



# A triangular representation of the volumetric properties of asphalt mixtures

Marjan Tušar<sup>a,b</sup>, Mojca Ravnikar Turk<sup>b</sup>, Lidija Ržek<sup>b,\*</sup>

<sup>a</sup> National Institute of Chemistry, Hajdrihova 19, Ljubljana, Slovenia

<sup>b</sup> Slovenian National Building and Civil Engineering Institute, Dimičeva 12, Ljubljana, Slovenia

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## ABSTRACT

The volumetric properties of asphalt mixtures are important parameters for evaluating the performance of asphalt. Volumetric properties (grading requirements, air void content, minimum and maximum voids filled with bitumen and voids in mineral aggregate) are usually presented first in specifications. The visualisation of volumetric properties is therefore important for the understanding of the composition of an asphalt mix. A triangular representation of requirements for the volumetric properties of asphalt mixtures is therefore a valuable method to illustrate the volumetric requirements. Such representation gives a better insight into the requirements and helps on decisions when authorities decide about these requirements.

## 1. Introduction

In the past, the specifications for the volumetric properties of asphalt mixtures were the most important parameters for evaluating the performance of asphalt. However, since the adoption of the modern European series of standards (EN 13108), the mechanical properties of asphalt mixtures are now considered more significant. Nevertheless, when asphalt mixtures are categorised into different types, they are still mostly described volumetrically. Visualisation is important in understanding the composition of an asphalt mix and two-dimensional graphical representations are often used for simple visualisation.

An asphalt mixture primarily consists of mineral aggregates, bitumen binder and air voids. Grading requirements, including air void content ( $V_{min}$  and  $V_{max}$ ), minimum and maximum voids filled with bitumen ( $V_{FBmin}$  and  $V_{FBmax}$ ), voids in mineral aggregate ( $V_{MAmin}$  and  $V_{MAmax}$ ) and bitumen content ( $V_{Bmin}$  and  $V_{Bmax}$ ), can therefore be specified for each type of asphalt mixture.

Asphalt mixtures represent highly versatile construction materials that are used for heavily trafficked roads and airport runways, as well as for pedestrian zones and playgrounds or as sealing elements for reservoirs and dams. To optimise an asphalt mixture for its planned use and prolong its service lifetime, it is essential to design it properly. The design of asphalt mixtures has been evolving for decades. To begin with, it was a relatively stochastic process. The first instructions for preparing an asphalt mixture were given as early as the end of the nineteenth century [1]. At that time, the design was focused on the proper selection of an aggregate gradation and the correct amount of bitumen to add. In

1905, Clifford Richardson described two types of asphalt mixtures for surfaces and lower courses. For lower courses, he calculated the void content in the mineral aggregate (VMA) to include the correct amount of bitumen and for the surface mixture he designed a simple “asphalt paper test” for the determination of the bitumen content [2].

In the 1920s and 1930s, a new mix design method, the Hubbard Field method, was developed. The volumetric analysis for this method is similar to that used today and it also uses a stability test as a criterion for asphalt mixtures. According to this method, the binder content is selected regarding air voids and stability [3]. In 1927, Francis Hveem developed a new mix design method. He proposed that a sufficient amount of binder is needed to satisfy aggregate absorption and to have a minimum film thickness on the surface of the aggregates. Furthermore, the mechanical strength of the mix is important for the design. However, the air void content is not part of the Hveem mix design [4]. In 1939, another systematic approach to mixture design was introduced by Bruce G. Marshall [5]. This was an upgrade of the Hubbard Field method, but instead of voids in the aggregate (VMA), he used voids filled with binder and the stability of the mixture as criteria. Later, the VMA was also added to the Marshall design method as a mix design criterion. The development of mix design methods is presented in Table 1. The Marshall method approach was seriously modified as late as the 1990s.

The Strategic Highway Research Program was established in 1987 to improve the performance and durability of highways in the USA [6]. As a result of this work, the Superpave mix design method was developed based on a sufficient amount of binder for durability, sufficient voids in the mineral aggregate and the paving mix as a whole, sufficient

\* Corresponding author.

E-mail address: [lidija.rzek@zag.si](mailto:lidija.rzek@zag.si) (L. Ržek).

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workability and also satisfactory performance characteristics over the service life of the pavement [7,8]. The original Superpave volumetric mix design procedure did not include any mechanical tests and solely relied on strict conformance to the material specifications and volumetric mix criteria to ensure satisfactory performance of the mix designs. As an addition, a simple performance test was added to the Superpave mix design [9]. The design of asphalt mixes in Europe is mainly based on the Marshall method. However, the design also uses material and performance tests to complement the design process with the fundamental properties of the asphalt mixes and performance-related properties [10]. Nowadays, asphalt mixtures are usually prepared according to the traditional Marshall asphalt design or the Superpave mix design method.

The newest mix design, balanced mix design [11] concept, was initially developed by researchers at the Texas A&M Transportation Institute using the Hamburg Wheel Tracking Test to evaluate rutting resistance and the Overlay Tester to evaluate cracking resistance. This approach uses performance tests to determine binder content and grade that provided adequate resistance to rutting and load associated cracking.

From Table 1 it can be seen that volumetric properties are important for performance of the asphalt mixtures.

The first aim of this study is to present a triangular representation of volumetric properties that can serve as a representation of the volumetric requirement additional to the ordinary representation in a table.

The second aim of this study is to visualise where in relation to requirements the asphalt mix design is, when the asphalt mix design is prepared according to the Marshall Procedure.

Finally, if the asphalt mixture's properties are related to volume, e.g., noise, this representation can serve as platform for optimising asphalt mixture with alternative methods (Simplex, experimental design procedure...).

## 2. A triangular representation of the volumetric properties of asphalt mixtures

In practice, a sieving curve of stone aggregates is still the only parameter that is usually graphically presented. A triangular representation of the volumetric properties enables graphical presentation among the three main components of asphalt: stone aggregate content designated 'stone', the bitumen content 'bitumen' and the air void content 'void' (Fig. 1). Mineral stone aggregates or 'stone' are presented as a whole (including filler). In the equilateral triangle presented in Fig. 1, it was assumed that the 100% content of the 'stone' is on the vertex, 100% content of 'air' is on the right side and 100% 'bitumen' content is on the left side. In the centre of the triangle, each component represents one third (33%) of the volume (red dot in Fig. 1). The void

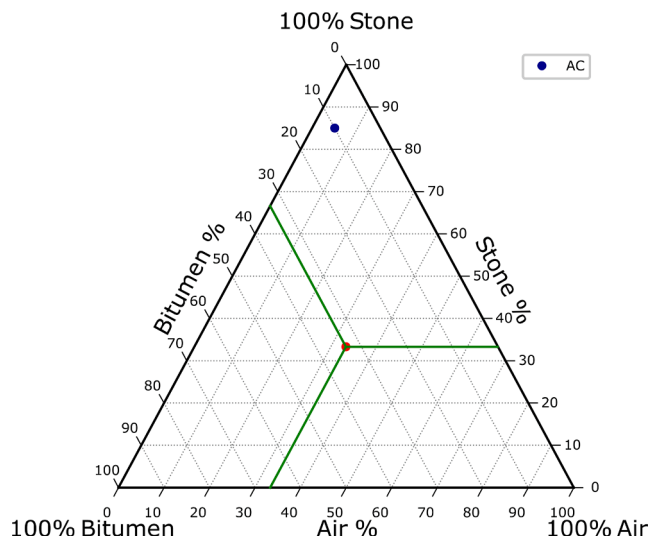


Fig. 1. Basic asphalt components represented by volume as a triangle.

content (V), bitumen content (VB) and voids filled with bitumen (VFB) in requirements are commonly presented only as numbers. In triangular representation void content (V), bitumen content (VB) can be graphically represented as two main axis of triangle and voids filled with bitumen (VFB) as ratio between void content (V) and bitumen content (VB). Certain values of void content (V), bitumen content (in volume percent) and stone aggregate contents (in volume percent) in such representation are lines parallel to the main sides of the triangle. Certain asphalt mixture design is in such representation illustrated as a point. Certain value of voids filled with bitumen (VFB) is line connecting the upper vertex of the triangle and the lower side of the triangle.

Typical asphalt mixtures for European roads include asphalt concrete (AC), porous asphalt (PA) and stone mastic asphalt (SMA). It is well known that AC consists of ~ 85 vol% of stone aggregate, 5 vol% of air voids and 10 vol% of bitumen. Such an asphalt mixture composition (designated 'AC base') is represented as a blue dot in Fig. 1. For all three components (stone aggregate, air voids and bitumen), there are limitations to their quantity. Henceforward, the asphalt mixture design will only be presented in a small part of the equilateral triangle. These limits will be defined in the following part of the study.

### 2.1. Stone aggregates

Stone aggregates (including filler) in asphalt mixtures generally have irregular shapes, which make it difficult to predict their packing. On the

Table 1  
Development of asphalt mix design methods.

Design method	Volumetric properties				Mechanical tests		
	In target asphalt mixture				In target asphalt mixture		
	Void content Vmin and Vmax % (V/V)	Bitumen content VBmin and VBmax % (V/V)	Voids in aggregate VMAmin and VMAmax % (V/V)	Bitumen content in voids VFBmin and VFBmax % (V/V)	Stability test kN	Simple performance test	Performance tests* %, kN, °C
Clifford Richardson (1905)		x	x				
Hubbard Field (1920s and 1930s)	x		x		x		
Hveem mix design (1927)		x			x		
Marshall mix design (1939, modified in 1990s)	x	x	x	x	x		
Superpave mix design (1987)	x	x	x	x		x	
European mix design (2006)	x	x	x	x	x		x
Balanced mix design (2018)	x	x	x	x			x

\*Permanent deformation, resistance to fatigue, failure temperature and so on.

contrary, the packing for spheres can be exactly determined. The densest packing of equal spheres in three dimensions uses ~ 74% of the volume [12]. The random packing of equal spheres generally has a density of ~ 64%. For unequal sphere packing, especially for spheres of different sizes, the problem rapidly becomes intractable. Theoretical determination of the densest packing of stone aggregates, with many sizes of irregular stones, is very demanding. The volume of stone aggregates in asphalt mixtures is usually calculated from their maximal density, bitumen content, bitumen density and the density of the asphalt specimen prepared with a compactor.

The Slovenian National Building and Civil Engineering Institute (ZAG) has analysed asphalt mixes for over 70 years. Their database for the years 2000 to 2005 consists of ~ 800 different asphalt mixtures. The maximal share of stone aggregates takes 86.4% of the volume [13]. For this study, it was assumed that the quantity of stone aggregates in different types of asphalt mixture takes between 64% and 90% of the volume. In Fig. 2, the area where all asphalt mixtures should be placed, according to this limitation for stone aggregates, is marked in yellow. We must be aware that the upper limit (90%) is determined only from data. With the addition of improper filler (very fine particles smaller than 0.063 mm), it is possible to fill the voids with stone aggregate to a larger extent than 90%.

For different types of asphalt mixtures, different limits for ‘stone’, ‘air’ and ‘bitumen’ exist. In the ZAG database for AC mixtures (N = 589) (Table 2), the maximal share of the ‘stone’ volume is 86.4% and the minimum ‘stone’ volume is 80.3%. Therefore, the real limitations for the volume content of stone aggregates in AC mixtures are relatively narrow. In the ZAG database with N = 205 different SMA mixtures, the maximal packing used 83.1% of the volume and the minimal packing used 78.8% of the volume. For PA, sufficient data does not exist to perform a statistical evaluation.

For comparison with the data from the ZAG database (Table 2), Table 3 presents the Slovenian national requirements for asphalt mixtures and layers for heavily trafficked roads.

In the current national specifications for an asphalt mixture, the volumetric limits for AC mixtures are only set for the void content and bitumen in voids. According to EN 12697-8, the repeatability for the determination of voids for AC11 is 1.1%, so the requirements for the void content in the laboratory specimen AC base are too narrow. This is why, in this study, the requirements for the void content in layers (last column in Table 3) are used for volumetric presentation.

The densest packing of stone aggregates can also be obtained also with an analysis of aggregate packing and discrete-element modelling

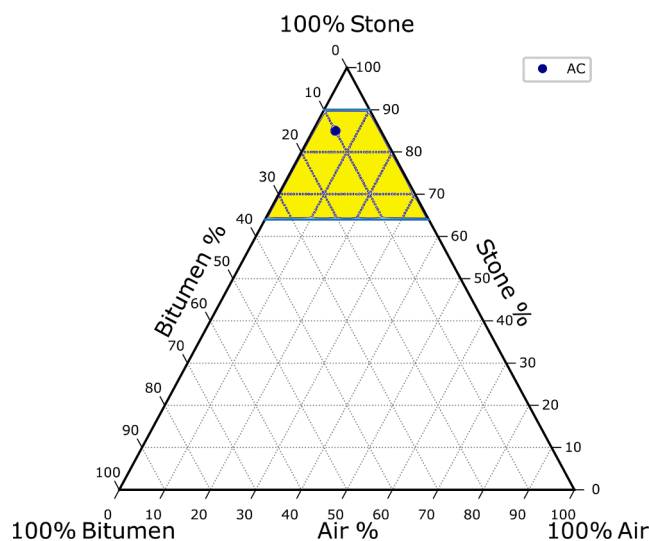


Fig. 2. Volumetric asphalt mixture design with assumed extreme limits for the volume content of stone aggregates.

Table 2 Results of tested samples in ZAG laboratory.

Type of asphalt mixture	AC surf	AC bin and AC base	SMA	PA (low noise)	
Number of samples	228	361	205	3	
Void content vol %	average	4.3	6.4	3.6	18.7
	minimum	2.0	2.0	2.7	17.8
	maximum	6.6	7.7	4.3	19.3
	st.	0.7	1.4	0.5	0.8
	deviation				
Stone content vol %	average	82.9	84.7	80.8	64.8
	minimum	80.5	80.3	78.8	63.6
	maximum	86.3	86.4	83.1	65.6
	st.	1.1	0.8	0.8	1.1
	deviation				
Bitumen content vol %	average	12.7	8.8	15.6	16.4
	minimum	9.6	7.4	14.0	15.1
	maximum	15.0	14.0	17.7	17.3
	st.	1.0	1.6	0.7	1.2
	deviation				

Table 3 Parameters for asphalt mix design with national standards for AC [14], SMA [15] and for layers [16].

Asphalt mixture	Void content Vmin, Vmax	Bitumen content VBmin, VBmax	Bitumen content in voids VFBmin, VFBmax	Void content in layer VminL, VmaxL
Asphalt mixture	% (V/V)	m.-%	% (V/V)	% (V/V)
AC surf	2.0–9.0	NA–NA	50–75	2.0–8.5
AC base	5.0–7.0	3.8*–NA	50–68	4.0–9.0
SMA11	2.5–4.5	6.3–NA	75–88	1.5–7.5

\*New proposed value.

simulation [17]. With this approach the link between the aggregate gradation property and the voids in mineral aggregates can be determined.

Another method to determine the stone aggregate volume limits for an asphalt mixture is with a test of compactability according to the European Standard EN 12697-10 [18]. After the sieving curve for AC type of asphalt mixture is determined, the stone aggregate volume limits for concrete AC mixture can be determined with a compactability test. During the compactability test, the height of the asphalt specimen is recorded. We calculate the compactability of the asphalt sample by determining the density. For the upper limit of stone aggregate volume, the minimum achievable thickness of the specimen ( $t_{\infty}$ ) can be calculated from Eq. (1) (the second method described in [18]):

$$\frac{1}{t(E)} = \frac{1}{t_{\infty}} - \left[ \frac{1}{t_{\infty}} - \frac{1}{t_0} \right] * e^{-\frac{E}{T}} \tag{1}$$

where:

- $t(E)$  is the thickness of the specimen compacted at compaction energy  $E$ ;
- $t_{\infty}$  is the calculated minimum achievable specimen thickness;
- $t_0$  is the calculated initial specimen thickness;
- $E$  is the compaction energy;
- $T$  is the compaction resistance.

It was discovered that Eq. (1) is not the optimal method to model the compaction process [19] and an alternative mathematical model was suggested to predict the minimum achievable thickness (Eq. (2)):

$$\frac{1}{h(E)} = \frac{1}{h_{\infty}} - \left[ \frac{1}{h_{\infty}} - \frac{1}{h_0} \right] * e^{\frac{E}{T_1}} + F * e^{\frac{E}{T_2}} \tag{2}$$

where:

- $h(E)$  is the thickness of the specimen compacted at compaction energy  $E$ ;
- $h_{\infty}$  is the calculated minimum achievable specimen thickness;
- $h_0$  is the calculated initial specimen thickness;
- $E$  is the compaction energy;
- $F$  is a factor connected with specimen thickness;
- $T_1, T_2$  are the parameters of the equation.

The calculated minimum achievable specimen thickness ( $h_{\infty}$ ) value can be used to calculate the upper limit of the volume content of stone aggregates. A lower limit of volume content of stone aggregates can be obtained from  $h_0$ . Eq. (2) can also be used for gyratory compactors [20].

Examples [19] for the asphalt mixture AC 8 show average values of  $h_0 = 79.9$  mm and  $h_{\infty} = 60.9$  mm from four measurements. In the Marshall specimen, with a height of 63.4 mm and prepared with standard 100 impacts, 4.1 vol% air voids, 13.1 vol% bitumen and 82.8 vol% stone aggregates were determined. From these data, it can be calculated with Eq. (2) that at  $h_0 = 79.9$  mm, there are 23.9 vol% air voids, 10.4 vol% bitumen and 65.7 vol% stone aggregates. At  $h_{\infty} = 60.9$  mm, there are 0.2 vol% air voids, 13.6 vol% bitumen and 86.2 vol% stone aggregates. For asphalt mixture AC 8, it can be concluded that the volume content of stone aggregates in asphalt layers is in the range of 65.7–86.2 vol%. For the AC type of asphalt mixture, it was assumed that the thin film of bitumen around grains of stone can be omitted. Therefore, it is possible to determine stone aggregates volume limits from the compaction behaviour for each concrete asphalt mixture.

### 2.2. Air voids in asphalt mixtures and layers ( $V_{min}$ and $V_{max}$ ) in vol %

For different types of asphalt mixtures, different requirements for the air void content in the asphalt layer are determined. Specifications for laid asphalt in Slovenia [16] require the air void content in an asphalt layer to be from 4 to 9 vol% for AC base. Fig. 3 shows a light green area where asphalt mixtures according to the limitations of stone aggregates and the requirements for the air void content for AC asphalt layers should be placed.

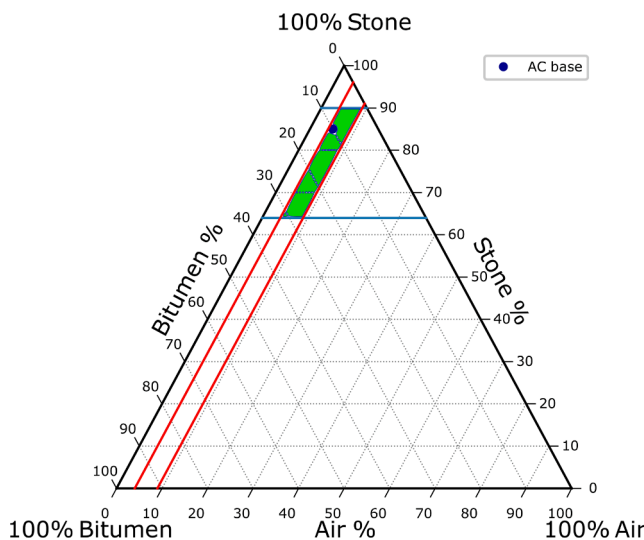


Fig. 3. Volumetric limits for stone aggregates (blue lines) and required limits for air void content (red lines) for AC base asphalt layers. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

For other asphalt mixtures, the requirements for void content are different, e.g., the PA limits for void content in the asphalt layer are from 15% to 28% and for SMA, the limits are from 1.0% to 7.5%.

### 2.3. Voids filled with bitumen (VFB) in %

Many European specifications, including the Slovenian national standards [14] and [15], have requirements for voids in a stone aggregate filled with bitumen. A minimum of 50% and a maximum of 75% of voids should be filled with bitumen for AC surf and 50% to 68% for AC base (Table 3). The requirements for voids filled with bitumen for AC base are designated with green lines in Fig. 4. This demand is graphically represented as a ratio between bitumen and air; e.g., the requirement of 50% VFB lies exactly perpendicular to the baseline.

In the requirements for asphalt mixtures in most European countries, limits for minimum and maximum void content and for voids filled with bitumen are set. Fig. 4 presents these limits (green lines) in the triangular volumetric representation. According to the requirements, AC base mixture design limits are depicted in light blue coloured area.

### 2.4. Bitumen content (VB) in m.-% and vol %

Requirements for minimum bitumen content can be also presented in the triangular volumetric representation. In Slovenia the limits for minimum bitumen content are set in mass percent for SMA mixtures. The minimum bitumen content is set to 6.5 m.-% for SMA 8 and 6.3 m.-% for SMA 11. These values should be of course corrected for realistic densities of stone aggregates with the reference density of stone aggregates ( $2.650 \text{ Mg/m}^3$ ) in the asphalt mixture. There is requirement for mass percentage and no requirement for the volume content of bitumen. The requirement for mass percentage is not constant in volumetric asphalt mixture design. The requirement for volume percentage of bitumen should be line parallel to right side of triangle, but the requirement for mass percentage of bitumen is line connecting the right corner of the triangle and the left side of the triangle.

For AC 22 base, the national specification limits for minimum bitumen content are set to 3.8 m.-%. The air void limits in asphalt layers are 4 to 9 vol%. Assuming the densities of bitumen ( $1.01 \text{ Mg/m}^3$ ) and stone aggregates ( $2.650 \text{ Mg/m}^3$ ), we determined the line representing the volumetric requirement for the bitumen content. In the case of AC 22 base, the calculated minimum requirement is ~8.8 vol% of bitumen. In

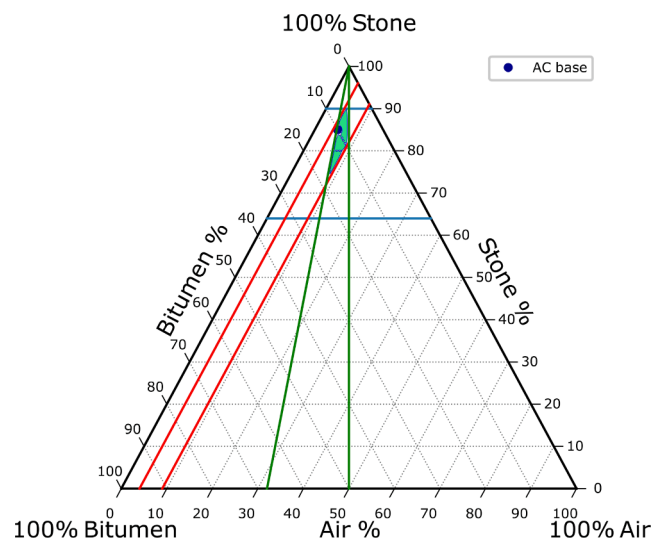


Fig. 4. Volumetric limits for stone aggregates (blue lines), air voids (red lines) and voids filled with bitumen (green lines) for AC (base) in asphalt layers. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



the triangular volumetric representation this limit is depicted as the purple line in Fig. 5. The requirement for minimum bitumen content narrows the area where asphalt mixture design is allowed.

In Fig. 5 there is a case how these limitations look like in the triangular volumetric representation. It can be seen that according to the requirements AC base mixture design must only be placed in the closed blue coloured area. In Fig. 5, the blue coloured area shows the suitable area where the properties of the AC base should be placed according to all requirements (for the mixture and layer). From the comparison between Fig. 4 and Fig. 5, it can be seen that the suitable area was only slightly reduced after the bitumen content requirement was added. As mentioned in the comment for Fig. 2 with the addition of filler containing very small particles, it is possible to fill the voids with stone aggregate to a greater extent than 90%.

The limitations for the content of stone aggregate are not shown in Fig. 6a. The blue shape in Fig. 6a designates the volumetric limits for asphalt mixture design. From the ZAG database [13], it was found for the AC mixture design that the quantity of stone aggregates in the asphalt mixture takes a minimum of 78.8% of the volume. If we take real data from the database, it can be seen that all AC asphalt mixture designs are in the upper half of the blue area in Fig. 6a. Similar areas can be determined for all other types of asphalt mixtures (SMA, PA and so on) that contain air voids. For mastic asphalt, which does not contain air voids, in the triangular representation of volumetric properties, the air voids should be replaced with filler content in volume percent as the third component.

For comparison it was calculated position of line representing requirement for 2 m.-% of bitumen. In Fig. 6b it can be seen that such requirement would not make sense. The “allowed area” determined with requirements for void content and VFB would not change much with additional requirement for 2 m.-% of bitumen.

### 3. Volumetric optimisation of asphalt mixture design by triangular representation

In this section, how triangular representation can also be, with some limitations, a very good tool for presenting process of asphalt mixture optimisation is presented. When we have a database of several asphalt mixtures with known mechanical properties, the triangular representation of volumetric properties can be a helpful tool to determine asphalt mixture design with optimal points of such mechanical properties. In the next section, the possibility to optimise low noise asphalt layers is represented in more detail.

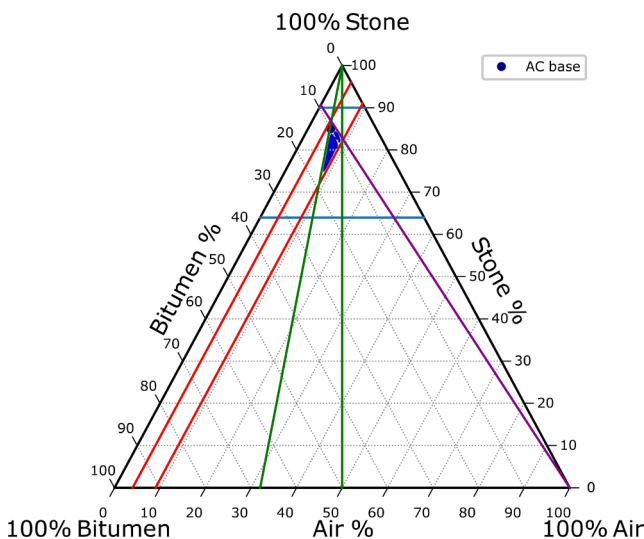


Fig. 5. Volumetric asphalt mixture design with required limits for air void content for AC asphalt layers, voids filled with bitumen and bitumen content.

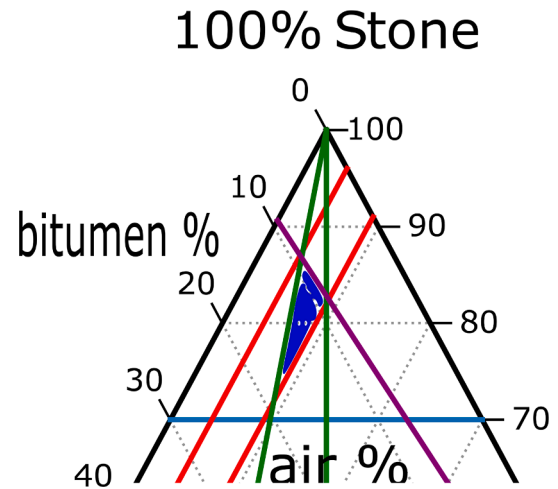


Fig. 6a. Enlargement of the area concerned in Fig. 5.

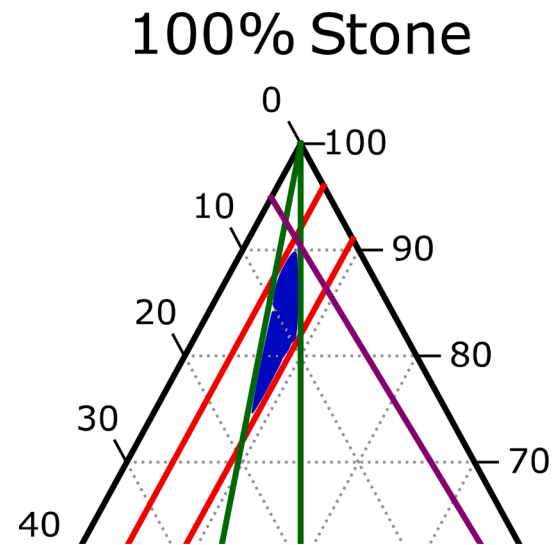


Fig. 6b. Requirement for minimal bitumen content changed to 2 m.-%.

In recent years, there is an increased need to reduce noise. For this purpose, many studies have been performed to reduce noise coming from the contact between asphalt pavement and car tyres [21–26]. To reduce such car noise, tyres and/or the surface layer of asphalt pavements must be optimised. Noise can be reduced through the addition of rubber in bitumen or through the change in the volumetric properties of the asphalt mixture (larger void content). It was proven that the content and shape of air voids in the asphalt layer have an important role in noise reduction [27–29]. A triangular representation of volumetric properties is a good tool for visualising the optimisation process of volumetric properties of an asphalt layer to reduce noise.

Fig. 7 and Fig. 8 show the average positions from the ZAG database of the AC surf, AC bin (and AC base), SMA and PA asphalt mixtures. In Fig. 7, the position of the AC surf is an average of 228 mixtures, AC bin, and AC base are and average of 361 mixtures, and SMA is an average of 205 mixtures, as presented in Table 2. The PA position is an average of only three mixtures from the ZAG database [13]. Fig. 8 presents the range, from minimal to maximal, for all asphalt mixtures from Table 2. All asphalt mixes were tested in the ZAG laboratory. The results are graphically presented as volumetric asphalt mixture design with approximate extreme limits for the volume content of stone aggregates and the average positions of different types of asphalt mixtures.

From Fig. 7, it can be seen that position of low noise PA asphalt

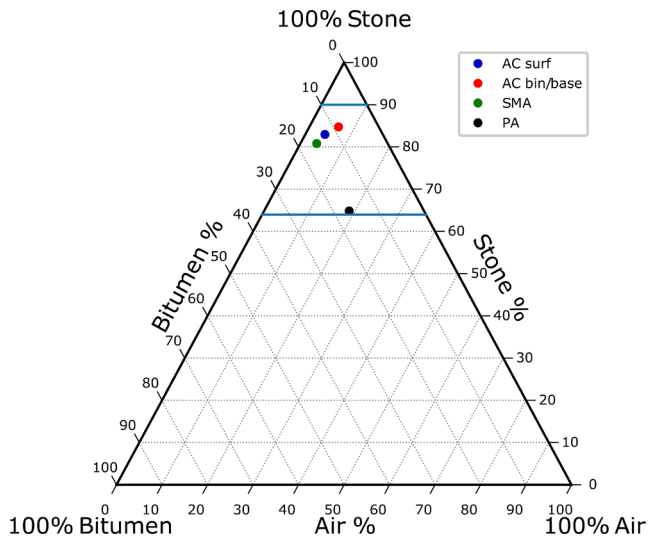


Fig. 7. Volumetric position for an average of asphalt mixtures AC surf, AC bin/base, SMA and PA from the ZAG database.

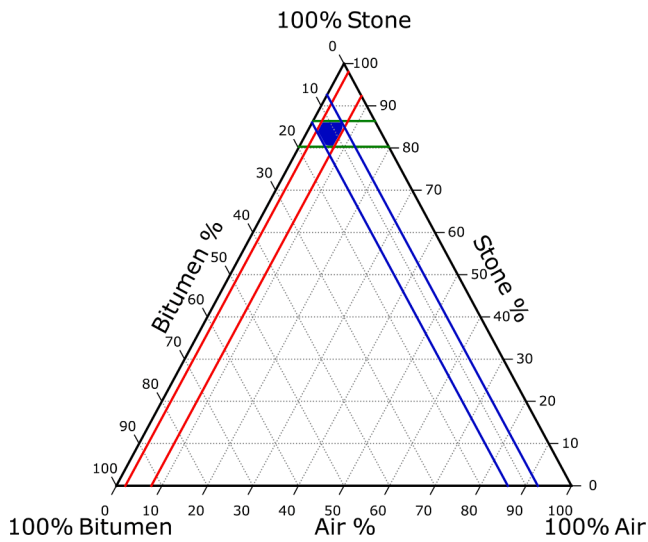


Fig. 8. The range of maximum and minimum values of ‘stone’, ‘bitumen’ and ‘air’ for AC bin/base from the ZAG database.

mixtures is relatively far from the position of other asphalt mixtures. Somewhere near the position of PA mixtures is the expected position of the optimal low noise asphalt mixture.

Fig. 8 presents the minimum and maximum values for ‘stone’, ‘bitumen’ and ‘air’, as presented in Table 2. For AC bin/base, the bitumen content is from 7.4% to 14.0% (blue lines), the stone content is from 80.3% to 86.4% (green lines) and the void content is from 2.0% to 7.7% (red lines).

Other case studies can be presented by triangular representation of the volumetric properties.

Fig. 9 presents the Marshall Procedure mix design to determine AC 32 with 6 different binder contents (from 3.0 mass % to 4.6 mass %). Three of the six mixtures with bitumen content 3.0, 3.4 and 4.6 mass % were out of the specification limits and they lay practically in one line. There was only a narrow space where mixtures with selected stone grading could be placed within the limits of specifications. These mixtures had bitumen contents of 3.8, 4.0 and 4.2 mass %. The optimum asphalt mix design had 4.0 mass % of bitumen and is marked with a blue dot.

With ordinary Marshall Procedure we can determine optimal mixture design practically only in one line (Fig. 9) and we don’t get any information about properties of asphalt in other parts of “allowed area” (Fig. 6a).

Further in this paper we are briefly presenting two alternative procedures (so called “Simplex algorithm” and “Experimental design”) which enable the searching for asphalt mixture optimum in whole “allowed area” and also outside this area. The result of these alternative procedures is the global maximum in the triangle, but the result of the Marshall Procedure is just local maximum.

Fig. 10 presents how the Simplex procedure [30] can be used to determine mix design. Asphalt mixtures must be designed so that the volumetric properties lie in the corners of the first (green) triangle. To achieve design of these mixtures, the filler content must also vary. After performing asphalt tests on all three mixtures (one test like simple noise trials or combination of asphalt tests like stiffness, fatigue...), the mixture with the worst properties must be determined and symmetrically, a new experimental point is determined. After performing tests on the fourth asphalt mixture, the next experimental point is determined according to the Simplex rules. In Fig. 11 the second and the third steps of the Simplex procedure are presented.

Another option is design of “Experimental design”. Fig. 11 shows a plan of experiments (according to Simplex Centroid Design) for a new asphalt mixture in triangular volumetric presentation [31–33]. Each point represents one of the planned experimental mixtures. To achieve design of these mixtures, the filler content must also be varied. After all asphalt tests are performed, the results of experiments can be used to perform modelling with a polynomial model. With a good polynomial model, the properties of all asphalt mixtures within a triangle are relatively well determined.

#### 4. From a triangular representation to representations of four and more dimensions

To obtain a new dimension, filler content is considered as a fourth dimension. Simultaneously, stone aggregate, as the first dimension, is presumed to be without filler. We can represent such a design as a tetrahedron (Fig. 12).

Fractions from the aggregate sieving curve can be further dimensions in more dimensional space. When more than four dimensions are used to represent asphalt composition, a problem occurs in that we cannot visualise all dimensions at once. Principal component analysis is a good tool that enables us to visualise more than four dimensions of asphalt mixture and evaluate the effect of those dimensions on the mechanical properties of the asphalt mixture. A promising tool to deal with more than four dimensions of asphalt mixture are neural networks [13,34,35].

#### 5. Conclusions

The main goal of this study was to demonstrate the applicability of graphical representations for asphalt mixture design. Visualisation of volumetric properties is important for the understanding of the composition of a granular asphalt mix. Three basic constituents of asphalt mixture (stone aggregates, air voids and bitumen content) are graphically represented by a triangular representation. This allows for a visual presentation of the true meaning of a particular requirement. For a demonstration, we used Slovenian specifications for AC base asphalt mixtures and proved that the triangular representation of requirements for volumetric properties of asphalt mixtures is the most useful way to represent the meaning of different volumetric requirements from specifications.

We found that this type of graphic presentation is also useful for the optimisation of asphalt mixture design. For example, it can be used to design the composition of asphalt mixture for a road layer with



## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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