On Formulating the Ground Scheduling Problem as a Multi-objective Bilevel Problem

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Abstract. In this paper, a bilevel multi-objective formulation of the Ground Scheduling Problem is presented. First, the problem is formulated as a bilevel optimisation problem (BOP), wherein the upper level (UL) is a biobjective problem determining the pairs of Ground Station (GS) to Spacecraft (SC) and the starting time of each event with objectives the maximisation of the access windows and the minimisation of the communication clashes of each GS. These two objectives of the UL can be assumed as a measure of the violation of the feasibility of a schedule. The lower level (LL) consists of a single objective optimisation problem that determines the duration of each event, with objectives the communication time requirement of SCs with GS and the total ground station usage, combined together to a weighted sum function. The approach used to solve this multi-objective BOP is a nested approach, where the Pareto front of the upper level is obtained by a multi-objective optimisation algorithm (NSGA2) and the lower level is solved using a GA. The formulation is tested on one small test case from literature and the relevant results are reported.

Keywords: Bilevel Optimisation · Multi-objective Optimisation · Satellite Scheduling.

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1 Introduction

The Ground Station Scheduling (GSS) refers to the problem of planning the communication between satellites (or spacecraft) and the ground stations. The importance of the GSS problem relies on finding optimal allocation of the communication of many satellites to a limited number of ground stations. The problem is very complex, highly constrained, and proved to be NP-hard [7]. Therefore, since only near-optimal solutions are expected to be found the use of EAs and other metaheuristics has become popular, e.g. [3], [9]. The problem is most of the time formulated as a multi-objective problem, consisting of several and conflicting objectives [11], [12]. Moreover, the optimisation of each of the objectives can be modeled in a hierarchical or simultaneous fashion [13]. In the simultaneous optimisation, the objectives are optimised at the same time, obtaining a Pareto front of the solutions, ignoring their hierarchy. In this way, the solutions of the optimisation might not be representative of the final schedule, since some communications will be omitted as they don't satisfy some of the constraints.

The main scope of this paper is to present and test a bilevel multi-objective formulation of the Ground Scheduling Problem. The problem is formulated as a bilevel optimisation problem (BOP), wherein the upper level is a biobjective problem determining the pairs of Ground Station (GS) to Spacecraft(SC) and the starting time of each event with objectives the maximisation of the access windows and the minimisation of the communication clashes of each GS. These two objectives of the UL can be assumed as a measure of the violation of the feasibility of a schedule. The lower level consists of a single objective optimisation problem that determines the duration of each event, with objectives the communication time requirement of SCs with GS and the total ground station usage, formulated as a weighted sum function. The approach used to solve this multi-objective BOP is a nested approach, where the Pareto front of the upper level is obtained by NSGA2 and the lower level is solved using a GA. The expected results of this approach are to have more representative solutions to the optimization of the final schedule.

The remainder of this paper is organized as follows. In Section 2 the multi-objective bilevel optimisation problem is described along with its mathematical representation. In Section 3 the ground scheduling problem is presented, with the notations and the objective functions taken into account in this implementation, and the formulation as a multi-objective bilevel problem is defined. The nested evolutionary approach adopted for the optimisation is shortly described in Section 4. The experimental setup and the obtained results are discussed in Section 5. Finally, Section 6 concludes the paper, giving some future steps of the research.

2 Multi-objective Bilevel Optimisation Problem

The general bilevel optimisation problem (BOP) consists of two levels of optimisation problems referred to as the upper and lower level (UL and LL). The lower level works as a constraint to the upper, meaning that the feasible space of the upper level is determined by the optimal solution of the lower level problem. The mathematical representation of a BOP can be described as follows:

$$\begin{aligned} \min_{x\in X} \quad & F(x,y) \\ \text{subject to} \quad & G(x,y) \leq 0, \\ & \min_{y\in Y} \quad f(x,y) \\ & \text{subject to} \quad g(x,y) \leq 0 \end{aligned} \tag{1}$$

where y is the solution of the LL problem from the set of solutions $Y \subseteq R^n$, with regard to the solution from UL, x from the set of solutions $X \subseteq R^m$. This means that the LL problem is optimised only with respect to y, while x is kept fixed. The highest level of the hierarchy is the UL optimisation problem where F is its objective, while f corresponds to the objective of the LL of the optimisation problem, the lowest level in the hierarchy [4]. Moreover, $G(x,y)$ and $g(x,y)$ correspond to the inequality constraints of the UL and LL respectively.

If F and f are vector functions ($F : R^n \times R^m \to R^N$ and $f : R^n \times R^m \to R^M$), then the problem is called a multi-objective bilevel problem. A general Multi-objective Bilevel Optimisation Problem (MBOP) can be described as follows [6]:

$$\begin{aligned} \min_{x\in X, y\in Y} \quad & F(x,y) = (F_1(x,y),\ldots,F_N(x,y)) \\ \text{subject to} \quad & G(x,y) \leq 0, \\ & \min_{y\in Y} \quad f(x,y) = (f_1(x,y),\ldots,f_M(x,y)) \\ & \text{subject to} \quad g(x,y) \leq 0 \end{aligned} \tag{2}$$

In the above formulation, $F_1(x,y),\ldots,F_N(x,y)$ and $f_1(x,y),\ldots,f_M(x,y)$ are the UL and LL objective functions respectively. A solution is a feasible solution to the UL problem, only if it is a Pareto-optimal solution of the LL optimisation problem. More about the basic notations and theoretical results about the MBOP can be found in [2]. Note that in our formulation we make the optimistic assumption, meaning in case there is more than one optimal LL solution, we assume that is the one that is optimal for the UL as well [2].

3 Satellite and Ground Scheduling Problem

In this section, the satellite scheduling problem and more specifically the ground scheduling problem are described shortly. Then, the mathematical formulation and notation of the problem are presented, defining its parameters, variables and objective functions used. Last, the interpretation of a bilevel multi-objective problem and its mathematical formulation is given.

3.1 Problem Description

The Ground Station Scheduling optimises the plan of the communication between satellites (or spacecraft) and the ground stations. The problem can be formulated with many different objectives. In this paper, a benchmark instance of the ground station scheduling generated with the STK simulation toolkit from Xhafa et. al [12], [13] is used. Therefore, the same formulation of objectives is implemented –with some small modifications of the quantification of some objectives– to correspond to the same input and output parameters. The main objectives taken into account in this formulation are 1. maximising the visibility windows of SCs and GSs 2. minimising the clashes of the time windows between different SCs to the same GS, 3. satisfying the required communication time between SC with GSs, 4. minimising the idle time of the GSs.

3.2 Mathematical Formulation

The notation and the mathematical problem is presented as follows:

Parameters

- $s \in 1, \ldots, S$ satellite set, index s
- $g \in 1, \ldots, G$ ground station set, index g
- $h \in 1, \ldots, H$ set of available Access Windows for a specific g and a specific s for all days of the schedule, index h
- $d \in 1, \ldots, D$ set of days, index d
- tw_{sg}^{h}: h^{th} time window between a specific g and a specific s
- $T_{AOS}(tw_{sg}^{h}), T_{LOS}(tw_{sg}^{h})$ are the visibility and losing signal times of a g from a s
- $\forall g \in G, s \in S \; AW_{s,g} = \bigcup_{h=1}^{H} [T_{AOS}(tw_{sg}^{h}), T_{LOS}(tw_{sg}^{h})]$ where AW defines all the time periods s and g can communicate
- $k_s^d \in 1, \ldots, K$ are requirements for each s each day d
- $T_{beg}(k_s^d), T_{end}(k_s^d)$ are the beginning and ending time of a requirement where connection has to be established for at least $T_{req}(k_s^d)$ during a specified period d.

Decision variables

- $n_{sg}^{m} \in N$ an event of the schedule, where $m \in M$ is the consecutive number of event when a specific g communicates with a specific s, N is the total number of events of the schedule
- $T_{start}(n_{sg}^{m}), T_{dur}(n_{sg}^{m})$ Starting and Duration time between s and g.

Objective Functions

Access windows fitness function: Access windows or visibility windows are the time windows during which a g can establish communication with an s. Therefore in the schedule, we want to maximize the number of events that fall into these time windows. $\forall g \in G, s \in S, m \in M$

$$f_{AW}(n_{sg}^{m}) = \begin{cases} 1 \text{ if } [T_{start}(n_{sg}^{m}), T_{start}(n_{sg}^{m}) + T_{dur}(n_{sg}^{m})] \subseteq AW_{s,g} \\ 0 \text{ else} \end{cases} \tag{3}$$

$$Fit_{AW} = \frac{\sum_{m=1}^{M} \sum_{g=1}^{G} \sum_{s=1}^{S} f_{AW}(n_{sg}^{m}) * 100}{N} \tag{4}$$

Communication clash fitness function: A communication clash occurs when two satellites are trying to communicate with the same Ground Station at the same time. In this case, the solutions are infeasible. The goal here is to minimise the clashes that are produced between the several SCs to one GS. Let from n_{gs}^{m} create the sets $\forall s : n_g^l \in N$ where $l \in L \subset N$ is the index of the m^{th} event of a specific g to all the s, after its events are sorted in ascending order for a fixed g and $\forall s$ according to their $T_{start}(n_s^m)$ Then:

$$f_{sc}(n_g^l) = \begin{cases} -1 \text{ if } T_{start}(n_s^{l+1}) < T_{start}(n_s^l) + T_{dur}(n_s^l) \\ 0 \text{ else} \end{cases} \tag{5}$$

$$Fit_{CS} = \frac{N + \sum_{g=1}^{G} \sum_{l=1}^{L} f(n_g^l) * 100}{N} \tag{6}$$

Communication time requirements fitness function: In order for TTC (Telemetry, Tracking, and Command) tasks to be completed, such as data download tasks, there exist some minimum time requirements. These periodical tasks are given as an input to the problem, in a matrix of their starting and ending times for each period (day) for each satellite. The objective is to satisfy as much as possible these requirements in the whole schedule. The fitness function is computed as follows:
$\forall n \in N$ and $\forall k \in K$

$$f(k_s^d, n_{sg}^m) = \|[T_{start}(n_{sg}^m), T_{start}(n_{sg}^m) + T_{dur}(n_{sg}^m)] \cap [T_{beg}(k_s^d), T_{end}(k_s^d)]\| \tag{7}$$

In the reference paper, the problem was formulated as follows:

$$f_{TR}(k_s^d) = \begin{cases} 1 \text{ if } (\sum_{g=1}^{G} \sum_{m=1}^{M} f(k_s^d, n_{sg}^m)) \geq T_{Req}(k_s^d) \\ 0 \text{ else} \end{cases} \tag{8}$$

This formulation (Eq. 8), where the percentage of violation of the requirement is calculated, may pose a problem in some bilevel formulations. When this objective is optimized without any other constraints on the same level, it may happen that all of the event duration are increased to their maximum limit (here e.g. the maximum days of the schedule). To prevent this from happening, the fitness function was reformulated as follows:

$$f_{TR}(k_s^d) = \begin{cases} 0 \text{ if } (\sum_{g=1}^{G} \sum_{m=1}^{M} f(k_s^d, n_{sg}^m)) < T_{Req}(k_s^d) \\ T_{Req}(k_s^d) / (\sum_{g=1}^{G} \sum_{m=1}^{M} f(k_s^d, n_{sg}^m)) \text{ else} \end{cases} \tag{9}$$

$$Fit_{TR} = \frac{\sum_{s=1}^{S} \sum_{g=1}^{G} f_{TR}(k_s^d)}{K} * 100 \tag{10}$$

The new formulation assigns a small penalty to the events that establish communication for a period that is longer than the required amount of time. This penalty is proportional to the length of the requirement.

Ground Station usage fitness function: This fitness function is maximizing the busy time of a GS (minimising its idle time). This is expressed as a percentage of the GSs busy time and the total available communication time of a GS.

$$Fit_{GU} = \frac{\| \bigcup_{m=1}^{M} \bigcup_{g=1}^{G} \bigcup_{s=1}^{S} [T_{start}(n_{gs}^m), T_{start}(n_{gs}^m) + T_{dur}(n_{gs}^m)] \|}{\sum_{g=1}^{G} T_{total}(g)} * 100 \tag{11}$$

where $T_{total}(g)$ is the total available time of a GS, in this case the number of days of the schedule.

3.3 Interpretation as a Bilevel Multi-objective Problem

In the reference paper, the problem was approached by combining the fitness objectives into one, by assigning weights to each fitness functions as follows:

$$Fit_{Total} = 1.5 * Fit_{AW} + Fit_{TR} + 0.1 * Fit_{CS} + 0.01 * Fit_{GU}$$

It should be noted that the fitness functions can be grouped into modules, of serial and parallel, according to the dependencies among the fitness functions. In this problem, Fit_{AW} and Fit_{CS} belong to the serial fitness module, while the rest two objectives to the parallel module. From a hierarchical point of view, the objectives of Access windows Eq. 4 and Communication Clash Eq. 6 (the serial module) are the ones that should be evaluated first, as they are also defining the violation of the feasibility of each event. Also, these two objectives are most of the time conflicting, making the problem difficult to find an optimal solution. In this paper, we take into account this hierarchy, by decoupling the problem into two levels. We define the UL as a biobjective optimisation problem and the LL as a single objective, defined by a weighted function of the rest two objectives. The mathematical representation of the problem is as follows:

$$\begin{aligned} &\max_{x \in X, y \in Y} \quad F(x,y) = (F_{AW}(x,y), F_{CS}(x,y)) \\ &\text{subject to} \\ &\qquad \max_{y \in Y} \quad f(x,y) = (F_{TR}(x,y) + 0.1 * F_{GU}(x,y)) \end{aligned} \tag{12}$$

where the decision variables of the upper level are $x = SC, GS, T_{start}$, and the decision variables of the lower level is $y = T_{dur}$.
Therefore, most of the variables of the scheduling problem are defined by the UL, while the LL defines only the duration of each event. The optimal $Tdur$ defined by the LL is then used to evaluate the Pareto front of the UL. The rationale behind this formulation lies in firstly the assumed hierarchy of the objectives and then the influence of the decision variables to each objective. One can notice that the ground station usage fitness (Eq. 11) and the communication time requirement fitness (Eq. 9) are mostly influenced by the variable T_{dur} of each event. Moreover, the objectives of the UL are actually a measure of the violation of the feasibility of the events. In this way, we take into account LL optimal objectives that are violating as less as possible these constraints, making the solution more representative to the final schedule. Finally, by keeping the number of decision variables low at the LL, the optimisation problem becomes of lower dimensionality and relatively easier to optimise. The general structure of the bilevel model is presented in Fig. 2.

4 Evolutionary Algorithms for MBOP

To solve the MBOP that we formulated, we adopt a nested optimisation approach [10]. A similar nested approach has been applied in [8], where they implemented a nested Genetic Algorithm (GA) for solving an integrated long-term staffing and scheduling problem. In the UL, the biobjective problem is solved with the NSGA-II multi-objective algorithm [1]. To evaluate each individual of the UL, the LL is solved to optimality with a GA. Each optimisation of the LL for each UL individual is independent, so these processes were parallelized to reduce the computational cost. Finally, the result is a Pareto front of all the non-dominated solutions of the upper-level. The general pseudocode of the nested algorithm is shown in Algorithm 1. For solving the problem with GA and NSGA2, the following representation of the chromosome was adopted. Each chromosome encodes a schedule as a list of communication events, where each event is represented by 5 binary variables, as seen in Figure 1. I is a binary variable that indicates whether the specific schedule is taken into account or not, while the rest are representing the SCs with their corresponding GSs and their starting and duration times. One chromosome consists of many of these tuples to create a whole schedule. As a crossover operator, HUX was selected, while BitFlip mutation was used as a mutation strategy for the specific implementation, both for NSGA2 and GA.

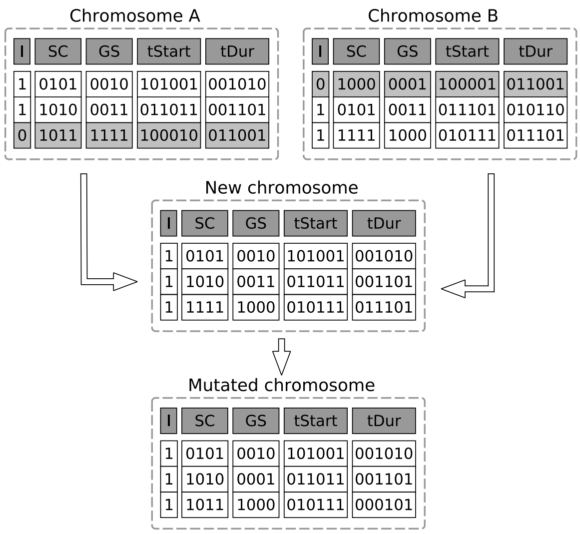

Fig. 1. Chromosome encoding and its mutation and crossover operators.

Algorithm 1 Nested MBOP

Input: $AW, S, G, Days, TReq$
Output: $GS - SC, n, fitnessfunctions$

procedure NESTEDMBOP
Initialize Population X_0
Best $\leftarrow \{\}$
    for number of generations **do**:
        for all *Individuals* $X_i \in X$ **do in parallel**:
            call lower-level GA with X_i *as an input, obtain* Y_{ibest} *as an output*
        end for
        Evaluate $Fitness(X)$
        A $\leftarrow$ *Pareto Front*$(X) \cup Best$
        Best $\leftarrow$ *Pareto Front*(A)
        reproduction (selection, mutation, recombination)
    return *Best non dominated solutions*

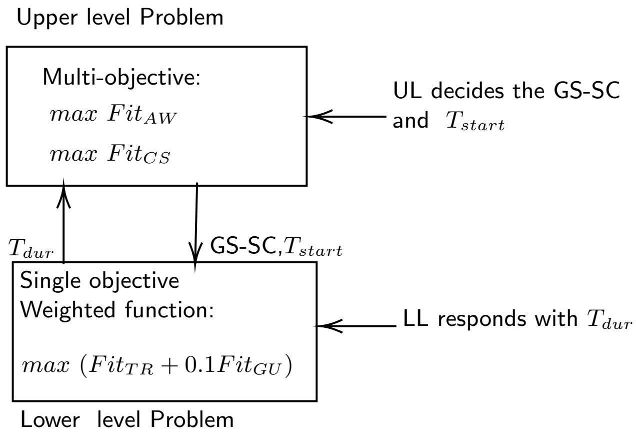

Fig. 2. Bilevel model structure of the Ground Scheduling Problem

5 Experiments

5.1 Experimental Setup and Problem Instance

For the implementation of the NSGA-II and GA, the platypus[3] framework in python is used. For the upper level, a population size of 30 and 500 generations was used, while for the LL a population size of 50 and 20 generations. The problem instance corresponds to the first small size of the benchmarks generated by Xhafa et al.from STK toolkit[4], where there are 5 Ground Stations, 10 Spacecraft and the number of days is 10. The preliminary results refer to one run of the nested approach to test the mathematical formulation of the problem. The control parameters values used are reported in Table 1. Our implementation of the objective functions and the related code can be found at [5].

Table 1. Selected control parameters that are used in all of the reported results.

	UL	LL
HUX crossover rate	0.3	0.3
BitFlip mutation rate	0.001	0.001
Population size	30	50
Number of generations	500	20

[3] https://platypus.readthedocs.io/en/latest/getting-started.html
[4] https://www.researchgate.net/publication/260086344_GS_Scheduling_Inputs

5.2 Results and Discussion

In Figure 3 the convergence of the hypervolume indicator of the UL NSGA-II is presented with respect to the number of generations. The results show that the algorithm converges as the generations evolve. In Figure 4 the scatter plot of the solutions of the final UL generations is depicted. Orange dots represent the obtained Pareto front, while the blue dots are the dominated solutions of the population. In figure 5 the final objective values of the Pareto front solutions and their corresponding LL objective values are depicted. It is interesting to note, that the objective values of the Fit_{TR} seem to be concentrated with a small variance around 20. These low values are most probably an implication of the reformulation of the objective function that was implemented and described in Section 3 and is a topic for further research. Also with this formulation, we only accept solutions that violate as less as possible the constraints of the AW and Clashes and this explains further the low values of the lower level objective.

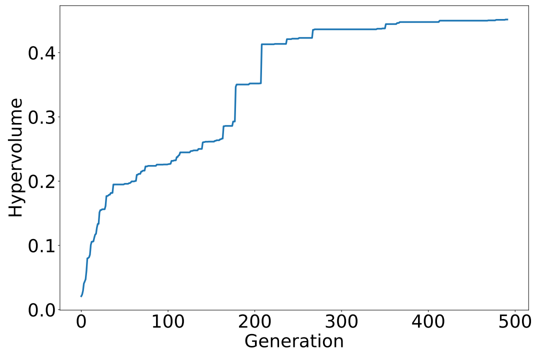

Fig. 3. Hypervolume indicator convergence of Upper Level NSGA-II.

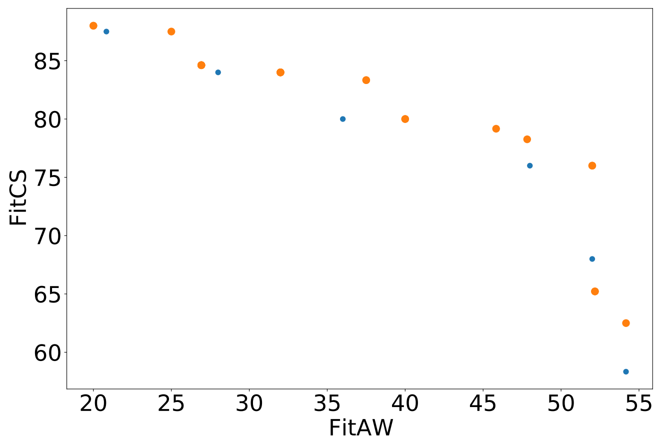

Fig. 4. Scatter plot of the non-dominated (orange) and dominated (blue) solutions of the final UL generation and the obtained Pareto front for case I_S_01.

6 Conclusion and Future Work

We formulated for the first time the GSP as a multiobjective bilevel problem and tested in one benchmark instance. In the final schedule of a GSP, only the feasible solutions are taken into account and the

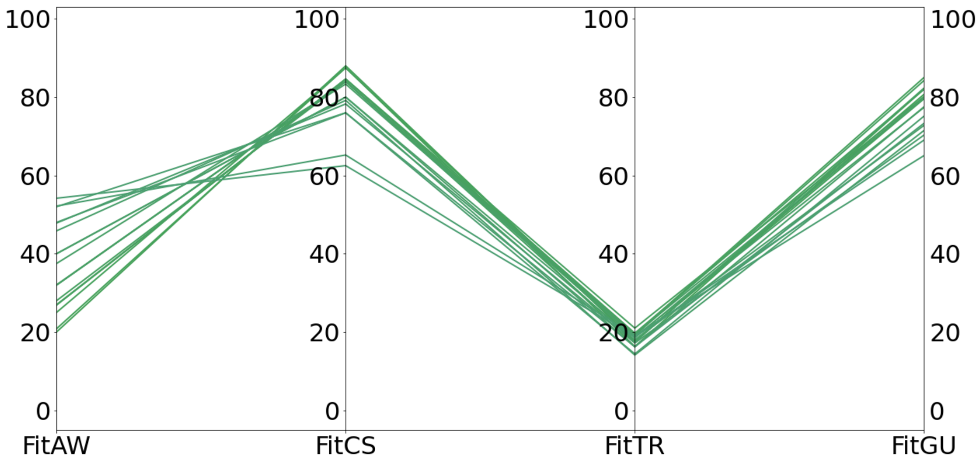

Fig. 5. Parallel coordinate plot of the approximate Pareto front on the case I_S_01 with bilevel algorithms (green).

values of some of the objectives are not optimal anymore. The proposed formulation of the GSP aims to give more representative results of this final schedule. It can take advantage of the hierarchy of the objectives, without using weights, giving more than a single optimal schedule at the end. Finally, the bilevel formulation can be useful for modeling the problem with other objectives as well, especially when the lower level is cheaper to evaluate.

In the next steps, the possible implications of the time requirement objective reformulation will be examined and improved. Moreover, a different formulation of the problem, meaning different UL and LL objectives and/or decision variables can be interesting, by exploring deeper the hierarchies of the problem. Last but not least, algorithmic-wise, ways to reduce further the computational cost of the bilevel approach will be examined, such as additional parallelization, use the knowledge of the previous runs by using them as an initial population of each lower level run, and the possibility of using approximation functions.

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