Solving Min-Max Optimisation Problems by Means of Bilevel Evolutionary Algorithms

A Preliminary Study

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ABSTRACT

Min-max optimisation is a special instance of a bilevel problem. It deals with the minimisation of the maximum output in all scenarios of a given problem. In this paper, numerical experiments are conducted to assess the accuracy and efficiency of three bilevel algorithms - known to perform well in general bilevel problems – on 13 unconstrained min-max test-functions. This study aims to bring the bilevel and min-max evolutionary community together and create a common ground for both optimisation problems.

CCS CONCEPTS

• **Mathematical Optimisation** → Bio-inspired Optimisation;

KEYWORDS

Bilevel optimisation, Min-max optimisation, Worst-case scenario optimisation, Evolutionary Algorithms, Mathematical Programming

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1 INTRODUCTION AND DEFINITIONS

The bilevel optimisation problem (BOP) is an optimisation problem, operating as the upper-level (UL), which has another optimisation problem as a constraint, namely the lower-level (LL) [4]. This problem is usually challenging and complex to solve. Due to this complexity, where a solution is hard to find when no assumptions about the problem are made, the community has attempted to solve them both with classical and evolutionary approaches.

Min-max optimisation is a special instance of a BOP. It deals with the minimisation of the maximum output in all scenarios of a given problem. These problems arise naturally in optimisation under uncertainty, where the LL plays the role of nature, which reacts to the UL's decisions in the most destructive way. Though there are evolutionary algorithms designed specifically for min-max problems, the evolutionary bilevel algorithms have -to the best knowledge of the authors- never been tested on this kind of problems. This study aims to bring bilevel and min-max evolutionary community and create the first steps for a common ground for both optimisation problems. Furthermore, min-max synthetic and real-world problems can be used to extend the bilevel benchmark functions.

Bilevel Optimisation Problem: The mathematical representation is as follows:

$$\min_{x\in X, y\in Y} \quad F(x,y) \quad \text{subject to} \quad G_k(x,y) \leq 0,\ k=1,...,K, \tag{1}$$

where K is number of constraint functions of UL and y is the solution of the LL problem from the set of solutions $Y \subseteq R^n$, with regard to solution from UL, x from set of solutions $X \subseteq R^m$,where

$$y \in \arg\min_{y\in Y} \quad f(x,y) \quad \text{subject to} \quad g_j(x,y) \leq 0,\ j=1,...,J, \tag{2}$$

where J is the number of constraint functions of the LL. F represents UL' s objective function, while f represents the LL' s objective function. The problem becomes ill-defined when more than one LL optimal solutions for all or some UL variables exist. One common approach for the researchers to tackle this problem is to identify two possible positions, namely the optimistic and the pessimistic position [4].

Min-max Optimisation Problem: The general unconstrained min-max problem can be described as:

$$\min_{x\in X} \max_{y\in Y} f(x,y) \tag{3}$$

and $X \subseteq R^m$ represents the set of candidate solutions and $Y \subseteq R^n$ the set of all possible scenarios. UL and LL are sharing the same objective function $f(x,y)$, where UL is minimising according to the variables x of the solution space and LL is maximising according to the uncertain parameters y of the scenario space [3]. They can be symmetrical or asymmetrical [5].

2 EXPERIMENTAL SETUP

Three algorithms are considered to evaluate the performance of bilevel evolutionary approaches to min-max problems. BLDE is a

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completely nested approach using the Differential Evolution algorithm (DE) in both levels found in [1], while BLEAQ2 [4] is using the approximations of 2 mappings; the LL rational reaction mapping and the LL optimal value function mapping along with a GA. Last, BLCMAES[2], is a nested CMA-ES with a sharing distribution mechanism that allows a priori knowledge of the LL optimisation from the UL optimisation procedure. The algorithms were selected for the following reasons:

- The performance of these algorithms on numerous bilevel test functions is available and one can compare their performance also for the min-max test functions.
- The original code for BLEAQ2 and BLCMAES can be found online, which is valuable for the reproducibility of the experiments. BLDE code was implemented by the authors since the original is not available.
- The approach of the 2 mappings and CMA-ES with sharing distribution, has -to the best of our knowledge- never been implemented in min-max algorithms and can be beneficial in reducing the cost of a nested approach.

The evolutionary methods in this paper assumed an optimistic approach. This, nonetheless, is not affecting the min-max optimisation problems. Min-max problems are sharing the same function and are pessimistic by default. That means that even in the case of multiple LL global optima, whichever is taken into account is a UL worst-case scenario.

We tested 13 unconstrained min-max test problems as collected in [3]. The first $f_1 - f_7$ test functions are convex in the upper level and concave in the lower. The dimensions range from 1 to 5 both for the upper and lower levels. All functions are symmetrical except for the functions $f_9 - f_{11}$, which are asymmetrical.

Please note this is not meant to be a benchmarking assessment among them, but a preliminary idea on how they perform. All algorithms are independently run 31 times on each test function. For the test function f_{10} and algorithm BLEAQ2 one run is reported, as an error was stopping the runs. Further investigation on why is this happening is ongoing.

3 RESULTS

Table 1 reports the numerical results obtained from the 3 algorithms in terms of median accuracy. Accuracy is $|f - f^*|$, where f is the optimal function value and f^* is the one obtained during a run. In the same table, the median number of total function evaluations needed is reported. In most of the test instances, the algorithms manage to converge to the optimal min-max solution. What is interesting, is that for f_{10}, except for the non-convergence/ error of BLEAQ2, BLCMAES also performs poorly. BLDE manages to converge to near-optimal solutions wasting all the computational budget it had. f_{10} is a function with many local optima. BLCMAES seems to be trapped to local optima of the upper level, agreeing with the observation made in [2], where it states that the efficiency of the algorithm is low on problems with many local optima in the UL, as CMAES, in general, prefers exploitation rather than exploration. BLEAQ2' s one run yields to a solution far from the real optimal as well.

[1]Results for BLEAQ2 after only 1 run for $f10$

Problems		BLEAQ2	BL-CMAES	BL-DE
f_1	Accuracy	3.33e-05	3.33e-05	3.33e-05
	TotalFEs	4767	51884	1552440
f_2	Accuracy	1.69e-05	1.69e-05	1.69e-05
	TotalFEs	5998	73098	1548570
f_3	Accuracy	3.31e-05	2.46e-05	1.67e-07
	TotalFEs	7123	71865	1147080
f_4	Accuracy	3.39e-05	3.39e-05	3.39e-05
	TotalFEs	8644	69624	1553370
f_5	Accuracy	2.99e-04	2.99e-04	6.35e-07
	TotalFEs	8823	76725	1468560
f_6	Accuracy	3.02e-05	3.03e-05	3.59e-04
	TotalFEs	11978	92829	1561620
f_7	Accuracy	7.90e-2	7.90e-2	9.36e-2
	TotalFEs	35526	182646	1561620
f_8	Accuracy	5.09e-16	1.51e-07	4.55e-07
	TotalFEs	2849	9066	366180
f_9	Accuracy	5.47e-3	6.40e-07	0
	TotalFEs	42435	12605	87450
f_{10}	Accuracy	[2.25e-2][1]	1.89e-1	3.027e-7
	TotalFEs	[129803]	50639	1561620
f_{11}	Accuracy	7.6e-3	5.34e-4	4.00e-3
	TotalFEs	121803	29828	1561620
f_{12}	Accuracy	5.55e-15	7.64e-07	2.08e-07
	TotalFEs	5525	45749	733500
f_{13}	Accuracy	6.45e-3	6.78e-07	4.09e-04
	TotalFEs	11249	142950	1561620

Table 1: Median Accuracy and Function Evaluations Results

4 CONCLUSIONS

In this paper, we presented a preliminary study on solving min-max problems with evolutionary bilevel algorithms. The fundamental idea was that since min-max is a special instance of bilevel problems, the evolutionary bilevel algorithms should be able to perform well also in this kind of problem. To the best of our knowledge, this is the first time bilevel evolutionary algorithms are tested on min-max test functions. It was shown that in most of the test instances the algorithms manage to converge to the optimal min-max solution. Further investigating and interpreting the current results of this study is the next step of our research. Future work includes testing the bilevel algorithms to more complex min-max functions, such as with higher dimensionality and constraints. Since some knowledge about the problem is available for min-max problems, modifying the bilevel algorithms to perform more efficiently on this kind of problem is also an important aspect to be further investigated.

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