. It demonstrates that the PLI significantly effects on decision results. Thus, we cannot neglect the experts' confidence levels.

4.2.3 Validity test

In the traditional decision-making model (Mandal et al. 2020) the ranking results are derived from the aggregate decision matrix of experts' opinions. In this context, according to Ref. (Mandal et al. 2020), the Pythagorean linguistic weighted averaging aggregation operator is employed to aggregate the opinions of five experts as follows:

$$Agg(\mathscr{V}_1, \cdots, \mathscr{V}_5) = \begin{array}{c} \\ \mathscr{V}_1 \\ \mathscr{V}_2 \\ \mathscr{V}_3 \\ \mathscr{V}_4 \\ \mathscr{V}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{C}_1 & \mathscr{C}_2 & \mathscr{C}_3 & \mathscr{C}_4 \end{array} \\ \begin{bmatrix} \langle \ell_{-0.81}, (0.69, 0.41) \rangle & \langle \ell_{0.98}, (0.78, 0.27) \rangle & \langle \ell_{-0.04}, (0.78, 0.25) \rangle & \langle \ell_{1.23}, (0.8, 0.3) \rangle \\ \langle \ell_{1.58}, (0.78, 0.4) \rangle & \langle \ell_{0.61}, (0.78, 0.43) \rangle & \langle \ell_{0.62}, (0.69, 0.35) \rangle & \langle \ell_{0.17}, (0.74, 0.28) \rangle \\ \langle \ell_{2.21}, (0.83, 0.28) \rangle & \langle \ell_{0.42}, (0.66, 0.43) \rangle & \langle \ell_{1.61}, (0.60, 0.55) \rangle & \langle \ell_{1.6}, (0.64, 0.53) \rangle \\ \langle \ell_{0}, (0.7, 0.4) \rangle & \langle \ell_{-0.79}, (0.85, 0.20) \rangle & \langle \ell_{-0.23}, (0.81, 0.36) \rangle & \langle \ell_{-0.18}, (0.73, 0.47) \rangle \\ \langle \ell_{1.38}, (0.8, 0.22) \rangle & \langle \ell_{0.4}, (0.8, 0.19) \rangle & \langle \ell_{1.82}, (0.83, 0.23) \rangle & \langle \ell_{1.81}, (0.7, 0.23) \rangle \end{bmatrix} \end{array}. \tag{40}$$

So, using the MULTIMOORA method and the aggregated decision matrix (40), the order of the alternatives is $\mathscr{V}_5 \succ \mathscr{V}_3 \succ \mathscr{V}_1 \succ \mathscr{V}_2 \succ \mathscr{V}_4$, using ranking values Γ_i^{Agg} that were found by Eq. (20). Thus, $\mathscr{V}_5$ is the ideal alternative. When the suggested model is implemented without accounting for the interface impact, the best alternative remains $\mathscr{V}_5$, which is shown in Fig. 7. This suggests that when we ignore the influence of experts, the best alternative of the proposed technique aligns with traditional approaches, thereby confirming the robustness of the recommended model. Nonetheless, when assessing the quantum interference among experts, the results of this model indicate that $\mathscr{V}_3$ represents the ideal approach, which is shown in Fig. 7. This discrepancy stems from the traditional framework's omission of the interference effect among experts, resulting in subjective biases in the assessment of alternatives. Consequently, the results of this model more accurately reflect real-world complexity.

4.3 Comparative study

We do a comparative study in this section to show how well the extended quantum-like MULTIMOORA-based MCGDM approach works in PLI situations. This section conducts a comparative analysis based on both qualitative and quantitative studies.

4.3.1 According qualitative analysis

Table 9 illustrates that the majority of current MCGDM models are either deficient or presuppose predetermined weights for experts and criteria. This study used PLNs to charac-

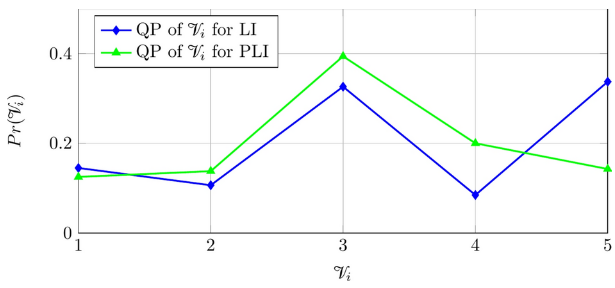

Fig. 6 Output of decision results for PLI and LI

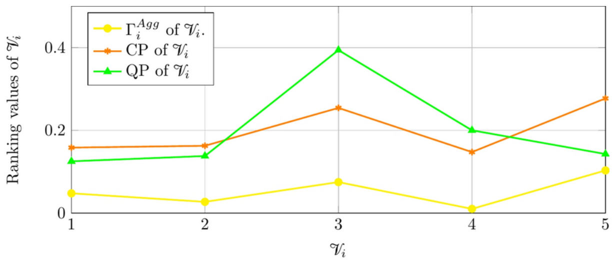

Fig. 7 Output of decision results based on aggregation of experts opinion, CP and QP of alternatives

Table 9 A comparison of the suggested approach with prior decision models inside QSBN

Approaches	Types of information	Weights of criteria	Computation of first layer probability	Computation of second layer probability
Han and Liu (2023)	Fuzzy	Subjective weights are given in advanced then applied QSBN	Given in advanced	QSBN with Deng entropy
He et al. (2018)	LI	Given in advanced	Computed by Liu et al. approach Liu et al. (2016)	linguistic weighted arithmetic averaging operator with evolution of phase angle
Jiang and Liu (2022)	Crisp	Calculate by AHP approach	Given in advanced	Scores-based approach with similarity heuristic approach
Mandal et al. (2024)	Linguistic Z number	Calculate by Shannon entropy	Computed by Mandal et al. approach Mandal et al. (2024)	Calculate by TODIM and PROMETHEE II approaches with Deng entropy
She et al. (2021)	Crisp	Given in advanced	Suggest any existing approach	QSBN with Deng entropy
Wu et al. (2021)	Linguistic distribution assessments	Suggest any existing approach	Suggest any existing approach	TODIM-based exponential approach with evolution of phase angle
Wu et al. (2022)	Linguistic distribution assessments	Calculate by Shannon entropy	Calculate by arithmetic average-based similarity approach	Calculate by prospect theory-based transformation of each expert opinion with evolution of phase angle
Zheng and Liu (2025)	2-tuple LI		Given in advanced	Calculate by GLDS approach with evolution of phase angle
In this paper	PLI	Proposed Pearson coefficient of correlations-based approach	Proposed Shannon entropy with proximity of relative-based approach	Proposed MULTIMOORA-based approach with Deng entropy

terize assessment information in the context of GSCM, determined the weights of experts using an entropy method, calculated the weights of criteria using a coefficient of correlations among criteria, and employed Deng's entropy to quantify information uncertainty. This method more precisely represents the interactions of experts and fits more closely with real-world circumstances.

After conducting a thorough comparison with previous studies, Wu et al. (2018) [65] concluded that the determination of criteria weights requires consideration of the correlation coefficients between the criteria. A low weight leads to a reasonable assignment to criteria with high coefficients. As mentioned in (Harmati et al. 2022), we can quantify the quality of the expert's assessment using the appropriate entropy. However, as shown in Table 9, most studies do not include the computation of criteria weights and first-layer probability in the process of aggregating individual opinions under the QSBN framework. In this paper, we consider both.

4.3.2 According quantitative analysis

In this section, we compare our proposed MCGDM approach based on different aggregations of individual information. Initially, experts developed the QPT to aggregate their opinions. This section provides a comparative study to demonstrate the efficacy of QPT in information fusion. Eq. (20) calculates the ranking values of the alternatives in this comparative study, aggregating expert opinions through the PLWA, PLOWA, PLWG, and PLOWG operators (Mandal et al. 2020), along with our proposed MCGDM approach within the QSBN framework. The ranking values of the alternatives by different aggregation operators and our proposed MCGDM approach within the QSBN framework are shown in Fig. 8, and the ranking order is shown in Table 10.

From Fig. 8 and Table 10, we have the following observation:

(1) We conducted a comparison between the PLWA (PLOWA) operator and QSBN. It can be seen that the optimal alternatives $\mathscr{V}_5$ and $\mathscr{V}_3$, determined by the two methods are different. This difference arises from the fact that QSBN takes into account the interference effect between experts, whereas WA makes the assumption that experts are independent. However, there may be an interference effect between each pair of experts. In order to establish a dependable collective green supplier for GSCM, it is crucial to take into account the interference effect during the information fusion phase. Therefore, QSBN is more general than PLWA (PLOWA).

(2) We compared the PLWG (PLOWG) operator with QSBN. The ideal alternatives $\mathscr{V}_1$ and $\mathscr{V}_3$ identified by the two approaches are distinct. This discrepancy mostly arises from their distinct methods of consolidating individual GSCM assessment data. The PLWG (PLOWG) operator is more appropriate than alternative approaches for aggregating information as multiplicative preference relations. This study presents the GSCM assessment information as additive preference relations. Consequently, in comparison

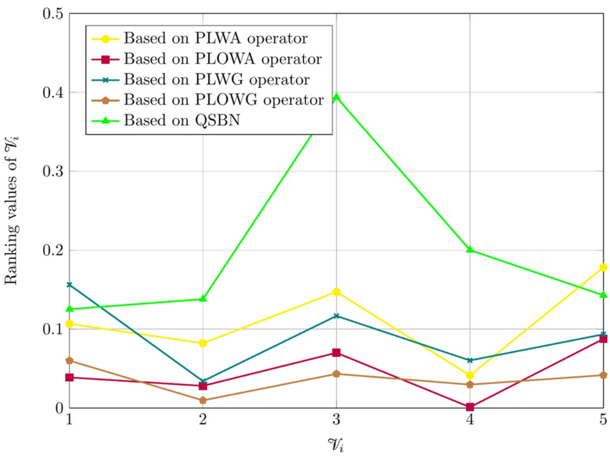

Fig. 8 Ranking values of alternatives according to different aggregation operators and our proposed MCGDM approach within the QSBN framework

Table 10 The ranking results of different aggregation operators and our approach within the QSBN framework

Aggregation operators	Ranking orders
PLWA	$\mathscr{V}_5 \succ \mathscr{V}_3 \succ \mathscr{V}_1 \succ \mathscr{V}_2 \succ \mathscr{V}_4$
PLOWA	$\mathscr{V}_5 \succ \mathscr{V}_3 \succ \mathscr{V}_1 \succ \mathscr{V}_2 \succ \mathscr{V}_4$
PLWG	$\mathscr{V}_1 \succ \mathscr{V}_3 \succ \mathscr{V}_5 \succ \mathscr{V}_4 \succ \mathscr{V}_2$
PLOWG	$\mathscr{V}_1 \succ \mathscr{V}_3 \succ \mathscr{V}_5 \succ \mathscr{V}_4 \succ \mathscr{V}_2$
QSBN	$\mathscr{V}_3 \succ \mathscr{V}_4 \succ \mathscr{V}_5 \succ \mathscr{V}_2 \succ \mathscr{V}_1$

to the PLWG (PLOWG) operator, QSBN demonstrates superior efficiency in consolidating GSCM assessment data.

5 Conclusion

This paper presents a quantum group decision model, designed to mitigate information ambiguity and opinion interference in GSCM. The approach uses PLIs to convey ambiguous information in complex decision-making contexts, employing Deng entropy to assess opinion interference by modeling the reciprocal effects among decision-makers in the MCGDM process. Unlike conventional methods that treat decision-makers as independent entities, this paradigm addresses interference by assigning appropriate weights to decision makers and regulating opinion interference through quantum decision-making procedures.

The proposed model enhances existing methodologies, offering more efficient solutions to challenges related to opinion interference and weight distribution. Our findings show that the model significantly improves decision-making in GSCM by providing a comprehensive framework to address information ambiguity and conflicting perspectives. However, the computational complexity increases with the inclusion of additional decision-makers and variables, highlighting the need for advanced computational techniques to maintain efficiency.

Future research will focus on integrating machine learning methods to enhance scalability and practicality, ensuring the models effectiveness in large-scale decision-making processes. This integration aims to manage rising complexity, foster consensus in quantum group decision-making, and ensure the models with long-term sustainability.

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Author contributions Prasenjit Mandal: Conceptualization of this study, methodology, validation, formal analysis, writing–original draft. Leo Mrsic: visualization, validation, writing-review and editing. Antonios Kalampakas: Visualization, review and formal analysis. Tofigh Allahviranloo: Visualization, formal analysis and supervision. Sovan Samanta: formal analysis, writing-review and editing.

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Data availability The data will be made available on request.

Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

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Authors and Affiliations

Prasenjit Mandal[1] · Leo Mrsic[1,2] · Antonios Kalampakas[3] · Tofigh Allahviranloo[4,5] · Sovan Samanta[1,4,6]

✉ Sovan Samanta
ssamantavu@gmail.com

Prasenjit Mandal
prasenjitmandal08@yahoo.com

Leo Mrsic
leo.mrsic@algebra.hr

Antonios Kalampakas
antonios.kalampakas@aum.edu.kw

Tofigh Allahviranloo
tofigh.allahviranloo@istinye.edu.tr

[1] Department of Technical Sciences, Algebra Bernays University, Gradiscanska 24, 10000 Zagreb, Croatia

[2] Rudolfovo Science and Technology Centre, Novo Mesto, Slovenia

[3] College of Engineering and Technology, American University of the Middle East, Egaila 54200, Kuwait

[4] Research Centre of Performance and Productivity Analysis, Istinye University, Istanbul, Türkiye

[5] Quantum Technologies Research Center (QTRC), Science and Research Branch, Islamic Azad University, Tehran, Iran

[6] Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk, WB 721636, India