

Article

Computational Fluid Dynamics, Transport, and Chemical Kinetics-Based Monolith Catalyst Dimensioning Methodology for Cost-Effective Performance

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Abstract: The newly developed computational fluid dynamics, transport, and chemical kinetics-based monolith catalyst dimensioning methodology consists of the following steps: (i) initial calculations, which generate some of the data, e.g., average inlet fluid velocity used in the (ii) computational fluid dynamics (CFD) modelling, which uses the laminar flow interface and the transport of diluted species interface while the user has to provide the kinetics of the reactions; (iii) the model order reduction uses a modified version of the plug flow reactor model and the linear pressure variation model; and (iv) the dimensioning optimization algorithm extracts the optimal monolith catalyst's channel geometry, which satisfies the user's performance constraints and reduces material consumption. Therefore, the methodology enables chemical engineers to quickly and efficiently design and dimension monolith catalysts for many different applications in an environmentally friendly way, which enables them to reduce both the material and operating costs while maintaining sufficient catalyst performance and, therefore, achieve its cost-effective performance.

Keywords: computational fluid dynamics; chemical kinetics; monolith catalyst; cost reduction; optimization

1. Introduction

The advantages of structured catalysts over fixed-bed reactors are their lower maldistribution and pressure drop [\[1\]](#page-11-0), which is why they are widely used in chemical industry. The monolith catalysts have been in use in chemical engineering applications both for gas phase and gas–liquid reactions [\[2](#page-11-1)[,3\]](#page-11-2). The gas phase reactions include (i) emission control of CO, unburned hydrocarbons (HC) and nitrogen oxides (NO_X) [\[4–](#page-12-0)[6\]](#page-12-1), (ii) catalytic combustion $[7-9]$ $[7-9]$, (iii) steam reforming $[10-12]$ $[10-12]$, (iv) methanation $[13,14]$ $[13,14]$, (v) hydrogenation/dehydrogenation [\[15](#page-12-8)[,16\]](#page-12-9), (vi) oxidation [\[17](#page-12-10)[–19\]](#page-12-11), and (vii) water gas shift reaction [\[20](#page-12-12)[,21\]](#page-12-13). Since many industrial processes depend on monolith catalysts, their optimized design, accurate modelling, effective performance, and reduced material toxicity are essential in multiple sectors of our society.

The computational fluid dynamics (CFD) modelling approach has been widely used to simulate processes of mass transfer and chemical kinetics inside monolith catalysts. Many studies have focused on either varying design/geometry [\[22](#page-12-14)[–24\]](#page-12-15) and/or predicting catalyst performance [\[25](#page-12-16)[–27\]](#page-12-17). However, not many authors, e.g., [\[28\]](#page-12-18), have used or combined their CFD results with one-dimensional (1D) monolith catalyst modelling. A one-dimensional modelling approach is often used for optimizing monolith design and configurations by manufactures [\[29\]](#page-12-19). Model order reduction (MOR) methodology could help monolith

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designers by converting often computationally costly CFD modelling approach to simple 1D model calculations.

Multiple reduced order models for modelling of washcoated monolithic catalysts are presented in the available literature. However, most of the reduced order models are quite complex with multiple partial differential equations [\[30\]](#page-12-20), transverse averaging using the Lyapunov–Schmidt method, look-up tables [\[31\]](#page-12-21), and/or multiple matrices, e.g., the Thiele matrix [\[32\]](#page-13-0). Our approach, on the other hand, used only two simple algebraic equations to model the cross-sectional average (perpendicular to the flow direction) values of concentration and pressure (computed via CFD approach), respectively.

Some of the commonly used active catalyst materials are expensive, e.g., Pt [\[33\]](#page-13-1), are toxic, e.g., V_2O_5 [\[34\]](#page-13-2), are hard or impossible to recycle [\[35\]](#page-13-3), and/or have no adequate alternative materials (e.g., Pt). Therefore, a reduction in the use of these crucial materials would reduce the manufacturing cost and the negative environmental impact of the monolith catalysts, considering their whole life cycle.

Chemical engineers who design and dimension a monolithic catalytic converter usually know the composition and the volumetric flow rate (*Q*) of their gas mixtures, the minimal required monolith catalyst performance (usually conversion of specific species or selectivity), maximal allowed pressure drop across the monolith channels, and the space constrains dedicated to the catalyst. Therefore, these elements are the input parameters for the developed dimensioning methodology. The chemical kinetics of the reactions occurring in the monolith catalyst should also be known in the form of rate law(s), as are, for example, for some selective catalytic reduction denitrification $(SCR\text{-}deNO_X)$ catalysts which use ammonia as a reducing agent [\[24,](#page-12-15)[36,](#page-13-4)[37\]](#page-13-5).

The usefulness of the CFD-based monolith catalyst dimensioning methodology was illustrated on a hypothetical case of an automotive $SCR\text{-}deNO_X$ washcoated monolithic catalyst (WMC) for the removal of nitrogen oxides from flue gasses emitted by a personal vehicle with a hydrogen-powered internal combustion engine drivetrain.

2. Methods

The monolith catalyst dimensioning methodology consists of four main consecutive steps: (i) initial calculations, (ii) CFD modelling, (iii) model order reduction; and (iv) the dimensioning optimization algorithm.

2.1. Initial Calculations

The typical automotive cylindrical monoliths have diameters (D_m) of 100 mm and lengths (*Lm*) of 150 mm [\[38\]](#page-13-6). These dimensions were also considered in our example of designing and dimensioning of an automotive $SCR\text{-}deNO_X$ catalyst. The share of effective flow area (*Sheff*) was assumed to be equal to 0.6 to account for the thickness of the monolith's walls which enable its structural integrity, load bearing capacity, and mechanical resistivity. The arrangement of the channels in the monolith catalyst was expected to follow the hexagonal-lattice-based circle packing pattern (of equally sized circles), which has been used in monoliths [\[39\]](#page-13-7) and can be seen in Figure [1](#page-2-0) [\[39\]](#page-13-7). The hexagonal-lattice-based circle packing is the densest circle packing in the plane with a packing density equal to $\frac{\pi}{2\sqrt{3}} \approx 0.9069$ [\[40\]](#page-13-8). This circle packing density is of course a theoretical limit, which, if applied to our case, would mean that the monolith's walls would have zero thickness and the monolith would have no extra material near its outer edges. Therefore, a considerably lower value than the 0.9069 theoretical limit was selected for the share of effective flow area (*Sheff*).

The average fluid velocity (u_{av}) flowing through the effective flow area (A_{eff}) was, therefore, determined using Equation (1).

$$
u_{av} = \frac{Q}{A_{eff}} = \frac{Q}{Sh_{eff} \frac{\pi}{4} D_m^2}
$$
 (1)

Figure 1. Cross-section (l**eft**) and drawing (ri**ght**) of a cylindrical monolith with the hexagonal-latticebased packing of cylindrical channels.

In case of the selected volumetric flow rate of 3.3 \times 10⁻² m³/s, the average velocity is, in this case, equal to 7 m/s. This value of velocity was used as an input for the subsequent CFD studies.

The radii (R) of the individual cylindrical monolith channels were determined with $\frac{1}{R}$ and $\frac{1}{R}$ a Fire ruan (1) of the marriagan cymratical monomic entrines were determined with
Equation (2), where *N* is the number of monolith channels and varied from 600 to 6000 by
steps of 50 (109 different radii, see Table A1) steps of 50 (109 different radii, see Table [A1\)](#page-11-3).

$$
R = \left| \sqrt{\frac{A_{eff}}{\pi N}} \right| \tag{2}
$$

The channels' walls are assumed to be coated with a uniformly thick layer of high surface area washcoat that contains dispersed active catalyst material $[41]$.

2.2. CFD Modelling

 ⁼ ቮඨ ^ቮ (2) The CFD studies were conducted using the COMSOL Multiphysics® (version 5.1, diluted species interface (tds, [\[43\]](#page-13-11)). COMSOL S.r.l., Brescia, Italy) simulation platform with two-dimensional (2D) axisymmetric stationary studies employing the laminar flow interface (spf, [\[42\]](#page-13-10)) and the transport of

The Navier–Stokes equation of single-phase steady state incompressible laminar flow *identity matrix,* μ *is the dynamic viscosity, and* F *represents the volume forces.* is Equation (3), where ρ denotes fluid density, u is the velocity, p is the pressure, I is the

$$
\rho(u \cdot \nabla)u = \nabla \left[-p\mathbf{I} + \mu \left(\nabla u + (\nabla u)^{\mathrm{T}} \right) \right] + F \tag{3}
$$

Equation (4) is the continuity equation.

$$
\rho \nabla u = 0 \tag{4}
$$

The inflow at the inlet of a single cylindrical catalyst channel was selected to be the laminar inflow with a parabolic velocity profile. The working fluid was assumed to be temperature 300 °C, which is within the optimal operating temperature range of multiple commercial de ${\rm NO}_{\rm X}$ catalysts [\[44\]](#page-13-12). The fluid's density was determined with the ideal gas was determined with the method of Wilke [\[45\]](#page-13-13). For the combustion products of hydrogen (with air) at the equivalence ratio of 1.15 (see Table B.1 of Voglar et al. [\[46\]](#page-13-14)) at 300 °C, the destiny equalled 0.5319 kg/m³ and the dynamic viscosity was 2.597 × 10⁻⁵ Pa·s. In all of the studies, the average inlet fluid velocity was set to 7 m/s. The outlet had the pressure (0 Pa) boundary condition with suppressed backflow. produced during combustion of hydrogen with air and to enter the monolith channel at the law and the calculation of the average gas mixture's molecular weight, while viscosity

The transport of diluted species interface models the chemical species' transport through diffusion (Fick's second law) and convection. This Equation (5) is used, where D_i denotes the diffusion coefficient of the species *i*, *cⁱ* is the concentration of the species *i*, and *Ri* is the reaction rate expression of the species *i*.

$$
\nabla \cdot (-D_i \nabla c_i) + u \cdot \nabla c_i = R_i \tag{5}
$$

The mass conservation equation for one or more chemical species *i* has a form of Equation (6), where *Nⁱ* denotes the flux of the species *i*.

$$
N_i = -D_i \nabla c_i + u c_i \tag{6}
$$

The velocity field was inserted from the laminar flow interface (spf) calculations. Diffusion coefficients of both species A and B were set to 1×10^{-5} m²/s, which is within the range typical for gaseous species [\[47\]](#page-13-15).

The initial (inlet) concentration of species A, simulating species of nitrogen oxides (NO_X), was set to 1×10^{-2} mol/m³, since the usual NO_X level is around 500 ppm [\[48,](#page-13-16)[49\]](#page-13-17), which equals to 1.06×10^{-2} mol/m³ (at 300 °C). The inlet concentration of species B, representing the molecular nitrogen (N_2) and water vapour (H_2O) produced by the SCRdeNO_X reactions, was set to 0 mol/m^3 .

This simple chemical reaction (Equation (7)) was assumed to take place at the fluid– cylinder wall interface (inside the monolith catalyst channel).

$$
A \to B \tag{7}
$$

The reaction was introduced as a flux boundary condition within the transport of diluted species interface (tds). Since the reaction is equimolar, it does not affect the flow regime by producing additional molecules and also simulates well the three main SCR $deNO_X$ reactions (standard, fast, and $NO₂ SCR$ reaction), which have their molar ratio between products and reactants close to one (from 1.111 to 1.357).

The reaction rate (*r*) equation is this Equation (8).

$$
r = k c_A \tag{8}
$$

The reaction constant *k* has a unit of m/s because it models a surface reaction. The value of the reaction rate coefficient, *k*, was set to 1×10^6 m/s.

The inward fluxes of the species A and B $(R_A \text{ and } R_B)$ at the cylinder wall were determined with Equations (9) and (10).

$$
R_A = -r \tag{9}
$$

$$
R_B = r \tag{10}
$$

Our CFD studies modelled the external mass transfer effects while the internal mass transfer (through the pores of the catalyst material) was neglected. The reaction kinetics did not present the majority of the resistance, while the transport of species to the catalyst surface presented the main conversion limitation.

The computational domain's meshing was conducted with the selection of a physicscontrolled mesh with a normal element size. Since our paper introduces a new catalyst dimensioning methodology and is not focused on generating accurate results, a grid independence study was, in our case, not performed. However, a grid independence study, verification, and experimental validation (on at least a couple of different monolith geometries) are advised to be carried out when our methodology is utilized in a real case of dimensioning of a monolith catalyst.

The temperature of the system was assumed to be constant 573.15 K (300 $^{\circ}$ C). The system was, therefore, adiabatic and isothermal, which is consistent with the other numerical studies, e.g., [\[50](#page-13-18)[,51\]](#page-13-19).

2.3. Model Order Reduction

When we extracted the relevant CFD data (pressure drop, concentrations of species), we continued our work with model order reduction (MOR) to simplify and enable our further methodological steps. The model order reduction methodology was applied to the CDF results. The model for determining the average concentration of species A inside the monolith channel varying with the monolith length (*l*) is the slightly modified version of the standard one-dimensional plug flow reactor (PFR) model (Equation (11)) [\[52\]](#page-13-20), where $c_{A,in}$ is the inlet concentration of species *A* and k_c is the model constant, which describes the mass transport phenomena (by convection and diffusion) and reaction kinetics.

$$
c_A = c_{A,in}e^{-k_c l} \tag{11}
$$

The model for the pressure variation across the channel's length is well known to be linear [\[53\]](#page-13-21), so Equation (12) was used, where p_{in} is the inlet pressure and k_p represents the model constant. We will name this model the linear pressure variation (LPV) model.

$$
p = p_{in} - k_p l \tag{12}
$$

The values of the model constants were extracted from the CFD results, where radially averaged values of the concentration of species A and pressure were sampled at the (equally spaced) nine lengths of the monolith channel. The model fitting was performed in the Python programming language (version 3.8.10) using the Spyder (version 5) integrated development environment (IDE).

2.4. Dimensioning Optimization Algorithm

The cost-effective monolith catalyst performance optimization algorithm was written in the Python programming language using the Spyder integrated development environment (IDE).

At this stage, the user has to provide the algorithm with two of the most relevant catalyst performance indicators: desired conversion (in our case 0.95 [\[54\]](#page-13-22)) and maximal allowed pressure drop (in our case 560 Pa [\[55\]](#page-13-23)).

The conversion of species A (*ConA*) is defined in Equation (13), where *cA,in* and *cA,out* are the inlet and outlet concentrations of the species A, respectively.

$$
Con_A = 1 - \frac{c_{A,out}}{c_{A,in}} \tag{13}
$$

The algorithm firstly extracted all of the geometries with conversions equal or higher than the desired conversion. The leftover channels would have to be longer than the initial length limitation (150 mm) to satisfy the conversion criterium. Secondly, the algorithm used the PFR model to calculate the optimal channel lengths (*L*) with conversions equal to the desired conversion and applied the LPV model to determine the corresponding pressure drops (∆*p*). The next step was the elimination of the channels with too large pressure drops. Finally, the cost of the monolith catalyst was considered.

The cost of washcoated monolith catalyst manufacturing is correlated with the quantity of the active catalyst material used, since it usually contains expansive materials, e.g., precious metals [\[56\]](#page-13-24). The volume (*V*) of the used active catalyst material, which is uniformly coated in a thin layer (of thickness *δ*) on the inner surface of the individual cylindrical monolith channel, can be determined with Equation (14).

$$
V = \pi \left((R + \delta)^2 - R^2 \right) L = \pi \left(R^2 + 2R\delta + \delta^2 - R^2 \right) L = \pi \left(2R\delta + \delta^2 \right) L \tag{14}
$$

In cases of thin coatings ($\delta \ll R$), the volume of the active catalyst material for an individual catalyst channel can be estimated using Equation (15).

$$
V \cong (2\pi R)\delta L \tag{15}
$$

Since the coating thickness (*δ*) is assumed to be constant for a specific type of an active catalyst material, the cost of the coating material for an individual catalyst channel is proportional to the product of the radius (R) and length (L) of the cylindrical monolith catalyst C ^r channel. Because the monolith catalyst has multiple (*N*) parallel cylindrical channels coated with an active catalyst material, the cost of the monolith catalyst (*Cost*) can be estimated with Equation (16). catalyst material, the cost of the coating material for an individual catalyst channel is probince the coating thickness (*v*) is assumed to be constant for a specific type of an ∝ (16)

$$
Cost \propto NRL \tag{16}
$$

The last step of the algorithm involves the optimization of the cost-effectiveness of the monolith catalyst. After calculating the estimated cost for the active catalyst material, the algorithm finds the case with the minimal cost from the already sifted geometries with adequate performance both in terms of conversion and pressure drop. Therefore, the algorithm enables chemical engineers to find the cost-effectiveness-optimized monolith catalyst geometry which satisfies their performance and space constraint criteria.

3. Results and Discussion 3. Results and Discussion

The initial assumptions of our hypothetical case, along with the results of the initial calculations, are already presented in Section [2.](#page-1-0) The main aim of the initial calculations was to extract the relevant data to be used as the boundary conditions of the following CFD studies. T_{total} and T_{total} case, along with the results of the initial case, along with the initial case, al calculations, are already presented in Section 2. The mutation 2. The mutation 2. The mutation 2. The initial case, along with the results of the initial case. The CFD studies were performed on the geometries of cylindrical monolith channels

The CFD studies were performed on the geometries of cylindrical monolith channels with lengths of 150 mm and 109 different radii (*R*) varying from 0.50 mm up to 1.58 mm. The Reynolds numbers (*Re*) of our CFD studies varied between 143 and 453 (see Table [A1\)](#page-11-3).
The axial direction of the changing with the changing with the changing with the changing with the changing wi with life CrD studies were performed on the geometries of Cylliterical monomiticianties

The velocity field was, as expected, unchanging with the axial direction of the channel, Fire velocity includes, as expected, diferrally with the axial difection of the enancer-
since the inlet velocity profile inserted as the boundary condition was already parabolic—a since the lifet velocity profile inserted as the boundary condition was already parabolic—a
fully developed laminar flow (Figure [2a](#page-5-0)). The pressure inside the monolith channels nels monotonically decreased with increasing channel length, while it had close to zero variation incholonidally decreased with increasing characterisqui, while it had close to zero variation.
in the radial direction (Figure [2b](#page-5-0)). This type of spatial pressure variation is typical for In the radial direction (rigure 20). The laminar flow inside a channel.

Figure 2. Velocity (a) and pressure (b) distributions inside the cylindrical monolith catalyst channel with *R* = 0.565 and *L* = 150 mm. with *R* = 0.565 and *L* = 150 mm.

The concentrations of species A and B have high gradients near the inlet, while they settle to a more uniform distribution at the outlet (Figure [3\)](#page-6-0). Since both of the diffusion coefficients are the same, the contours of both of the species are inverse to each other, as expected. The same of the same, the species are inverse to each of the species are inverse to each other, as expected.

Figure 3. Contours of concentrations of species A (a) and B (b) inside the cylindrical monolith catalyst channel with $R = 0.565$ and $L = 150$ mm.

The conversions of species A varied from 0.4540 (at $R = 1.581$) to 0.9812 (at $R = 0.500$), while the pressure drops varied from 88 Pa (at $R = 1.581$) to 874 Pa (at $R = 0.500$). Small diameter channels exhibited high conversion accompanied with high pressure drop, and diameter channels exhibited high conversion accompanied with high pressure drop, and large diameter channels had low conversion with low pressure drop. Therefore, the trade-off between a high conversion and a high pressure drop is clearly present (Table [A1\)](#page-11-3), so the off between a high conversion and a high pressure drop is clearly present (Table A1), so engineers have to be aware of it and consider it during the design and dimensioning of the $\frac{1}{2}$ monolith catalysts. large diameter channels had low conversion with low pressure drop. Therefore, the trade-

The model order reduction results are also presented in Table [A1.](#page-11-3) The values of the coefficients k_c and k_p both monotonically decrease as the channel radius increases. The coefficient of determination (R^2) values for the PFR model varied from 0.881 to 0.995, and for the LPV model from 0.999997 to 1.0. The R^2 values monotonically decreased as the channel radius increased, which could be explained for the PFR model by an increase in departure from the PFR model's assumptions by an increase in the radial gradients of velocity and species concentrations. Since the algorithm firstly eliminated the geometries with insufficient conversion (with index values from 36 to 109), the MOR models with the smallest values of R^2 used in further calculations of optimal channel lengths were 0.991 and 1.0 for the PFR and the LPV models, respectively.

An example of model order reduction fits to radially averaged concentration of species A and pressure is presented in Figure [4.](#page-7-0)

Figure 4. Model order reduction fitting results of c_A (a) and p (b) for the cylindrical monolith catalyst channel with *R* = 0.565 and *L* = 150 mm. channel with *R* = 0.565 and *L* = 150 mm.

Table [1](#page-8-0) contains the channel geometries which enable sufficient conversion of 0.95, Table 1 contains the channel geometries which enable sufficient conversion of 0.95, and are optimized in length based on the calculated parameters of the PRF model in the MOR stage of the method. The length optimization reduced both the cost and the pressure MOR stage of the method. The length optimization reduced both the cost and the pressure drops by up to 34.4%, since both the used monolith material volume and the pressure drops by up to 34.4%, since both the used monolith material volume and the pressure drop are linearly dependent on the channel's length, originating from Equation (15) and the LPV model, respectively.

Index $($)	R (mm)	N(Δp (Pa)	L (mm)	CR $\left(\frac{9}{6}\right)$
21	0.548	5000	567	116.7	22.2
22	0.550	4950	564	117.2	21.9
23	0.553	4900	564	118.3	21.1
24	0.556	4850	563	119.4	20.4
25	0.559	4800	561	120.2	19.9
26	0.562	4750	563	122.0	18.7
27	0.565	4700	550	120.3	19.8
28	0.568	4650	549	121.4	19.1
29	0.571	4600	546	122.0	18.7
30	0.574	4550	546	123.3	17.8
31	0.577	4500	545	124.4	17.1
32	0.581	4450	547	126.3	15.8
33	0.584	4400	546	127.5	15.0
34	0.587	4350	548	129.4	13.7
35	0.591	4300	548	131.1	12.6

Table 1. *Cont.*

The bold values relate to the optimal monolith catalyst geometry.

After consideration of the maximal allowed pressure drop of 560 Pa, the number of viable geometries reduces to 9 (Table [2\)](#page-8-1). The optimal geometry among the list was selected based on the material cost estimated with Equation (16). The optimal monolith catalyst geometry was the one with index number 27, which had 4700 parallel cylindrical channels with radii of 0.565 mm and lengths of 120.3 mm. Compared to the viable monolith geometry with the highest estimated material cost (index 35), the optimal geometry reduced the material cost by 4.1%.

Table 2. The list of the monolith catalysts' dimensional characteristics, calculated pressure drops, lengths, and estimated material cost for achievement of the target conversion of 0.95 and pressure drop below the set limit of 560 Pa with accompanied cost reductions.

Index $($)	$R \, (mm)$	$N(\Lambda)$	Δp (Pa)	L (mm)	Cost(CR $\left(\% \right)$
27	0.565	4700	550	120.3	0.3194	4.1
28	0.568	4650	549	121.4	0.3206	3.7
29	0.571	4600	546	122.0	0.3205	3.7
30	0.574	4550	546	123.3	0.3221	3.2
31	0.577	4500	545	124.4	0.3232	2.9
32	0.581	4450	547	126.3	0.3263	2.0
33	0.584	4400	546	127.5	0.3276	1.6
34	0.587	4350	548	129.4	0.3306	0.7
35	0.591	4300	548	131.1	0.3329	0.0

The bold values relate to the optimal monolith catalyst geometry.

If we retrace the optimal monolith geometry with index number 27, we can observe from Table [1](#page-8-0) that shortening the length from the initial 150 mm reduced its material cost and pressure drop by 19.8%. The total cost reduction enabled by our monolith dimensioning methodology is hard to predict in real conditions and is dependent on the many factors (e.g., initial/reference catalyst design and geometry) and specific characteristics of the case.

Our estimation is that our methodology would enable chemical engineers to save precious catalyst material of up to 20% and at the same time reduce the operating costs of monolith catalysts via a reduced pressure drop.

4. Conclusions

The presented MOR model was able to successfully reduce the order of the modelling approach with sufficient accuracy and therefore is suitable for modelling small diameter (<2 mm) cylindrical channels, typically used in monolith catalysts. The two-dimensional CFD modelling results from a single cylindrical monolith channel are described using two simple algebraic equations. The first one deals with cross-sectional average of concentration (of species A; modified PFR model) with *R* ² values ranging from 0.881 to 0.995. The second one (LPV model) models the pressure evolution across the channel's length, with accuracy related to *R* ² values equal to or above 0.999997.

Our approach includes the assessment of the washcoat monolith's material cost with a simple method for assessing the consumption of an active catalyst material which coats the catalyst's support in a relatively thin (compared to the channel's dimensions) and uniform manner. The calculated cost of the active catalyst material served us in the selection process within the dimensioning optimization algorithm.

The newly developed computational fluid dynamics, transport, and chemical kineticsbased monolith catalyst dimensioning methodology enables chemical engineers to design and dimension monolith catalysts for many different applications in silico, which enable them to reduce both the material and operation costs while maintaining sufficient catalyst performance and, therefore, achieve its cost-effective performance. According to our results and estimations, the material savings could be in the range of up to 20%.

The presented monolith catalyst dimensioning methodology was conducted in a case of a washcoated monolith catalyst (WMC). However, it could easily be adapted to dimensioning a bulk monolithic catalyst (BMC) with alterations to the estimated material cost equation (Equation (16)).

Our methodology is relatively simple and easy to implement since the user has to know just a few basic initial conditions like space constrains, inflow conditions, the range in the number of catalyst channels, and performance targets, e.g., conversion and pressure drop. From there forward, the only real requirement is having a working computer with sufficient performance characteristics (e.g., computational power, random-access memory (RAM) size), and installed CFD software and some other data processing software which enables the user to virtually test an array of multiple possible monolith catalyst designs and geometries to extract the optimal one for the designated purpose. Therefore, the developed methodology not only reduces the material and operation costs, as already mentioned, but also saves engineers the substantial amount of time and effort required to experimentally test multiple potential catalyst geometries, since only the tests required for validation of the CFD calculations and the tests of the monolith catalyst prototype are necessary.

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Data Availability Statement: Data will be available on a reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. CFD Results and MOR Data Summary

Table A1. All of the considered monolith catalysts with initial length of 150 mm, their integral dimensional and performance characteristics, Reynolds numbers, and MOR model parameters with corresponding coefficients of determination (*R* 2).

Table A1. *Cont.*

Index(R (mm)	$N($ / $)$	Δp (Pa)	$Con_A($	Re(k_c (1/m)	R^2 (/)	k_p (Pa/m)	$R^2($
$\bf 4$	0.506	5850	852	0.9793	145	29.56	0.994750	5683.9	1.000000
5	0.509	5800	845	0.9793	146	29.55	0.994571	5635.2	1.000000
6	0.511	5750	838	0.9786	146	29.22	0.994666	5586.6	1.000000
7	0.513	5700	831	0.9776	147	29.06	0.994395	5539.0	1.000000
8	0.515	5650	824	0.9774	148	28.79	0.994704	5490.8	1.000000
9	0.518	5600	816	0.9772	148	28.77	0.994608	5443.5	1.000000
$10\,$	0.520	5550	809	0.9764	149	28.50	0.994424	5394.4	1.000000
$11\,$	0.522	5500	802	0.9753	150	28.21	0.994275	5345.4	1.000000
12	0.525	5450	794	0.9743	150	27.89	0.994155	5297.1	1.000000
13	0.527	5400	787	0.9737	151	27.64	0.994292	5248.7	1.000000
$14\,$	0.530	5350	780	0.9724	152	27.41	0.993711	5199.0	1.000000
15	0.532	5300	773	0.9716	153	27.16	0.993869	5151.5	1.000000
16	0.535	5250	765	0.9704	153	26.84	0.993438	5102.4	1.000000
17	0.537	5200	758	0.9700	154	26.73	0.993488	5053.8	1.000000
18	0.540	5150	751	0.9690	155	26.39	0.993145	5006.0	1.000000
19	0.542	5100	743	0.9682	156	26.28	0.993280	4957.4	1.000000
$20\,$	0.545	5050	736	0.9669	156	25.90	0.993056	4908.4	1.000000
$21\,$	0.548	5000	729	0.9660	157	25.68	0.992873	4860.0	1.000000
$22\,$	0.550	4950	722	0.9654	158	25.56	0.992800	4812.1	1.000000
23	0.553	4900	715	0.9644	159	25.32	0.992759	4764.0	1.000000
$24\,$	0.556	4850	707	0.9631	160	25.09	0.992401	4714.6	1.000000
25	0.559	4800	700	0.9626	160	24.93	0.992461	4666.9	1.000000
$26\,$	0.562	4750	693	0.9610	161	24.56	0.991998	4618.3	1.000000
27	0.565	4700	686	0.9622	162	24.91	0.992596	4571.0	1.000000
$28\,$	0.568	4650	678	0.9612	163	24.68	0.992391	4523.2	1.000000
29	0.571	4600	671	0.9604	164	24.55	0.992352	4475.5	1.000000
$30\,$	0.574	4550	664	0.9596	165	24.30	0.992217	4426.9	1.000000
$31\,$	0.577	4500	657	0.9583	166	24.08	0.992139	4378.2	1.000000
32	0.581	4450	649	0.9563	167	23.72	0.991320	4329.4	1.000000
33	0.584	4400	642	0.9550	167	23.49	0.991487	4280.6	1.000000
34	0.587	4350	635	0.9524	168	23.15	0.991095	4232.2	1.000000
35	0.591	4300	627	0.9513	169	22.86	0.990751	4183.1	1.000000
36	0.594	4250	620	0.9495	170	22.72	0.990327	4133.9	1.000000
37	0.598	4200	613	0.9472	171	22.30	0.989910	4084.7	1.000000
38	0.601	4150	605	0.9454	172	22.04	0.989998	4036.0	1.000000
39	0.605	4100	598	0.9438	173	21.87	0.989740	3987.0	1.000000
$40\,$	0.609	4050	591	0.9411	175	21.49	0.989604	3938.0	1.000000
$41\,$	0.612	4000	583	0.9374	176	20.96	0.989082	3888.7	1.000000
42	0.616	3950	576	0.9341	177	20.62	0.988426	3839.5	1.000000
43	0.620	3900	568	0.9303	178	20.24	0.987491	3790.2	1.000000
44	0.624	3850	561	0.9279	179	19.93	0.987649	3741.2	1.000000
$45\,$	0.628	3800	554	0.9245	180	19.59	0.987004	3691.9	1.000000
46	0.632	3750	546	0.9223	181	19.37	0.986508	3643.1	1.000000
$47\,$	0.637	3700	539	0.9205	183	19.15	0.986923	3595.3	1.000000
$\rm 48$	0.641	3650	532	0.9178	184	18.94	0.986445	3546.3	1.000000
49	0.645	3600	525	0.9155	185	18.71	0.986071	3497.8	0.999999
$50\,$	0.650	3550	517	0.9126	186	18.44	0.985406	3449.2	0.999999
51	0.655	3500	510	0.9100	188	18.21	0.985677	3400.8	0.999999
52	0.659	3450	503	0.9072	189	17.96	0.984682	3352.3	0.999999
53	0.664	3400	496	0.9048	190	17.79	0.984604	3304.4	1.000000
54	0.669	3350	488	0.9025	192	17.58	0.984443	3255.7	0.999999
55	0.674	3300	481	0.9002	193	17.40	0.984127	3207.8	0.999999
56	0.679	3250	474	0.8975	195	17.19	0.983736	3159.5	0.999999
57	0.685	3200	467	0.8938	196	16.93	0.983234	3110.9	0.999999
58	0.690	3150	459		198	16.86	0.983105	3062.5	0.999999
59	0.696	3100	452	0.8924 0.8877	200	16.51	0.982191	3013.9	0.999999
	0.701	3050	445	0.8867	201	16.43	0.982221	2966.3	0.999999
60									

Table A1. *Cont.*

The bold values relate to the optimal monolith catalyst geometry.

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