The core of a vertex-transitive complementary prism

Marko Orel [*]

University of Primorska, FAMNIT, Glagoljaška 8, Koper, Slovenia and
University of Primorska, IAM, Muzejski trg 2, Koper, Slovenia and
IMFM, Jadranska 19, Ljubljana, Slovenia

Abstract

The complementary prism $\Gamma\overline{\Gamma}$ is obtained from the union of a graph Γ and its complement $\overline{\Gamma}$ where each pair of identical vertices in Γ and $\overline{\Gamma}$ is joined by an edge. It generalizes the Petersen graph, which is the complementary prism of the pentagon. The core of a vertex-transitive complementary prism is studied. In particular, it is shown that a vertex-transitive complementary prism $\Gamma\overline{\Gamma}$ is a core, i.e. all its endomorphisms are automorphisms, whenever Γ is a core or its core is a complete graph.

Keywords: Graph homomorphism, core, complementary prism, self-complementary graph, vertex-transitive graph.

Math. Subj. Class. (2020): 05C60, 05C76

1 Introduction

In the study of graph homomorphism a basic object is a *core* (a.k.a. *unretractive graph*), which is a graph such that all its endomorphisms are automorphisms. A subgraph Γ' of a graph Γ is its core, if there exists some graph homomorphism $\varphi\colon \Gamma \to \Gamma'$ and Γ' is a core. Equivalently, Γ' is the minimal retract of Γ (cf. [17]). Despite that each graph has its core, which is unique up to isomorphism, it can be often very difficult to determine if a given graph is a core or not (cf. [6, 16, 30]). From this point of view, graphs that have either high degree of symmetry (i.e. 'large' automorphism group) or some 'nice' combinatorial properties are the most interesting. Many of such classes of graphs are *core-complete*, which

---

means that they are either cores or their cores are complete graphs. Among them we can find all non-edge-transitive graphs [6, Corollary 2.2], connected regular graphs with the automorphism group that acts transitively on unordered pairs of vertices at distance two [16, Theorem 4.1], and all primitive strongly regular graphs [36, Corollary 3.6]. Given a core-complete graph, it can be extremely complicated to decide if the graph is a core or its core is complete. For some graphs, this task is equivalent to some of the longstanding open problems in finite geometry (see [6, 30]). A well known core is the Petersen graph, which has both 'large' automorphism group and 'nice' combinatorial properties. Given a family of graphs that (naturally) generalize the Petersen graph it is interesting to study if its members are cores or not. Kneser graphs $K(n, r)$, with $2r < n$, are all cores [15, Theorem 7.9.1]. The graph $HGL_n(\mathbb{F}_4)$, whose vertex set is formed by all $n \times n$ invertible hermitian matrices over the field with four elements and with the edge set $\{\{A, B\} : \text{rank}(A - B) = 1\}$, is a core whenever $n \geq 2$ [29]. The core of a generalized Petersen graph $G(n, k)$ was studied very recently in [13]. The complementary prism $\Gamma\bar{\Gamma}$, whose definition in full details is given in Section 2, is another generalization of the Petersen graph, which is obtained if Γ is the 5-cycle C_5[1]. Graph $\Gamma\bar{\Gamma}$ was introduced in [18] and is the main mater of research in several papers (see for example [1, 3, 7, 10, 19, 26]). Recall that C_5 is strongly regular vertex-transitive self-complementary graph. The core of $\Gamma\bar{\Gamma}$ was recently studied by the author [28] (see also the arXiv version [31]). In particular, it was shown that $\Gamma\bar{\Gamma}$ is a core whenever Γ is strongly regular and self-complementary. In this paper we build on a result from [28] and investigate vertex-transitive self-complementary graphs Γ. These are precisely the graphs that provide vertex-transitive complementary prisms [31, Corollary 3.8]. For such graphs we prove that $\Gamma\bar{\Gamma}$ is a core whenever Γ is core-complete (see Theorem 3.3), and state an open problem, which asks if there exists a vertex-transitive self-complementary graph Γ such that $\Gamma\bar{\Gamma}$ is not a core (Problem 3.6). The main results are presented in Section 3. In Section 2 we recall some tools and definitions that we need in what follows.

2   Preliminaries

All graphs in this paper are finite and simple. The vertex set and the edge set of a graph Γ are denoted by $V(\Gamma)$ and $E(\Gamma)$, respectively. A subset of pairwise adjacent vertices in $V(\Gamma)$ is a *clique*, while a set of pairwise nonadjacent vertices in $V(\Gamma)$ is an *independent set*. The *clique number* $\omega(\Gamma)$ and the *independence number* $\alpha(\Gamma)$ are the orders of the largest clique and the largest independent set in Γ, respectively. In particular, $\alpha(\Gamma) = \omega(\bar{\Gamma})$, where $\bar{\Gamma}$ is the complement of the graph Γ. The chromatic number of a graph is denoted by $\chi(\Gamma)$. It is well known that $\chi(\Gamma) \geq \omega(\Gamma)$ and $\chi(\Gamma) \geq \frac{n}{\alpha(\Gamma)}$, where $n = |V(\Gamma)|$ (cf. [5]). A *graph homomorphism* between graphs Γ_1, Γ_2 is a map $\varphi \colon V(\Gamma_1) \to V(\Gamma_2)$ such that $\{\varphi(u), \varphi(v)\} \in E(\Gamma_2)$ whenever $\{u, v\} \in E(\Gamma_1)$. If in addition φ is bijective and $\{u, v\} \in E(\Gamma_1) \iff \{\varphi(u), \varphi(v)\} \in E(\Gamma_2)$, then φ is a *graph isomorphism* and graphs Γ_1, Γ_2 are *isomorphic*, which we denote by $\Gamma_1 \cong \Gamma_2$. If $\Gamma_1 = \Gamma_2$, then a graph homomorphism/graph isomorphism is a graph *endomorphism/automorphism*, respectively. A graph Γ is *self-complementary* if there exists a graph isomorphism $\sigma \colon \Gamma \to \bar{\Gamma}$, which is referred to as *antimorphism* or a *complementing permutation*. Observe that in this case σ is also an antimorphism as a map $\bar{\Gamma} \to \Gamma$. A graph Γ is regular if each vertex has the same number of neighbors. If this number equals k, then we say that it is k-*regular*.

---

[1]Graphs $K(5, 2)$, $HGL_2(\mathbb{F}_4)$, $G(5, 2)$, $C_5\overline{C_5}$ are all isomorphic to the Petersen graph.

If for each pair of vertices $u, v \in V(\Gamma)$ there exists an automorphism φ of Γ such that $u = \varphi(v)$, then Γ is a *vertex-transitive* graph. Clearly, each vertex-transitive graph is regular. Lemma 2.1 can be found in [14, Corollaries 2.1.2, 2.1.3], where it is stated in a more general settings.

Lemma 2.1. *If a graph Γ is vertex-transitive, then $\alpha(\Gamma)\omega(\Gamma) \leq |V(\Gamma)|$. If the equality holds, then $|C \cap I| = 1$ for each clique C and each independent set I that provide the equality.*

If a graph on n vertices is $(\frac{n-1}{2})$-regular, then n must be odd. Moreover, it follows from the hand-shaking lemma that $n = 4m + 1$ for some integer $m \geq 0$. In particular, this is true for all regular self-complementary graphs. By a result of Sachs [37] or Ringel [35], each cycle in an antimorphism of a self-complementary graph has the length divisible by four, except for one cycle of length one, in the case the order of the graph equals 1 modulo 4 (a proof in English can be found in [11, page 12]). Lemma 2.2 is a special case of this fact, and is crucial in the proof of Theorem 3.3.

Lemma 2.2. *If σ is an antimorphism of a regular self-complementary graph Γ, then there exists a unique vertex $v \in V(\Gamma)$ such that $\sigma(v) = v$.*

A graph Γ is a *core* if each its endomorphism is an automorphism. Given a graph Γ, we use core(Γ) to denote any subgraph of Γ that is a core and such that there exists some graph homomorphism $\varphi \colon \Gamma \to \text{core}(\Gamma)$. Graph core$(\Gamma)$ is referred to as the *core of* Γ. It is always an induced subgraph and unique up to isomorphism [15, Lemma 6.2.2]. Clearly, a graph Γ is a core if and only if $\Gamma = \text{core}(\Gamma)$. On the other hand, core(Γ) is a complete graph if and only if $\chi(\Gamma) = \omega(\Gamma)$. We remark that there always exists a *retraction* $\psi \colon \Gamma \to \text{core}(\Gamma)$, i.e. a graph homomorphism that fixes each vertex in core(Γ). In fact, if $\varphi \colon \Gamma \to \text{core}(\Gamma)$ is any graph homomorphism, then the restriction $\varphi|_{V(\text{core}(\Gamma))}$ is invertible and the composition $(\varphi|_{V(\text{core}(\Gamma))})^{-1} \circ \varphi$ is the required retraction. Lemma 2.3 is proved in [39, Theorem 3.2], where it is stated in an old terminology. Its proof can be found also in [15, 17].

Lemma 2.3. *If graph Γ is vertex-transitive, then core(Γ) is vertex-transitive.*

Let Γ be a graph with the vertex set $V(\Gamma) = \{v_1, \ldots, v_n\}$. The *complementary prism* of Γ is the graph $\Gamma\bar{\Gamma}$, which is obtained from the disjoint union of Γ and its complement $\bar{\Gamma}$, by adding an edge between each vertex in Γ and its copy in $\bar{\Gamma}$. In this paper we use the following notation. The vertex set of the complementary prism of graph Γ is the set $V(\Gamma\bar{\Gamma}) = W_1 \cup W_2$, where

$$W_1 = W_1(\Gamma\bar{\Gamma}) = \{(v_1, 1), \ldots, (v_n, 1)\} \text{ and } W_2 = W_2(\Gamma\bar{\Gamma}) = \{(v_1, 2), \ldots, (v_n, 2)\}.$$

The edge set $E(\Gamma\bar{\Gamma})$ is the union of the sets

$$\big\{\{(u, 1), (v, 1)\} : \{u, v\} \in E(\Gamma)\big\},$$
$$\big\{\{(u, 2), (v, 2)\} : \{u, v\} \in E(\bar{\Gamma})\big\},$$
$$\big\{\{(u, 1), (u, 2)\} : u \in V(\Gamma)\big\}.$$

It follows from the definition that a complementary prism $\Gamma\bar{\Gamma}$ is regular if and only if Γ is $(\frac{n-1}{2})$-regular (see also [7, Theorem 3.6]). The core of a complementary prism for general graph Γ was recently studied in [28] (see also the arXiv version [31]). For regular case, the following result was proved.

Lemma 2.4 ([28], Corollary 3.4])**.** *Let Γ be any graph on n vertices that is $\left(\frac{n-1}{2}\right)$-regular. Then one of the following three possibilities is true.*

(i) *Graph $\Gamma\bar{\Gamma}$ is a core.*

(ii) *All vertices of $core(\Gamma\bar{\Gamma})$ are contained in W_1, in which case*

$$core(\Gamma\bar{\Gamma}) \cong core(\Gamma).$$

(iii) *All vertices of $core(\Gamma\bar{\Gamma})$ are contained in W_2, in which case*

$$core(\Gamma\bar{\Gamma}) \cong core(\bar{\Gamma}).$$

The same conclusion can be obtained also if the core of $\Gamma\bar{\Gamma}$ is regular and we exclude some small graphs. Below, K_2 is a complete graph on two vertices, and P_3 is a path on three vertices.

Lemma 2.5 ([28], Corollary 3.6])**.** *Let Γ be any graph, which is not isomorphic to K_2, $\overline{K_2}$, P_3, or $\overline{P_3}$. If $core(\Gamma\bar{\Gamma})$ is regular, then one of the three possibilities in* Lemma 2.4 *is true.*

Clearly, Lemma 2.4 is valid for each regular self-complementary graph Γ. The study of such graphs and their vertex-transitive counterparts has origins in the papers [21, 37, 40], which influenced a lot of research related to vertex-transitive self-complementary graphs (see for example [2, 4, 8, 9, 12, 20, 22, 23, 24, 25, 27, 33, 34, 38, 41] and the references therein). In this paper the aim is to to study the core of a complementary prism $\Gamma\bar{\Gamma}$, where Γ is vertex-transitive and self-complementary graph. The following observation is obtained for free, with a double proof.

Corollary 2.6. *If graph Γ is vertex-transitive and self-complementary graph, then one of the three possibilities in* Lemma 2.4 *is true.*

Proof. The claim follows directly from Lemma 2.4. The same claim is deduced also if we combine Lemmas 2.5 and 2.3. □

In [28] some examples of regular self-complementary graphs Γ are provided such that the statement (ii) or (iii) in Lemma 2.4 is true. In this paper we show that this is not possible for a large class of vertex-transitive self-complementary graphs. It should be mentioned that it was recently proved that a complementary prism $\Gamma\bar{\Gamma}$ is vertex-transitive if and only if Γ is vertex-transitive and self-complementary [31, Corollary 3.8]. Despite our proofs do not rely on this result, it means that this paper studies the core of vertex-transitive complementary prisms.

3　Main results

The main result of this paper is Theorem 3.3. Propositions 3.1 and 3.2 are the stepping stones towards its proof.

Proposition 3.1. *Let Γ be a regular self-complementary graph. If Γ is a core, then $\Gamma\bar{\Gamma}$ is a core.*

Proof. Let core($\Gamma\bar{\Gamma}$) be any core of Γ and let $n = |V(\Gamma)|$. Since Γ is $\left(\frac{n-1}{2}\right)$-regular, one of the statements (i), (ii), (iii) in Lemma 2.4 is true. Suppose that (iii) is correct, that is,

$$V(\text{core}(\Gamma\bar{\Gamma})) \subseteq W_2 \tag{3.1}$$

and

$$\text{core}(\Gamma\bar{\Gamma}) \cong \text{core}(\bar{\Gamma}). \tag{3.2}$$

Since $\bar{\Gamma}$ is a core, (3.2) implies that core($\Gamma\bar{\Gamma}$) $\cong \bar{\Gamma}$. Hence, (3.1) yields

$$V(\text{core}(\Gamma\bar{\Gamma})) = W_2. \tag{3.3}$$

Let $\psi_1(v) = (v, 1)$, for $v \in V(\Gamma)$, be the canonical isomorphism between Γ and the subgraph of $\Gamma\bar{\Gamma}$, which is induced by the set W_1. Similarly, let $\psi_2(v) = (v, 2)$, for $v \in V(\Gamma)$, be the canonical isomorphism between $\bar{\Gamma}$ and the subgraph induced by W_2. If Ψ is any retraction from $\Gamma\bar{\Gamma}$ onto core($\Gamma\bar{\Gamma}$), and σ is any antimorphism between $\bar{\Gamma}$ and Γ, then the composition $\sigma \circ \psi_2^{-1} \circ (\Psi|_{W_1}) \circ \psi_1$ is an endomorphism of Γ. Since Γ is a core, the restriction $\Psi|_{W_1}$ is an isomorphism between the subgraphs in $\Gamma\bar{\Gamma}$ that are induced by the sets W_1 and W_2, respectively. Consequently $\psi_2^{-1} \circ (\Psi|_{W_1}) \circ \psi_1 \colon \Gamma \to \bar{\Gamma}$ is an antimorphism. By Lemma 2.2, there exists $v \in V(\Gamma)$ such that $\left(\psi_2^{-1} \circ (\Psi|_{W_1}) \circ \psi_1\right)(v) = v$. Consequently, $\Psi(v, 1) = (\Psi|_{W_1})(v, 1) = (v, 2)$. Since Ψ is a retraction, (3.3) implies that $\Psi(v, 2) = (v, 2)$. Since $\{(v, 1), (v, 2)\}$ is an edge in $\Gamma\bar{\Gamma}$, we have a contradiction.

In the same way we see that (ii) in Lemma 2.4 is not possible, which means that $\Gamma\bar{\Gamma}$ is a core. $\square$

Proposition 3.2. *If Γ is a vertex-transitive self-complementary graph on $n > 1$ vertices, then core($\Gamma\bar{\Gamma}$) is not a complete graph.*

Proof. We need to prove that $\chi(\Gamma\bar{\Gamma}) > \omega(\Gamma\bar{\Gamma})$. Since $n > 1$ and Γ is both self-complementary and vertex-transitive, Lemma 2.1 implies that

$$\omega(\Gamma\bar{\Gamma}) = \max\{\alpha(\Gamma), \omega(\Gamma)\} = \omega(\Gamma) \leq \sqrt{n}.$$

Let I be any independent set in $\Gamma\bar{\Gamma}$. Then I is a disjoint union of some sets $I_1 \subseteq W_1$ and $I_2 \subseteq W_2$. If we write $I_i = \{(u, i) : u \in J_i\}$ for $i \in \{1, 2\}$, where $J_1, J_2 \subseteq V(\Gamma)$, then

$$J_1 \cap J_2 = \emptyset. \tag{3.4}$$

Since J_1 and J_2 are an independent set and a clique in Γ, respectively, we have $|J_1| \leq \alpha(\Gamma) = \omega(\Gamma) \leq \sqrt{n}$ and $|J_2| \leq \omega(\Gamma) = \sqrt{n}$, while Lemma 2.1 and (3.4) imply that $|J_1| \cdot |J_2| < n$. Hence, $|I| = |J_1| + |J_2| < 2\sqrt{n}$, and therefore

$$\chi(\Gamma\bar{\Gamma}) \geq \frac{|V(\Gamma\bar{\Gamma})|}{\alpha(\Gamma\bar{\Gamma})} > \frac{2n}{2\sqrt{n}} = \sqrt{n} \geq \omega(\Gamma\bar{\Gamma}).$$

$\square$

Theorem 3.3. *Let Γ be a vertex-transitive self-complementary graph. If Γ is either a core or its core is a complete graph, then $\Gamma\bar{\Gamma}$ is a core.*

Proof. If Γ is a core, then the claim follows from Proposition 3.1. Hence, we may assume that Γ has a complete core and more than one vertex. By Corollary 2.6, one of the statements (i), (ii), (iii) in Lemma 2.4 is true for core($\Gamma\bar{\Gamma}$). If (ii) or (iii) is correct, then the self-complementarity implies that core($\Gamma\bar{\Gamma}$) is complete, which contradicts Proposition 3.2. □

Remark 3.4. The claims in Proposition 3.2 and Theorem 3.3 are not true for some regular self-complementary graphs. In fact, there exists a regular self-complementary graph Γ with a complete core such that core($\Gamma\bar{\Gamma}$) is complete (see [28, Example 3.5] or [31, Example 5.5]).

Recall that a k-regular graph on n vertices is *strongly regular* with parameters (n, k, λ, μ) if each pair of adjacent vertices has λ common neighbors and each pair of distinct non-adjacent vertices has μ common neighbors. Theorem 3.5 was very recently proved in [28] (see also the arXiv version [31, Theorem 5.7]). The proof relied on application of Lemma 2.4 together with several properties of the Lovász theta function and the graph spectrum. Here we provide a sketch of an alternative proof that essentially copies the proofs in this section and applies a remarkable result that Roberson recently proved [36].

Theorem 3.5 ([28]). *If Γ is a strongly regular self-complementary graph, then $\Gamma\bar{\Gamma}$ is a core.*

Sketch of a proof. To see that $\Gamma\bar{\Gamma}$ does not have a complete core, we copy the proof of Proposition 3.2, where we replace the application of Lemma 2.1 by its analog that can be found in [14, Corolarry 3.8.6 and Theorem 3.8.4] and is valid for all strongly regular graphs. From [36, Corollary 3.6] it follows that Γ is either a core or its core is complete. Then we just copy the proof of Theorem 3.3, where we rely directly on Lemma 2.4 instead on Corollary 2.6. □

Recall from the introduction that many 'nice' graphs are either cores or their cores are complete. Hence it is expected that many vertex-transitive self-complementary graphs fulfil the assumptions in Theorem 3.3. Consequently, we state the following open problem.

Problem 3.6. Does there exist a vertex-transitive self-complementary graph Γ such that $\Gamma\bar{\Gamma}$ is not a core?

Note that edge-transitive self-complementary graphs are also arc-transitive (see [11]). Since self-complementary graphs are always connected (cf. [11]), it follows that each edge-transitive self-complementary graph is also vertex-transitive. However, such graphs are always strongly regular (cf. [11]) and therefore their complementary prisms are cores by Theorem 3.5. Despite the orders of vertex-transitive self-complementary graphs were fully determined in [27], there is a major gap between the understanding of vertex-transitive self-complementary graphs and the understanding of edge-transitive self-complementary graphs. In fact, the later were completely characterized in [33]. Moreover, the first non-Cayley vertex-transitive self-complementary graph was constructed only in 2001 [24], and the construction is highly nontrivial. We believe that all these facts indicate that Problem 3.6 may be challenging.

In a distinct paper [32], we consider the only families of vertex-transitive self-complementary graphs the author is aware of, which are neither cores nor their cores are complete graphs (i.e. they do not satisfy the assumption in Theorem 3.3). Unfortunately they do not solve Problem 3.6.

ORCID iDs

Marko Orel ⓘ https://orcid.org/0000-0002-2544-2986

References

[1] A. Alhashim, W. J. Desormeaux and T. W. Haynes, Roman domination in complementary prisms, *Australas. J. Comb.* **68** (2017), 218–228, https://ajc.maths.uq.edu.au/pdf/68/ajc_v68_p218.pdf.

[2] B. Alspach, J. Morris and V. Vilfred, Self-complementary circulant graphs, *Ars Comb.* **53** (1999), 187–191.

[3] R. M. Barbosa, M. R. Cappelle and E. M. M. Coelho, Maximal independent sets in complementary prism graphs, *Ars Comb.* **137** (2018), 283–294.

[4] R. A. Beezer, Sylow subgraphs in self-complementary vertex transitive graphs, *Expo. Math.* **24** (2006), 185–194, doi:10.1016/j.exmath.2005.09.003, https://doi.org/10.1016/j.exmath.2005.09.003.

[5] A. E. Brouwer and W. H. Haemers, *Spectra of Graphs*, Universitext, Springer, New York, 2012.

[6] P. J. Cameron and P. A. Kazanidis, Cores of symmetric graphs, *J. Aust. Math. Soc.* **85** (2008), 145–154, doi:10.1017/S1446788708000815, https://doi.org/10.1017/S1446788708000815.

[7] D. M. Cardoso, P. Carvalho, M. A. A. de Freitas and C. T. M. Vinagre, Spectra, signless Laplacian and Laplacian spectra of complementary prisms of graphs, *Linear Algebra Appl.* **544** (2018), 325–338, doi:10.1016/j.laa.2018.01.020, https://doi.org/10.1016/j.laa.2018.01.020.

[8] E. Dobson, On self-complementary vertex-transitive graphs of order a product of distinct primes, *Ars Comb.* **71** (2004), 249–256.

[9] E. Dobson and M. Šajna, Almost self-complementary circulant graphs, *Discrete Math.* **278** (2004), 23–44, doi:10.1016/j.disc.2003.05.002, https://doi.org/10.1016/j.disc.2003.05.002.

[10] M. A. Duarte, L. Penso, D. Rautenbach and U. d. S. Souza, Complexity properties of complementary prisms, *J. Comb. Optim.* **33** (2017), 365–372, doi:10.1007/s10878-015-9968-5, https://doi.org/10.1007/s10878-015-9968-5.

[11] A. Farrugia, *Self-complementary graphs and generalisations: a comprehensive reference manual*, Master's thesis, University of Malta, Malta, 1999, http://www.alastairfarrugia.net/sc-graph.html.

[12] D. Fronček, A. Rosa and J. Širáň, The existence of selfcomplementary circulant graphs, *European J. Comb.* **17** (1996), 625–628, doi:10.1006/eujc.1996.0053, https://doi.org/10.1006/eujc.1996.0053.

[13] I. García-Marco and K. Knauer, Beyond symmetry in generalized petersen graphs, 2022, arXiv:2202.06785 [math.CO].

[14] C. Godsil and K. Meagher, *Erdős-Ko-Rado Theorems: Algebraic Approaches*, volume 149 of *Camb. Stud. Adv. Math.*, Cambridge University Press, Cambridge, 2016, doi:10.1017/CBO9781316414958, https://doi.org/10.1017/CBO9781316414958.

[15] C. Godsil and G. Royle, *Algebraic Graph Theory*, volume 207 of *Grad. Texts Math.*, Springer, New York, 2001.

[16] C. Godsil and G. F. Royle, Cores of geometric graphs, *Ann. Comb.* **15** (2011), 267–276, doi:10.1007/s00026-011-0094-5, https://doi.org/10.1007/s00026-011-0094-5.

[17] G. Hahn and C. Tardif, Graph homomorphisms: Structure and symmetry, in: *Graph Symmetry, Algebraic Methods and Applications. Proceedings of the NATO Advanced Study Institute and séminaire de mathématiques supérieures,Montréal, Canada*, Kluwer Academic Publishers, Dordrecht, pp. 107–166, 1997.

[18] T. W. Haynes, M. A. Henning, P. J. Slater and L. C. van der Merwe, The complementary product of two graphs, *Bull. Inst. Comb. Appl.* **51** (2007), 21–30.

[19] T. W. Haynes, M. A. Henning and L. C. van der Merwe, Domination and total domination in complementary prisms, *J. Comb. Optim.* **18** (2009), 23–37, doi:10.1007/s10878-007-9135-8, https://doi.org/10.1007/s10878-007-9135-8.

[20] R. Jajcay and C. H. Li, Constructions of self-complementary circulants with no multiplicative isomorphisms, *European J. Comb.* **22** (2001), 1093–1100, doi:10.1006/eujc.2001.0529, https://doi.org/10.1006/eujc.2001.0529.

[21] A. Kotzig, Selected open problems in graph theory, in: J. A. Bondy and U. S. R. Murty (eds.), *Graph Theory and Related Topics*, Academic Press, New York, pp. 358–367, 1979.

[22] C. H. Li, On self-complementary vertex-transitive graphs, *Comm. Algebra* **25** (1997), 3903–3908, doi:10.1080/00927879708826094, https://doi.org/10.1080/00927879708826094.

[23] C. H. Li, On finite graphs that are self-complementary and vertex-transitive, *Australas. J. Comb.* **18** (1998), 147–155, https://ajc.maths.uq.edu.au/pdf/18/ocr-ajc-v18-p147.pdf.

[24] C. H. Li and C. E. Praeger, Self-complementary vertex-transitive graphs need not be Cayley graphs, *Bull. Lond. Math. Soc.* **33** (2001), 653–661, doi:10.1112/s0024609301008505, https://doi.org/10.1112/s0024609301008505.

[25] C. H. Li, G. Rao and S. J. Song, On finite self-complementary metacirculants, *J. Algebraic Comb.* **40** (2014), 1135–1144, doi:10.1007/s10801-014-0522-9, https://doi.org/10.1007/s10801-014-0522-9.

[26] D. Meierling, F. Protti, D. Rautenbach and A. R. de Almeida, Cycles in complementary prisms, *Discrete Appl. Math.* **193** (2015), 180–186, doi:10.1016/j.dam.2015.04.016, https://doi.org/10.1016/j.dam.2015.04.016.

[27] M. Muzychuk, On Sylow subgraphs of vertex-transitive self-complementary graphs, *Bull. Lond. Math. Soc.* **31** (1999), 531–533, doi:10.1112/S0024609399005925, https://doi.org/10.1112/S0024609399005925.

[28] M. Orel, The core of a complementary prism, accepted for publication in Journal of Algebraic Combinatorics.

[29] M. Orel, On generalizations of the Petersen graph and the Coxeter graph, *Electron. J. Comb.* **22** (2015), research paper p4.27, 17 pp., doi:10.37236/3759, https://doi.org/10.37236/3759.

[30] M. Orel, On Minkowski space and finite geometry, *J. Comb. Theory, Ser. A* **148** (2017), 145–182, doi:10.1016/j.jcta.2016.12.004, https://doi.org/10.1016/j.jcta.2016.12.004.

[31] M. Orel, A family of non-Cayley cores based on vertex-transitive or strongly regular self-complementary graphs, 2021, arXiv:2110.10416 [math.CO].

[32] M. Orel, The core of a vertex-transitive complementary prism of a lexicographic product, *Art Discrete Appl. Math.* (2023), doi:10.26493/2590-9770.1623.7f0, https://doi.org/10.26493/2590-9770.1623.7f0.

[33] W. Peisert, All self-complementary symmetric graphs, *J. Algebra* **240** (2001), 209–229, doi: 10.1006/jabr.2000.8714, https://doi.org/10.1006/jabr.2000.8714.

[34] S. B. Rao, On regular and strongly-regular self-complementary graphs, *Discrete Math.* **54** (1985), 73–82, doi:10.1016/0012-365X(85)90063-9, https://doi.org/10.1016/0012-365X(85)90063-9.

[35] G. Ringel, Selbstkomplementäre Graphen, *Arch. Math. (Basel)* **14** (1963), 354–358, doi:10.1007/bf01234967, https://doi.org/10.1007/bf01234967.

[36] D. E. Roberson, Homomorphisms of strongly regular graphs, *Algebr. Comb.* **2** (2019), 481–497, doi:10.5802/alco.50, https://doi.org/10.5802/alco.50.

[37] H. Sachs, Über selbstkomplementäre Graphen, *Publ. Math. Debrecen* **9** (1962), 270–288.

[38] D. A. Suprunenko, Self-complementary graphs, *Cybern. Syst. Anal.* **21** (1985), 559–567, doi: 10.1007/bf01074707, https://doi.org/10.1007/bf01074707.

[39] E. Welzl, Symmetric graphs and interpretations, *J. Comb. Theory, Ser. B* **37** (1984), 235–244, doi:10.1016/0095-8956(84)90056-X, https://doi.org/10.1016/0095-8956(84)90056-X.

[40] B. Zelinka, Self-complementary vertex-transitive undirected graphs, *Math. Slovaca* **29** (1979), 91–95.

[41] H. Zhang, Self-complementary symmetric graphs, *J. Graph Theory* **16** (1992), 1–5, doi:10.1002/jgt.3190160102, https://doi.org/10.1002/jgt.3190160102.